

# Jet quenching in evolving anisotropic matter

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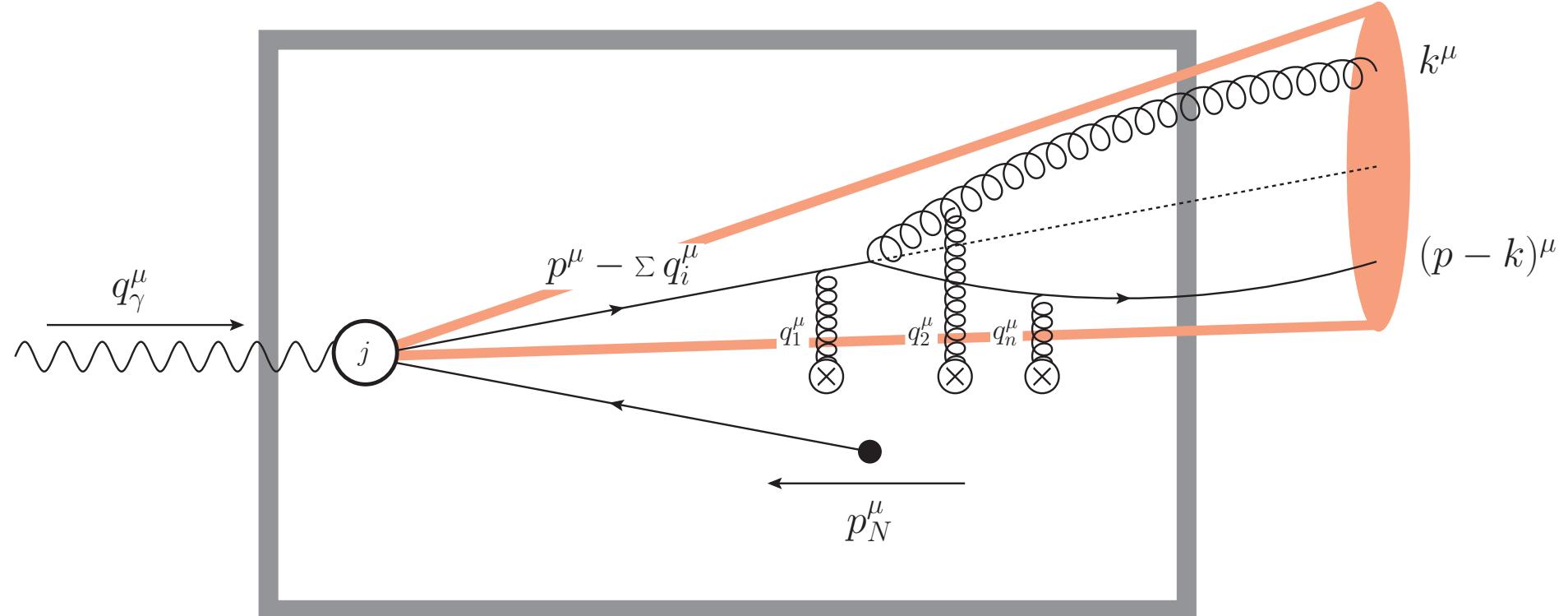
Exploring Quark-Gluon Plasma through soft and hard probes,  
29-31 May 2023, Belgrade, Serbia



## Jet tomography

- Jets see the matter in HIC at multiple scales, and essentially X-ray it;
- The existing jet quenching theory is based on multiple simplifying assumptions: large parton energy, static matter, no fluctuations, etc;
- There is a very recent progress on the medium motion and structure effects in jet quenching and this is the focus of this talk;
- The developed formalism can be also applied to include orbital motion of nucleons and some of the in-medium fluctuations to the energy loss in cold nuclear matter;



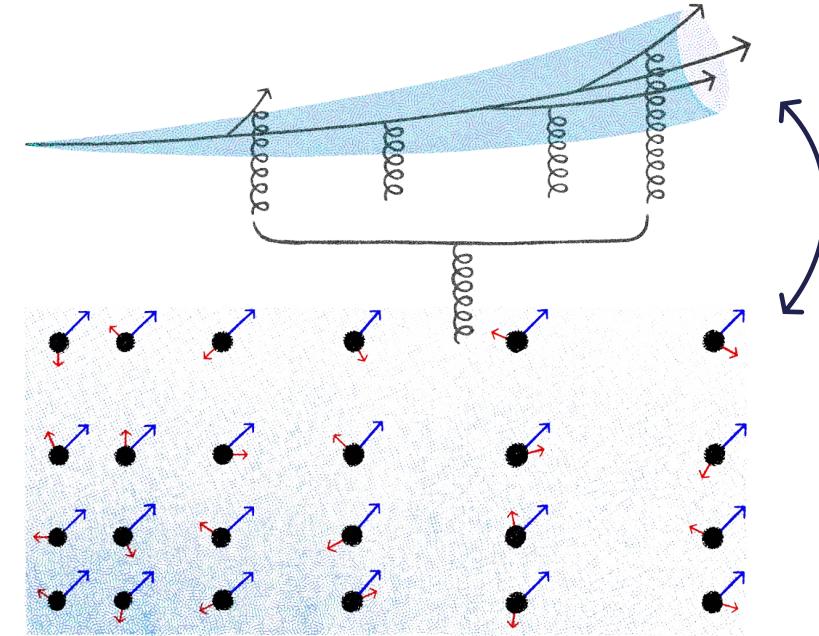
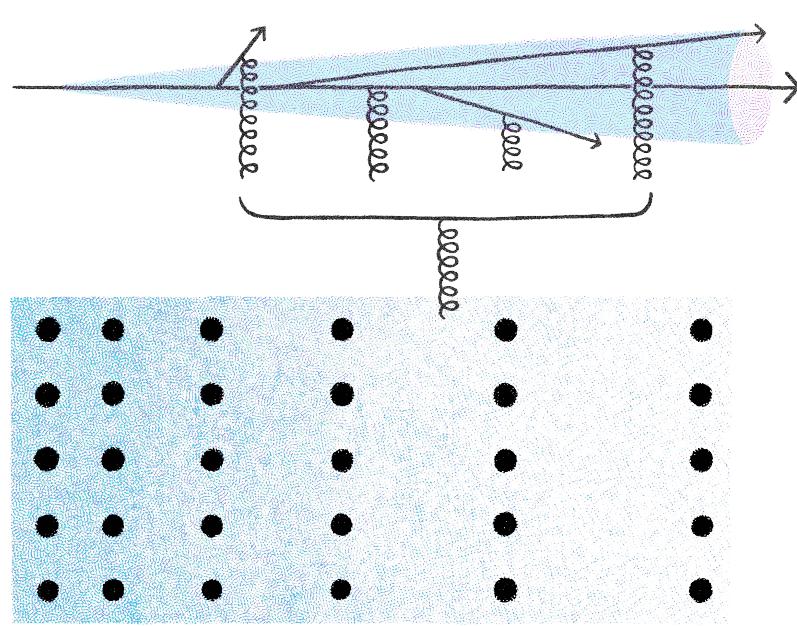




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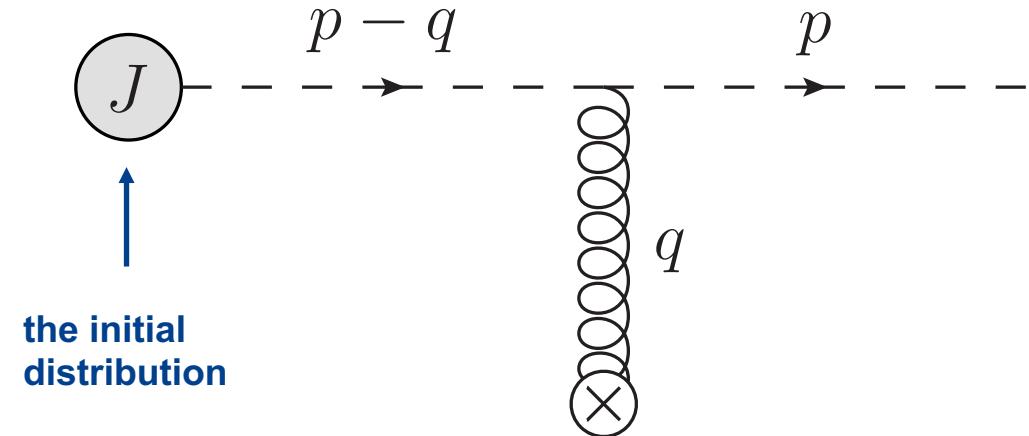
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# Color potential

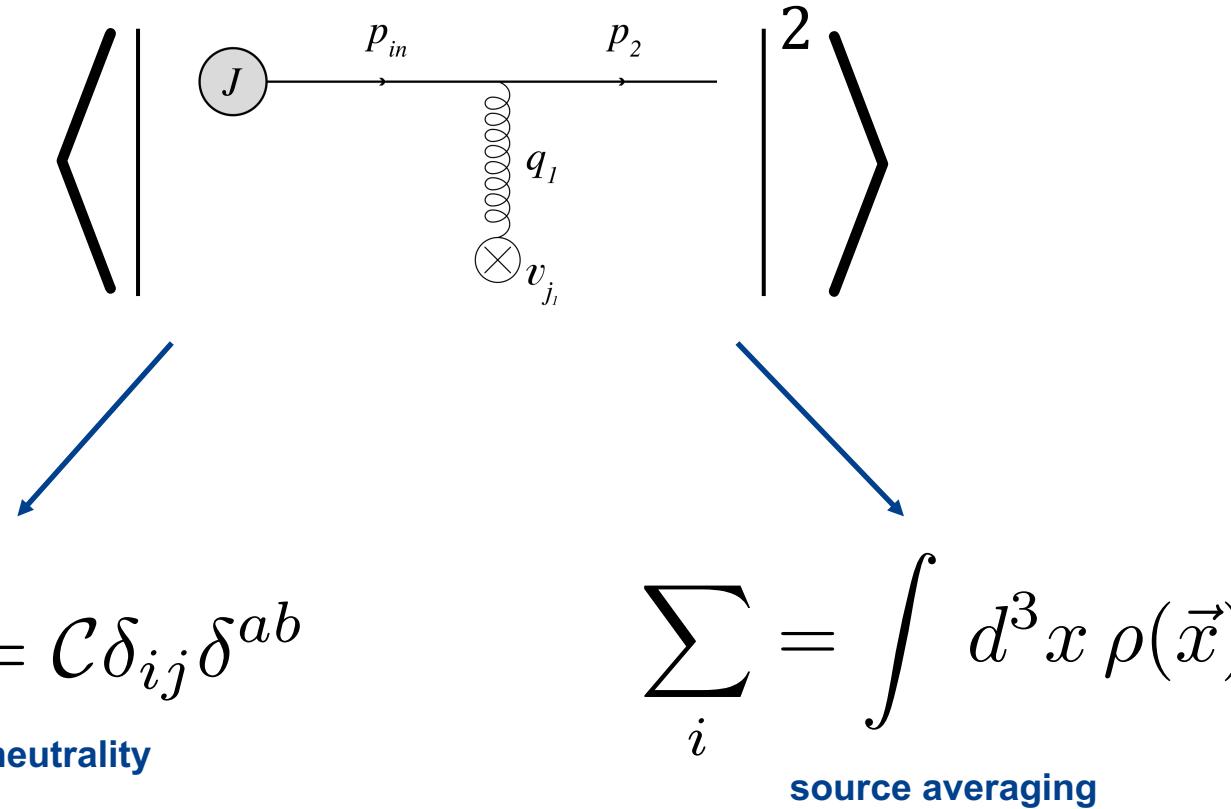


$$gA_{ext}^{\mu a}(q) = \sum_i e^{iq \cdot x_i} t_i^a u_i^\mu v_i(q) (2\pi) \delta(q^0 - \vec{u}_i \cdot \vec{q})$$

i inhomogeneity the fluid velocity

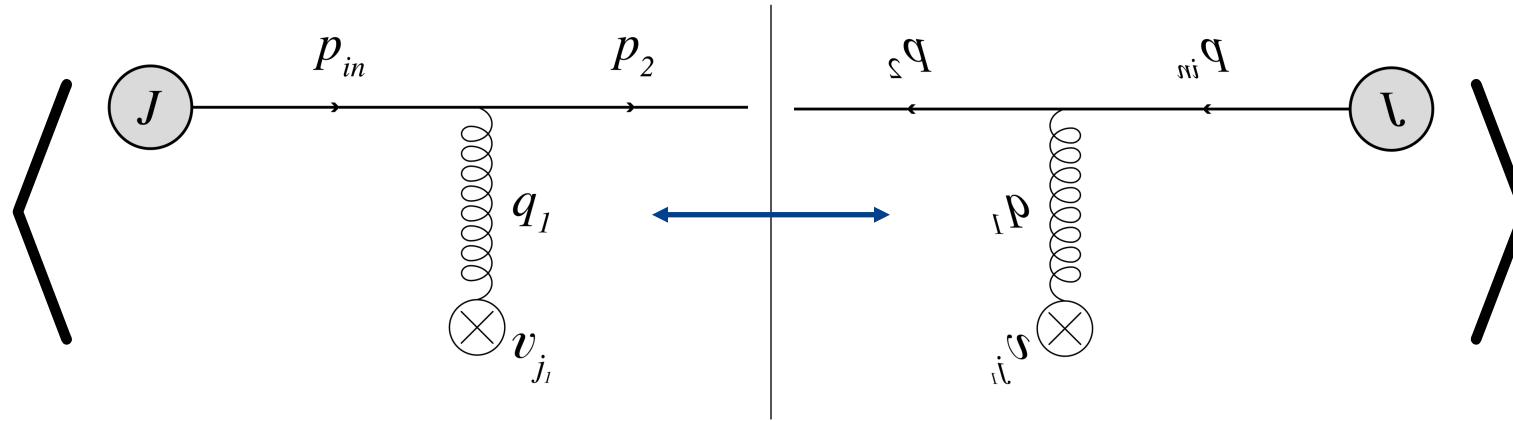


# Medium averaging





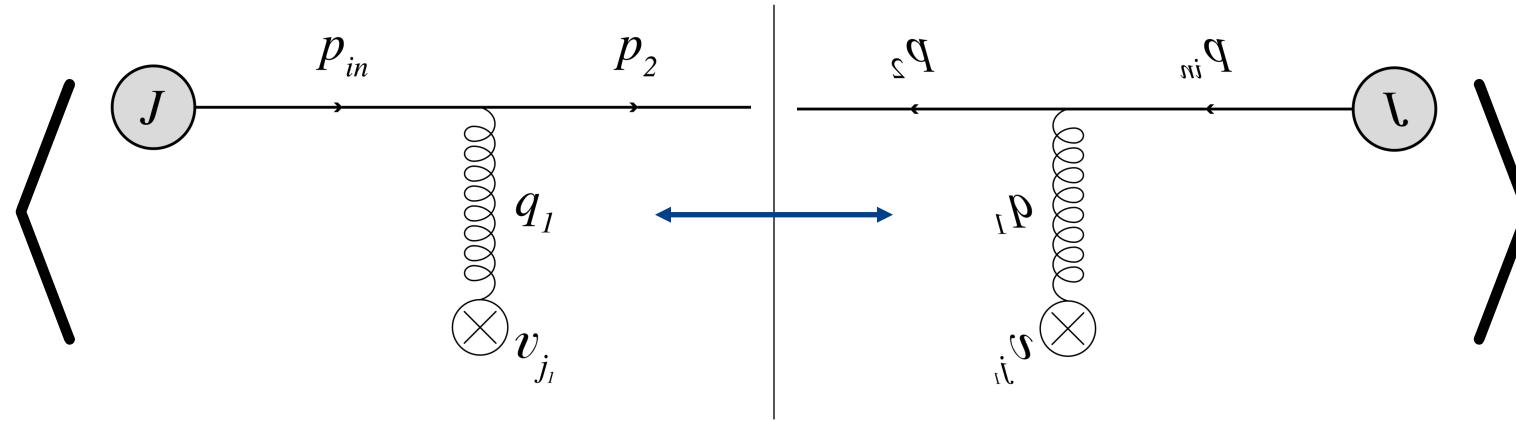
## Medium averaging



$$\int d^2 \mathbf{x}_n e^{-i(\mathbf{q}_n \pm \bar{\mathbf{q}}_n) \cdot \mathbf{x}_n} = (2\pi)^2 \delta^{(2)}(\mathbf{q}_n \pm \bar{\mathbf{q}}_n)$$

$\rho \sim T^3$

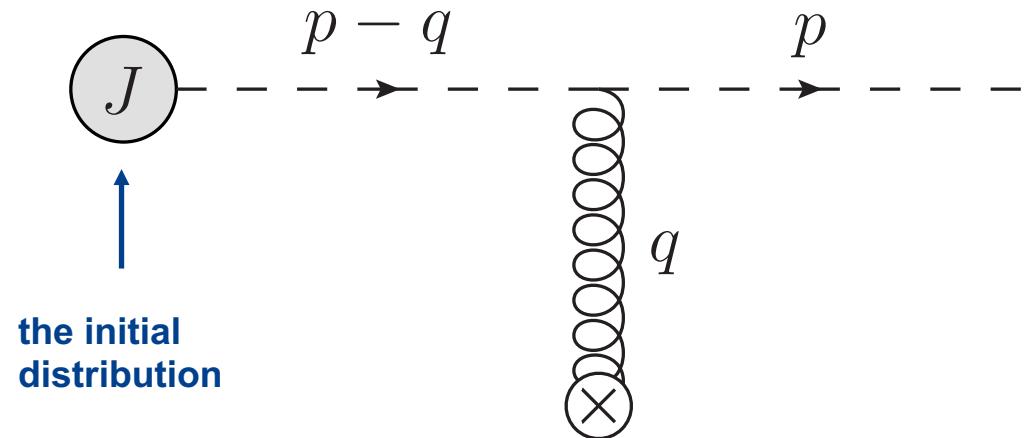
## Medium averaging



$$\int d^2 \mathbf{x}_n e^{-i(\mathbf{q}_n \pm \bar{\mathbf{q}}_n) \cdot \mathbf{x}_n} = (2\pi)^2 \delta^{(2)}(\mathbf{q}_n \pm \bar{\mathbf{q}}_n)$$

$$\int d^2 \mathbf{x}_n x_n^\alpha e^{-i(\mathbf{q}_n \pm \bar{\mathbf{q}}_n) \cdot \mathbf{x}_n} = i (2\pi)^2 \frac{\partial}{\partial(q_n \pm \bar{q}_n)_\alpha} \delta^{(2)}(\mathbf{q}_n \pm \bar{\mathbf{q}}_n)$$

## Color potential

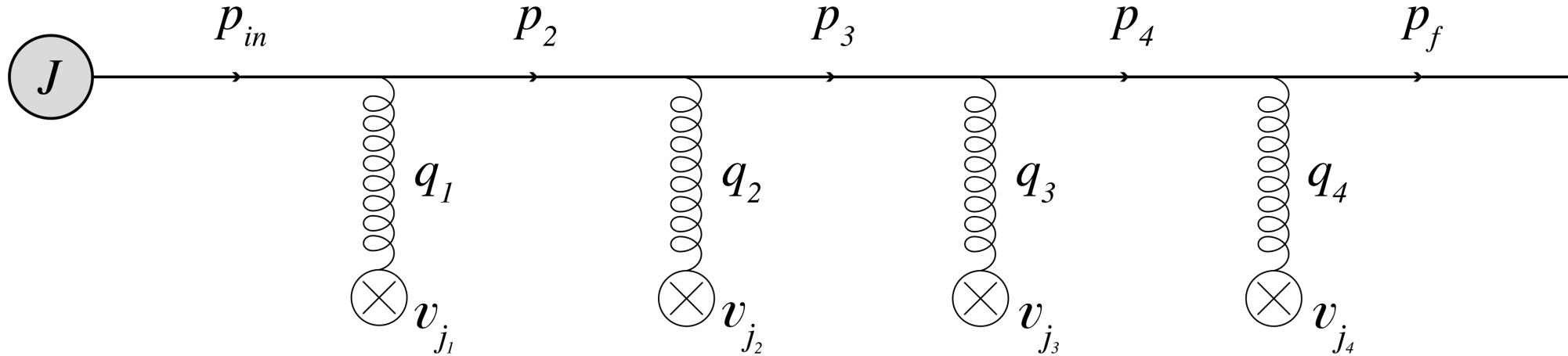


$$g A_{ext}^{\mu a}(q) = \sum_i e^{iq \cdot x_i} t_i^a u_i^\mu v_i(q) (2\pi) \delta(q^0 - \vec{w}_i \cdot \vec{q})$$

i      (1,0,0,0)  
**inhomogeneity**



## The broadening

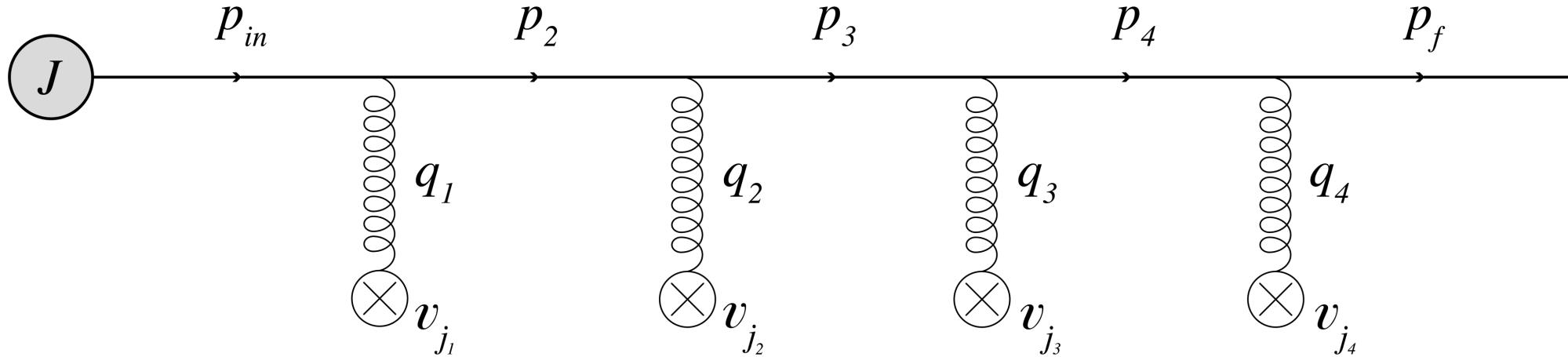


$$\begin{array}{c} E \rightarrow \infty \\ v(q) \\ \mu \ll E \quad \mu z \gg 1 \end{array} \longrightarrow iM(p) = \int \frac{d^2 \mathbf{p}_{in}}{(2\pi)^2} e^{i \frac{\mathbf{p}_f^2}{2E} L} G(\mathbf{p}_f, L; \mathbf{p}_{in}, 0) J(E, \mathbf{p}_{in})$$

↑  
**single particle propagator**



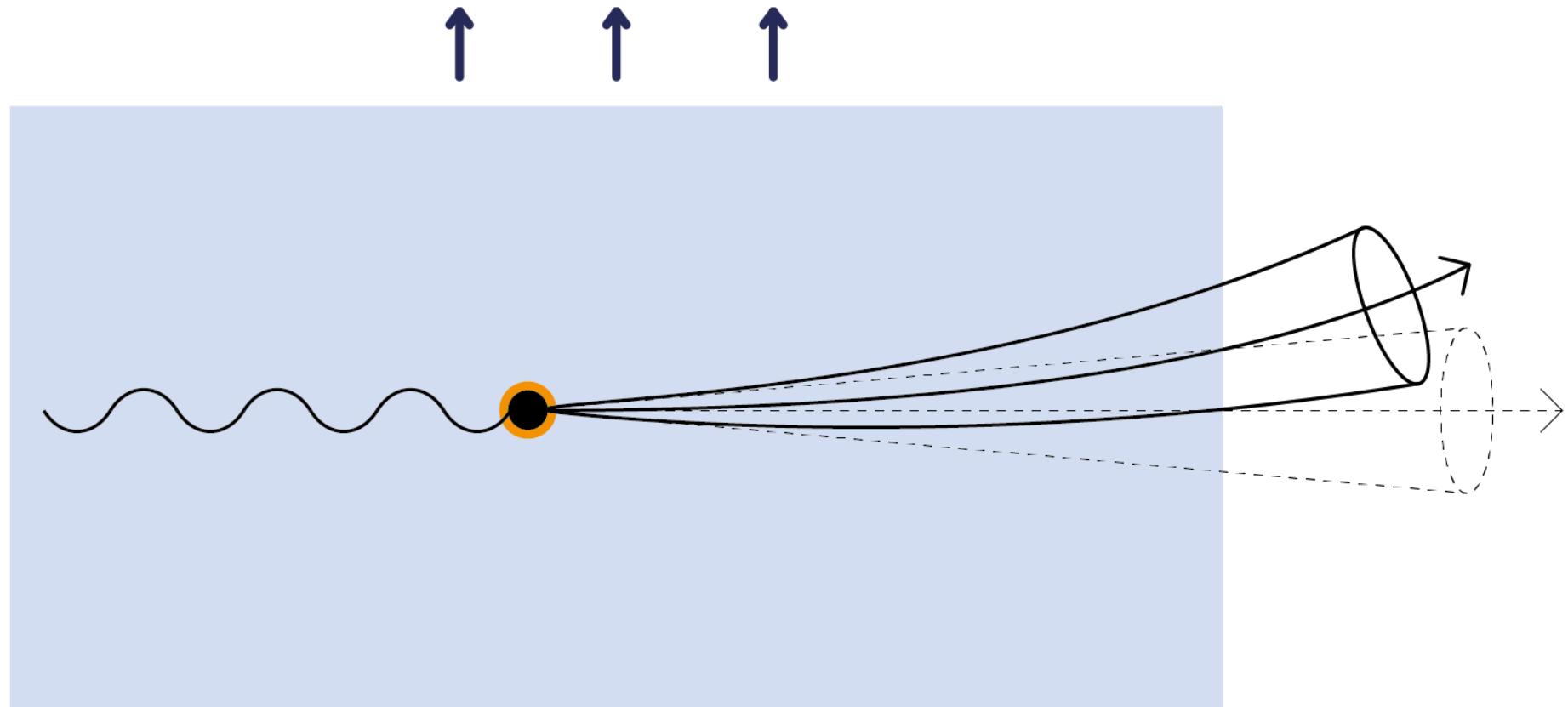
## The broadening



$$G(\mathbf{x}_L, L; \mathbf{x}_0, 0) = \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp \left( \frac{iE}{2} \int_0^L d\tau \dot{\mathbf{r}}^2 \right) \mathcal{P} \exp \left( -i \int_0^L d\tau t_{\text{proj}}^a v^a(\mathbf{r}(\tau), \tau) \right)$$

$$\begin{aligned}
 & \left\langle \mathcal{P} \exp \left( -i \int_0^L d\tau t_{\text{proj}}^a v^a(\mathbf{r}(\tau), \tau) \right) \mathcal{P} \exp \left( i \int_0^L d\bar{\tau} t_{\text{proj}}^b v^b(\bar{\mathbf{r}}(\bar{\tau}), \bar{\tau}) \right) \right\rangle \\
 &= \exp \left\{ - \int_0^L d\tau \left[ 1 + \frac{\mathbf{r}(\tau) + \bar{\mathbf{r}}(\tau)}{2} \cdot \hat{\mathbf{g}} \right] \mathcal{V}(\mathbf{r}(\tau) - \bar{\mathbf{r}}(\tau)) \right\} \\
 &\quad \downarrow \\
 & \frac{\mathcal{N}}{d^2 \mathbf{x} dE} = \mathcal{P}(\mathbf{x}) \hat{\mathcal{S}}(\mathbf{x}) \frac{\mathcal{N}^{(0)}}{d^2 \mathbf{x} dE} \\
 &\quad \swarrow \qquad \nwarrow \\
 & \frac{\mathcal{N}}{d^2 \mathbf{p} dE} \qquad \qquad \frac{\mathcal{N}^{(0)}}{d^2 \mathbf{p} dE}
 \end{aligned}$$

The diagram illustrates the derivation of the expression for the normalized distribution function  $\mathcal{N}/(d^2 \mathbf{x} dE)$ . It starts with a complex path integral expression involving position  $\mathbf{r}$  and momentum  $\bar{\mathbf{r}}$  fields, separated by a projection operator  $\mathcal{P}$ . This is simplified using a Gaussian ansatz for the interaction term, leading to an exponential form. The exponential is then expanded into a sum of terms, one of which is the final result  $\mathcal{N}/(d^2 \mathbf{x} dE)$ . The other term,  $\mathcal{N}^{(0)}/(d^2 \mathbf{x} dE)$ , is shown separately. Arrows indicate the flow from the initial expression through the Gaussian approximation and expansion to the final result.



$$\langle \mathbf{p} p_{\perp}^2 \rangle \simeq \chi^2 \frac{L \nabla T}{2T} \frac{\mu^4}{E} \left( \log \frac{E}{\mu} \right)^2$$

a limit of AMY KT  
(JHEP 2002/2003)

$$W_L(\mathbf{Y}, \mathbf{p}) \equiv \int_{\mathbf{y}, \mathbf{x}} e^{-i\mathbf{p} \cdot \mathbf{y}} \left\langle G\left(\mathbf{Y} + \frac{\mathbf{y}}{2}; \frac{\mathbf{x}}{2}\right) G^\dagger\left(\mathbf{Y} - \frac{\mathbf{y}}{2}; -\frac{\mathbf{x}}{2}\right) \right\rangle$$



$$\longrightarrow \left( \partial_L + \frac{\mathbf{p}}{E} \cdot \nabla_{\mathbf{Y}} - \frac{\hat{q}}{4} \partial_{\mathbf{p}}^2 \right) W_L(\mathbf{Y}, \mathbf{p}) = 0$$

in the harmonic  
approximation

$$W_L(\mathbf{Y}, \mathbf{p}) \equiv \int_{\mathbf{y}, \mathbf{x}} e^{-i\mathbf{p} \cdot \mathbf{y}} \left\langle G\left(\mathbf{Y} + \frac{\mathbf{y}}{2}; \frac{\mathbf{x}}{2}\right) G^\dagger\left(\mathbf{Y} - \frac{\mathbf{y}}{2}; -\frac{\mathbf{x}}{2}\right) \right\rangle$$



$$\left( \partial_L + \frac{\mathbf{p}}{E} \cdot \nabla_{\mathbf{Y}} - \frac{\hat{q}(\mathbf{Y})}{4} \partial_{\mathbf{p}}^2 \right) W_L(\mathbf{Y}, \mathbf{p}) = \nabla_i \nabla_j \rho$$

$$\times \int_{\mathbf{q}} \left[ \kappa \frac{\partial^2}{\partial p_i \partial p_j} \delta^{(2)}(\mathbf{q}) - V_{ij}(\mathbf{q}) \right] W_L(\mathbf{Y}, \mathbf{p} - \mathbf{q})$$

cannot be obtained from  
local collision kernel



quantum corrections to the  
Boltzmann-diffusion equation

$$W_L(\mathbf{Y}, \mathbf{p}) \equiv \int_{\mathbf{y}, \mathbf{x}} e^{-i\mathbf{p} \cdot \mathbf{y}} \left\langle G\left(\mathbf{Y} + \frac{\mathbf{y}}{2}; \frac{\mathbf{x}}{2}\right) G^\dagger\left(\mathbf{Y} - \frac{\mathbf{y}}{2}; -\frac{\mathbf{x}}{2}\right) \right\rangle$$



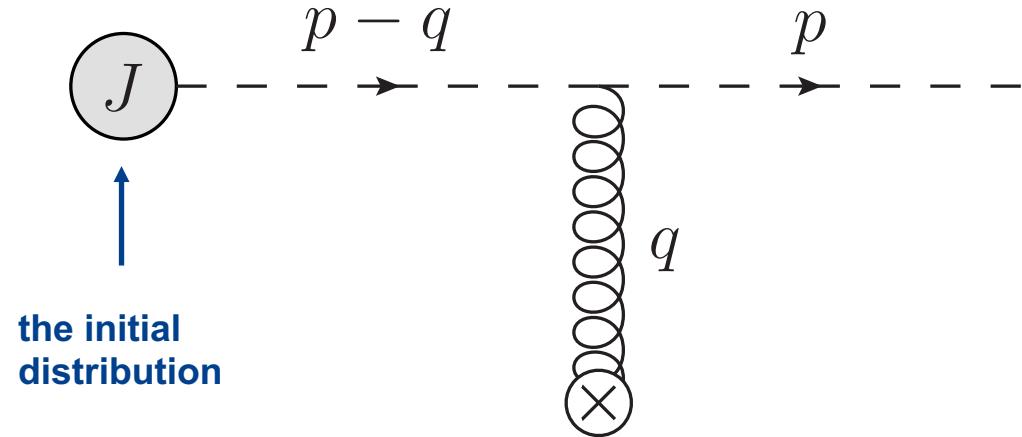
$$\left( \partial_L + \frac{\mathbf{p}}{E} \cdot \nabla_{\mathbf{Y}} - \frac{\hat{q}(\mathbf{Y})}{4} \partial_{\mathbf{p}}^2 \right) W_L(\mathbf{Y}, \mathbf{p}) = \boxed{\nabla_i \nabla_j \rho}$$

$$\times \int_{\mathbf{q}} \left[ \kappa \frac{\partial^2}{\partial p_i \partial p_j} \delta^{(2)}(\mathbf{q}) - V_{ij}(\mathbf{q}) \right] W_L(\mathbf{Y}, \mathbf{p} - \mathbf{q})$$



$$\hat{q}_r = \hat{q} + \nabla^2 \hat{q} \left( \frac{\hat{q} L^3}{12 E^2} + \boxed{\eta} \right)$$

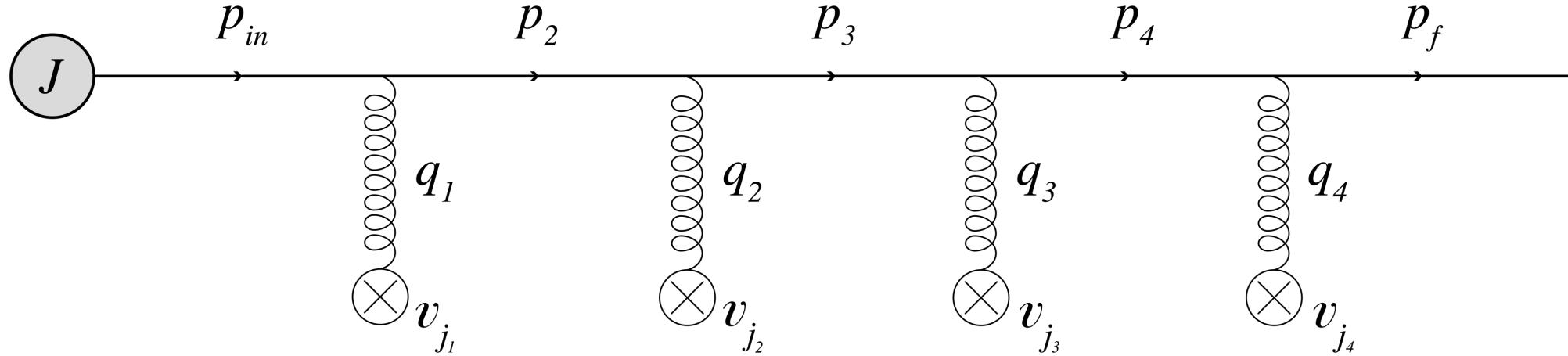
# Color potential



$$g A_{ext}^{\mu a}(q) = \sum_i e^{iq \cdot x_i} t_i^a u_i^\mu v_i(q) (2\pi) \delta(q^0 - \vec{u}_i \cdot \vec{q})$$

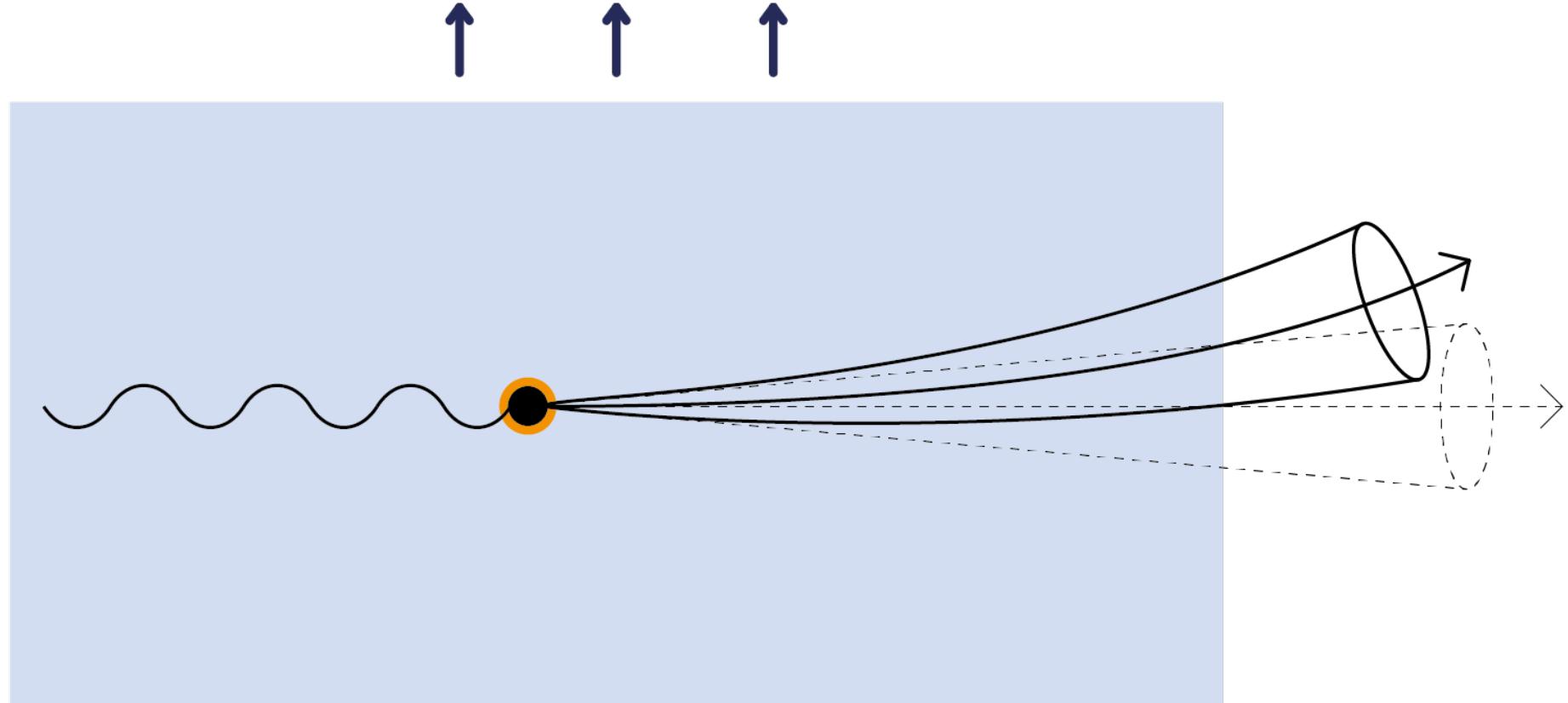


# Jet broadening



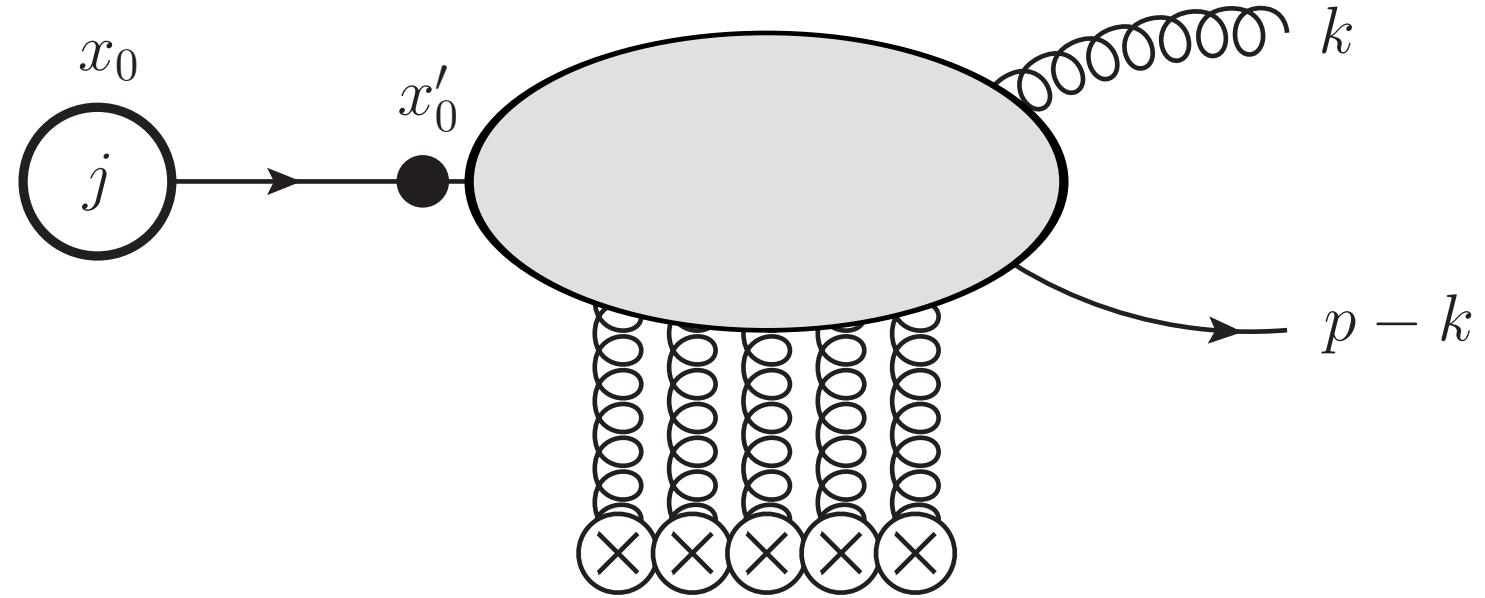
$$i\mathcal{M}_n(p) = e^{-i\left(\mathbf{u}\cdot\mathbf{p} - \frac{p_\perp^2}{2E}\right)L} \int \frac{d^2\mathbf{p}_0}{(2\pi)^2} \mathcal{G}_n(\mathbf{p}, L; \mathbf{p}_0, 0) J(E - \mathbf{u} \cdot (\mathbf{p} - \mathbf{p}_0), \mathbf{p}_0),$$





$$\langle \mathbf{p} \rangle \simeq 3 \chi \mathbf{u} \frac{\mu^2}{E} \log \frac{E}{\mu}$$

# Gluon emission



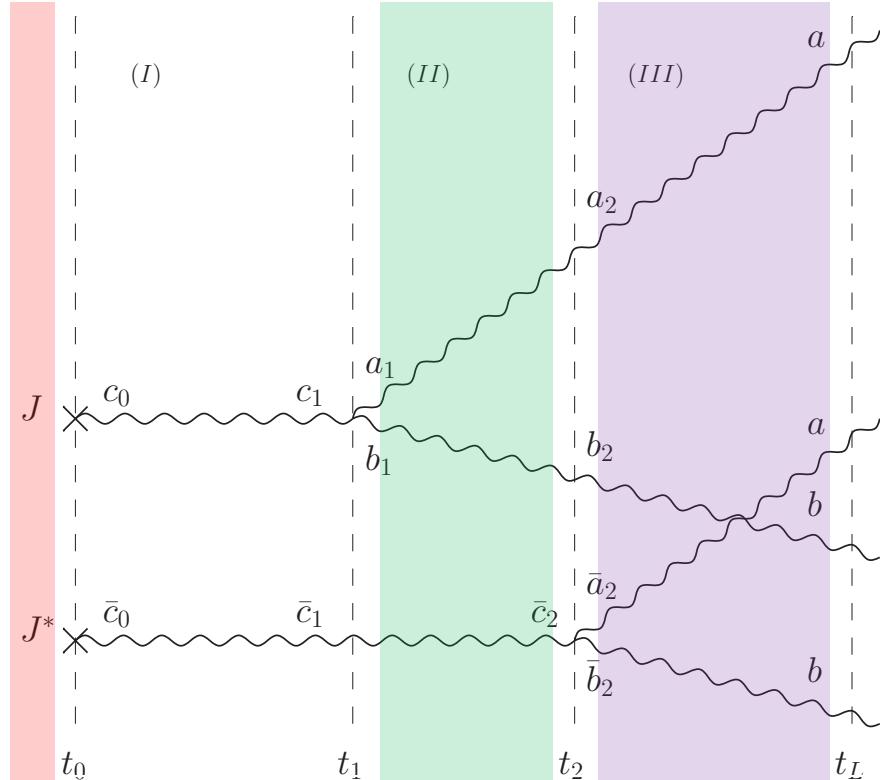
# Gluon emission

$$iR \simeq -\frac{g}{xE} \lim_{z_f \rightarrow \infty} \int_0^\infty dz_s \int d^2 x_0 e^{-i\mathbf{x}_0 \cdot \mathbf{l}_f} J(\mathbf{x}_0) \\ \times \mathcal{W}(\mathbf{x}_0; \infty, z_s) t_{proj}^a \mathcal{W}(\mathbf{x}_0; z_s, 0) e^{i \frac{\mathbf{k}_f^2}{2\omega} z_f} \left[ \epsilon \cdot \vec{\nabla}_{\mathbf{x}_0} \mathcal{G}^{ba} (\mathbf{k}_f, z_f; \mathbf{x}_0, z_s) \right]$$

↑ ↑  
**Wilson line ( $x \ll 1$ )**                            **the gluon (single-particle) propagator**  
↑ ↑  
**energy fraction**



## Gluon emission



$$(2\pi)^2 \omega \frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c \omega^2} \text{Re} \int_0^L d\bar{z} \int_0^{\bar{z}} dz \int_{\mathbf{x}_0, \mathbf{y}} |J(\mathbf{x}_0)|^2 \left[ (\nabla_{\mathbf{x}} \cdot \nabla_{\bar{\mathbf{x}}}) \right. \\ \times \left. \left\langle \mathcal{G}^{bc}(\mathbf{k}, L; \mathbf{y}, \bar{z}) \mathcal{G}^{\dagger \bar{a}b}(\mathbf{k}, L; \bar{\mathbf{x}}, \bar{z}) \right\rangle \left\langle \mathcal{G}^{ca}(\mathbf{y}, \bar{z}; \mathbf{x}, z) \mathcal{W}_A^{\dagger a\bar{a}}(\mathbf{x}_0; \bar{z}, z) \right\rangle \right] \Big|_{\mathbf{x}=\bar{\mathbf{x}}=\mathbf{x}_0}$$

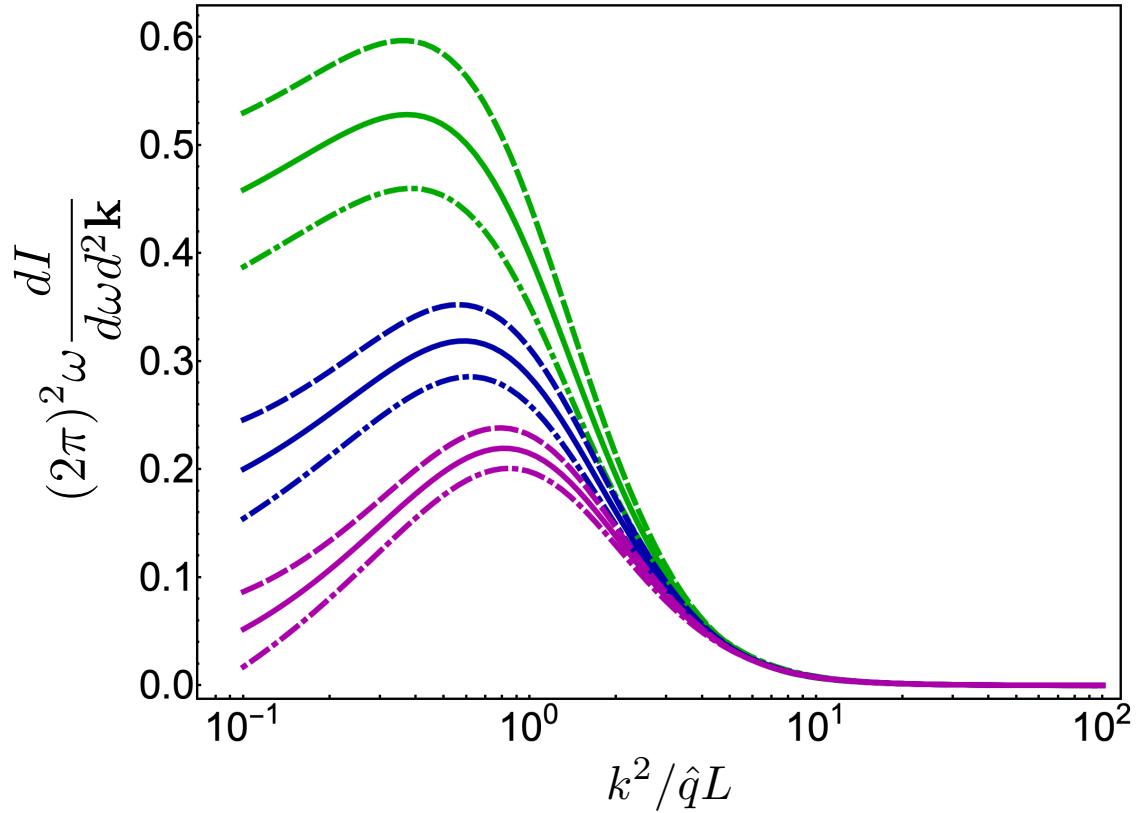
broadening of the gluon  
(already have it with  $\nabla$ )

A blue arrow pointing upwards, indicating a positive value or direction.

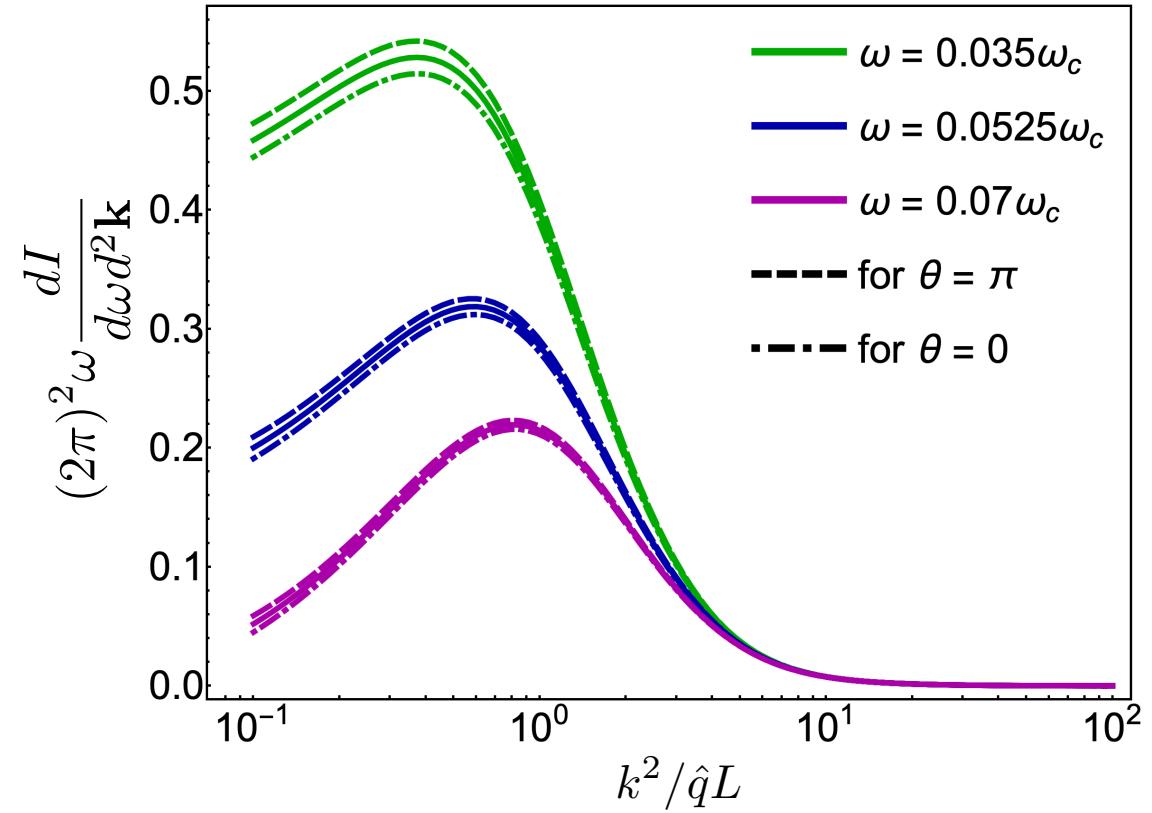
$$\delta\mathcal{K}(\mathbf{y}, \bar{z}; \mathbf{x}, z) = - \int_{\bar{z}}^{\bar{z}} ds \int_{\mathbf{w}} \mathcal{K}_0(\mathbf{y}, \bar{z}; \mathbf{w}, s) \delta\mathcal{V}(\mathbf{w}, s) \mathcal{K}_0(\mathbf{w}, s; \mathbf{x}, z)$$



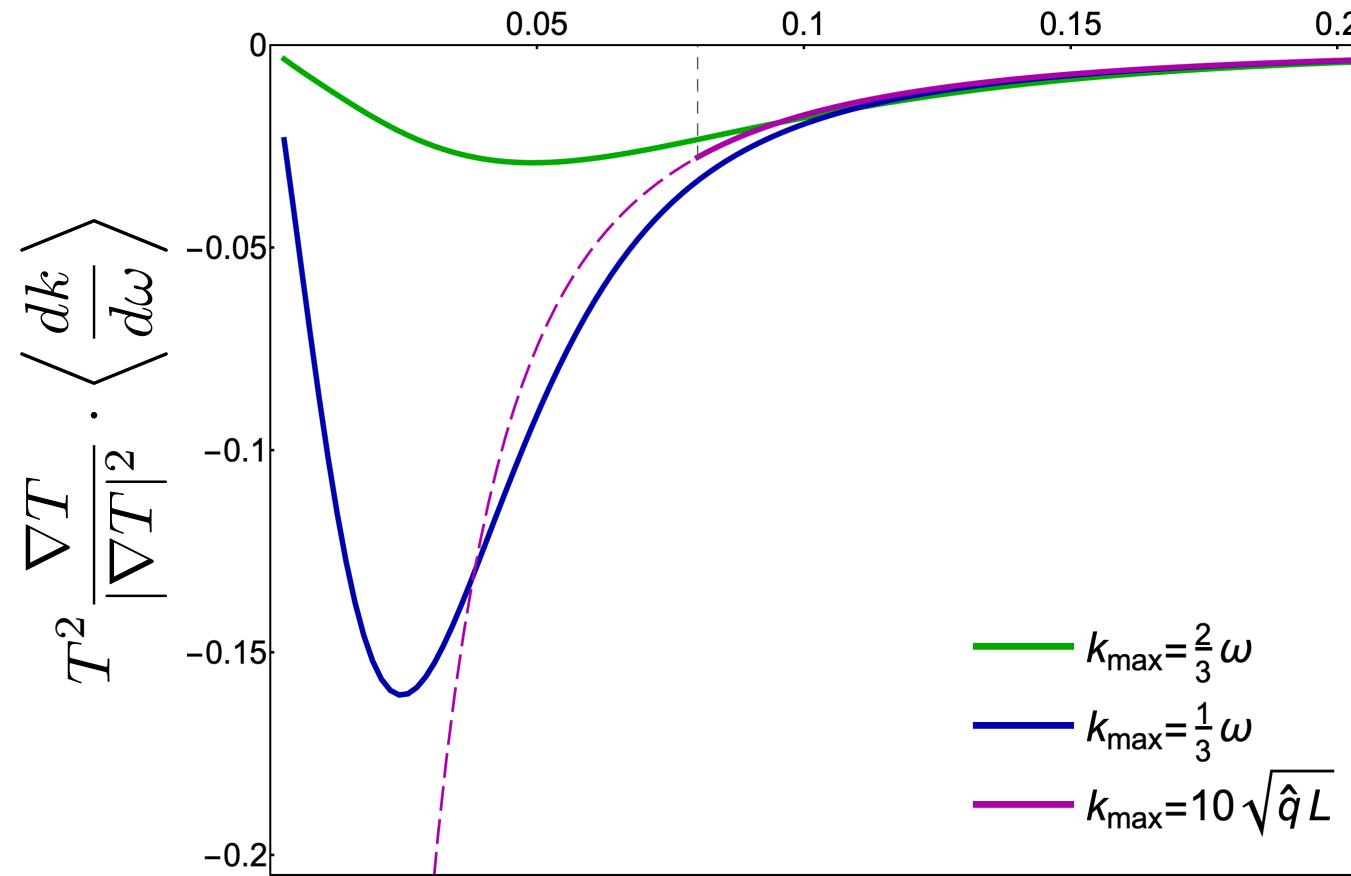
$$\left| \frac{\nabla T}{T^2} \right| = 0.05$$



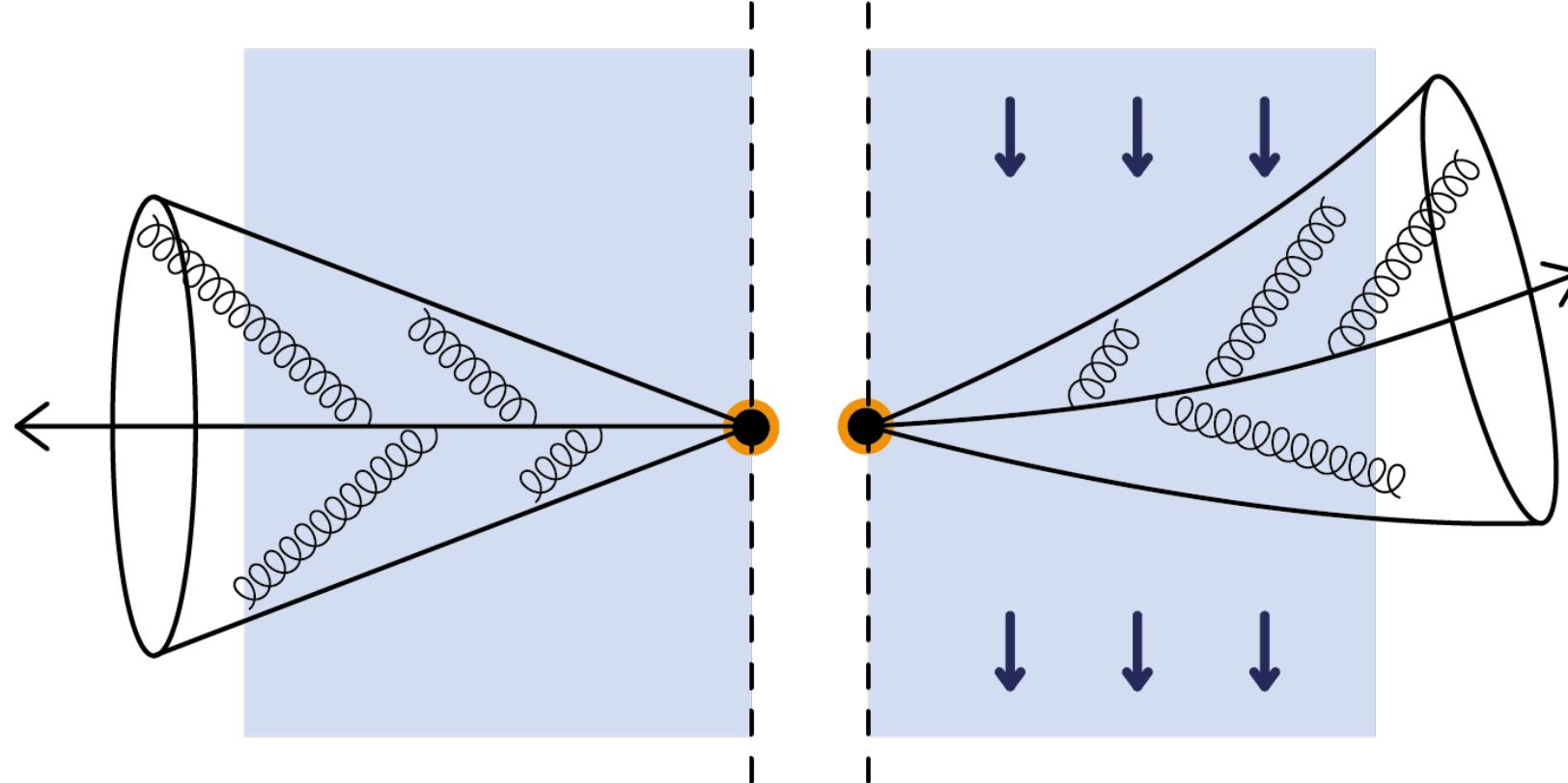
$$\left| \frac{\nabla T}{T^2} \right| = 0.01$$



$$L = 5\text{fm}, \quad T = 0.3\text{GeV}, \quad \hat{q} = 1\text{GeV}^2 \cdot \text{fm}^{-1}$$

 $\omega/\omega_c$ 

$$\left\langle \frac{d\mathbf{k}}{d\omega} \right\rangle \equiv \int_{\Gamma} d^2\mathbf{k} \mathbf{k} \frac{dI}{d\omega d^2\mathbf{k}}$$



## Summary

- Jets do feel the transverse flow and anisotropy, and get bended and distorted;
- The transverse flow and anisotropy do affect the medium-induced radiation, bending the substructure of jets;
- These effects can be in principle probed in experiment, leading us towards actual jet tomography;
- One should also expect similar evolution-induced effects for the other probes of nuclear matter;



# Jet broadening

uniform matter

- Opacity  $\chi \approx 4$
- $u \approx 0.7$  (about  $\pi/4$  to z-axis)
- $\mu = gT$  with  $g \approx 2$  and  $T \approx 500 \text{ MeV}$

$$\left\langle \frac{p_\perp}{E} \right\rangle \simeq 3 \chi \frac{u_\perp}{1 - u_z} \frac{\mu^2}{E^2} \log \frac{E}{\mu}$$

What jet energy corresponds to  $\langle \theta \rangle \approx 1^\circ$ ?



$E \sim 50 \text{ GeV}$

# Jet broadening

inhomogeneous matter

- Opacity  $\chi \approx 4$
- $\mu = gT$  with  $g \approx 2$  and  $T \approx 500 \text{ MeV}$
- $L\nabla T > T$

$$\left\langle \frac{\mathbf{p}}{E} \frac{p_\perp^2}{\mu^2} \right\rangle \simeq \chi^2 \frac{L\nabla T}{2T} \frac{\mu^2}{E^2} \left( \log \frac{E}{\mu} \right)^2$$

What jet energy corresponds to  $\langle |\theta| \rangle \approx 1^\circ$ ?



Jet energies:  $E < 100 \text{ GeV}$

# Energy-energy correlators

$$\begin{aligned} \frac{d\Sigma}{d\theta d\alpha} &= \int_0^1 dx \frac{d\sigma}{\sigma dz d\theta d\alpha} x(1-x) \\ &= \int_0^1 dz \left( \frac{\alpha_s C_F}{\pi^2} \frac{1}{x\theta} + \omega \frac{dI}{d\omega d^2\mathbf{k}} p_t^{\text{jet}} |\mathbf{k}| \right) x(1-x) \end{aligned}$$

- EEC for jets, see K. Lee et al, 2022
- EEC in HIC, see Carlota's slides
- The small-x BDMPS-Z formula is not strictly applicable, just an illustration
- The gradient effects can be clearly seen

