Recent developments in the theoretical modelling of quarkonia production

Pol B Gossiaux, SUBATECH (NANTES)

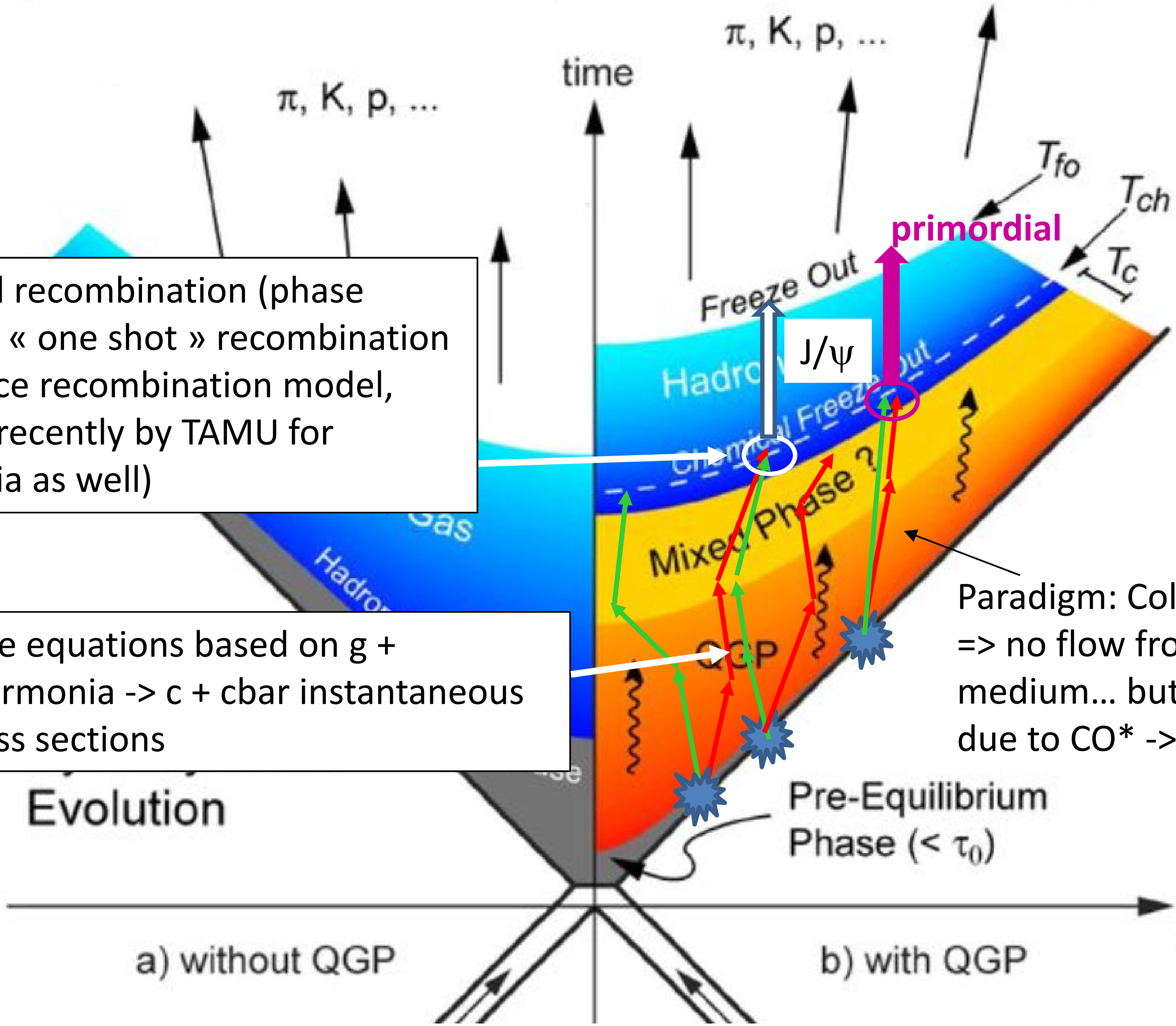
Explore QGP Workshop
29-31 May 2023
Belgrade, Serbia

With Th Gousset, Stéphane Delorme & J-P Blaizot

1. Introduction
2. The QME formalism
3. Some results

and Pays de la Loire

Quarkonia as a possible probe of the QGP



Statistical recombination (phase space) or « one shot » recombination (resonance recombination model, adopted recently by TAMU for charmonia as well)

Rate equations based on g + charmonia -> c + cbar instantaneous cross sections

Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

The best working horse today: Rate equations

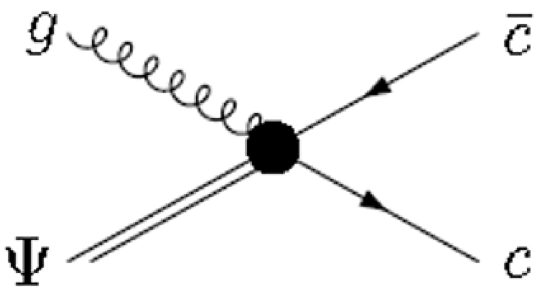
$$\frac{dN(t)}{dt} = -\Gamma(T(t))\,(N(t) - N^{\text{eq}}(T(t)))$$

Loss Gain

Statistical limit (canonical) assumed

For instance, Bhanot-Peskin gluo-dissociation

$$\sigma_{J/\psi}(\omega) = A_0 \frac{(\omega/\epsilon_{J/\psi} - 1)^{3/2}}{(\omega/\epsilon_{J/\psi})^5}$$



ω: gluon energy in the quarkonium rest frame

$$\Gamma_\Psi(T) \sim \int d^3k_g \sigma_\Psi(\omega) f_{BE}(T,\omega)$$ Dissociation rate

$$N_{\text{final}} = N_0 \times \underbrace{e^{-\int_{t_0}^{+\infty} \Gamma_\Psi(T(t))dt}}_{R_{AA}}$$ If just suppression

Various states are still decoupled in their evolution… while in principle, one could have some gluon-induced “conversion” (not implemented in any model to my knowledge)

Interactive talk…

$$\frac{dN(t)}{dt} \dot{=} -\Gamma(T(t))\,(N(t) - N^{\mathrm{eq}}(T(t)))$$

Loss — Gain

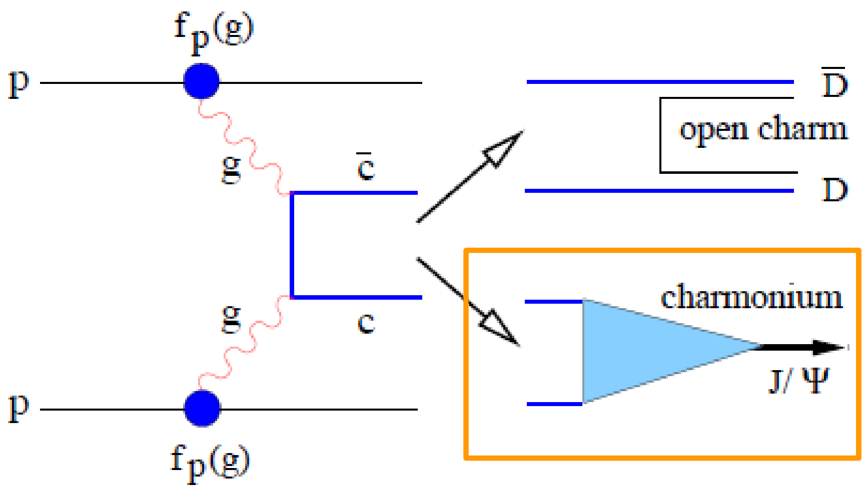
Statistical limit (canonical) assumed

Given any kind of transport theory, does the asymptotic limit when gain and loss equilibrate always correspond to the usual Boltzmann distribution ?

A. Yes of course !
B. Not necessarily
C. Dunno
D. Who cares, I hope we'll keep on time for the coffee break

Decoupled production of various HF mesons

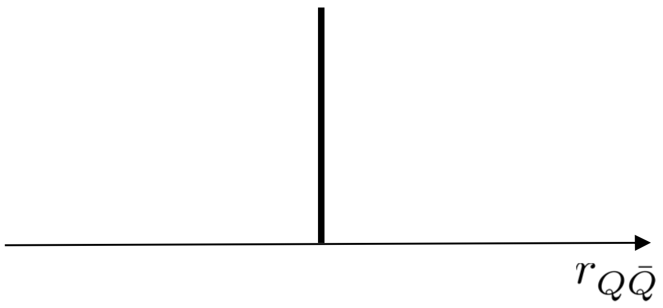
Picture behind transport theory :



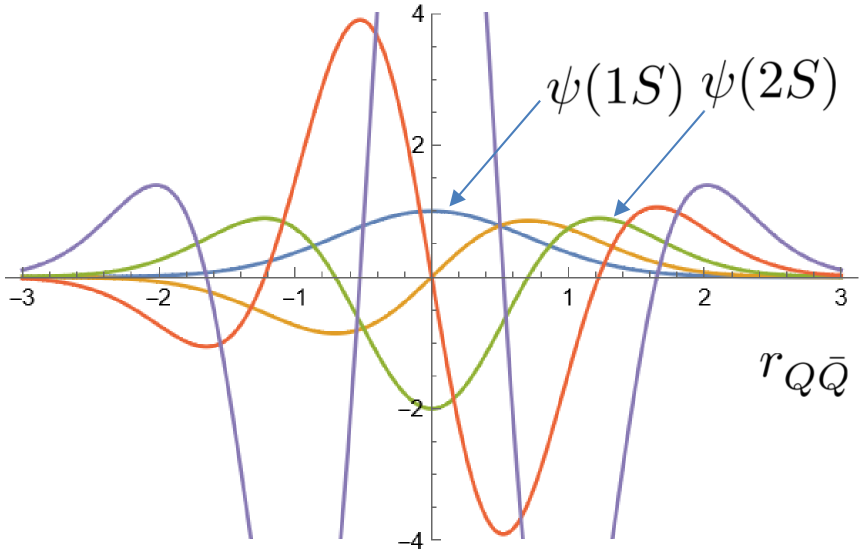
Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some “formation time” τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Quantum coherence at « early » time



$= \Sigma$



Initial compact state : δ

Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2$

Coherence

Neglect of coherence

$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^\star c_i \langle\psi_j|r^2|\psi_i\rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle\psi_i|r^2|\psi_i\rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Crucial to include coherence !

N.B. : one can model this effect by phenomenological formation time, but lack of control

How can we restore the quantumness of Quarkonia treatment (in interaction with some environment) ?

- Statistical mechanics is about averages… averaging wave functions makes no sense.

Stochastic-like Hamiltonians and ensemble average on the various realizations… even possibly integrating dissipative terms (Schroedinger Langevin with R. Katz)

Like : less computer demanding

Dislike : cannot be rigorously derived from fundamental principles

XOR

Deals with density matrix for the full state (quarkonia + environment)

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$|\psi\rangle = |\psi_{\mathrm{QGP}}\rangle \otimes |\psi_{Q\bar{Q}}\rangle$$

Possible to make statistical averages on the environment (« tracing out ») while still preserving all

Evolution equation for $\rho_{Q\bar{Q}}(x_Q, x_{\bar{Q}}, x'_Q, x'_{\bar{Q}})$

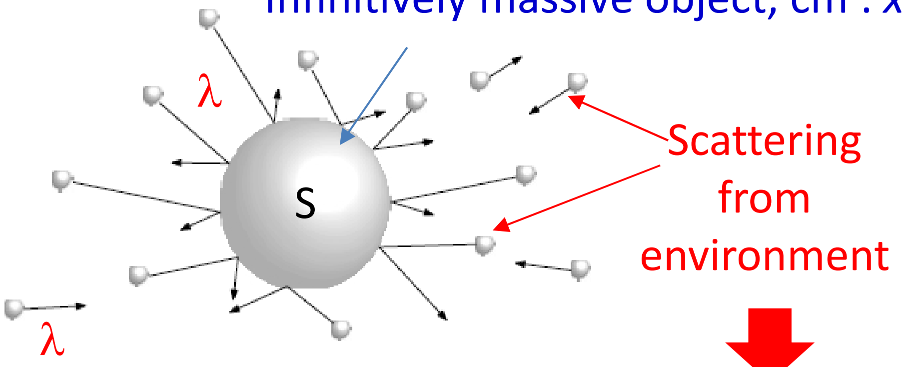
Quantum Master Equation

Like : rigorously derived from fundamental principles

Dislike : computer demanding… currently for several pairs

Decoherence from system-env. interaction

Quantitative model :



$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_{\text{int}}$$

Reduced density matrix $\rho_S(\mathbf{x}, \mathbf{x}')$

⇕

Wigner density $W_S(\mathbf{X} = \frac{\mathbf{x}+\mathbf{x}'}{2}, p)$

Fourier conjugate of $\mathbf{x} - \mathbf{x}'$

$$\frac{\partial \rho_S(\mathbf{x}, \mathbf{x}', t)}{\partial t} = -F(\mathbf{x} - \mathbf{x}')\rho_S(\mathbf{x}, \mathbf{x}', t)$$

Decoherence factor:

$$F(\mathbf{x} - \mathbf{x}') = \int dq \rho(q) v(q) \int \frac{d\hat{n} d\hat{n}'}{4\pi} \left(1 - e^{iq(\hat{n}-\hat{n}')\cdot(\mathbf{x}-\mathbf{x}')/\hbar}\right) \underbrace{|f(q\hat{n}, q\hat{n}')|^2}_{\frac{d\sigma}{d\Omega(\hat{n}, \hat{n}')}}$$

Short wave length (λ << Δx)

$$F(\mathbf{x} - \mathbf{x}') = \Gamma_{\text{tot}}$$

Total collision rate

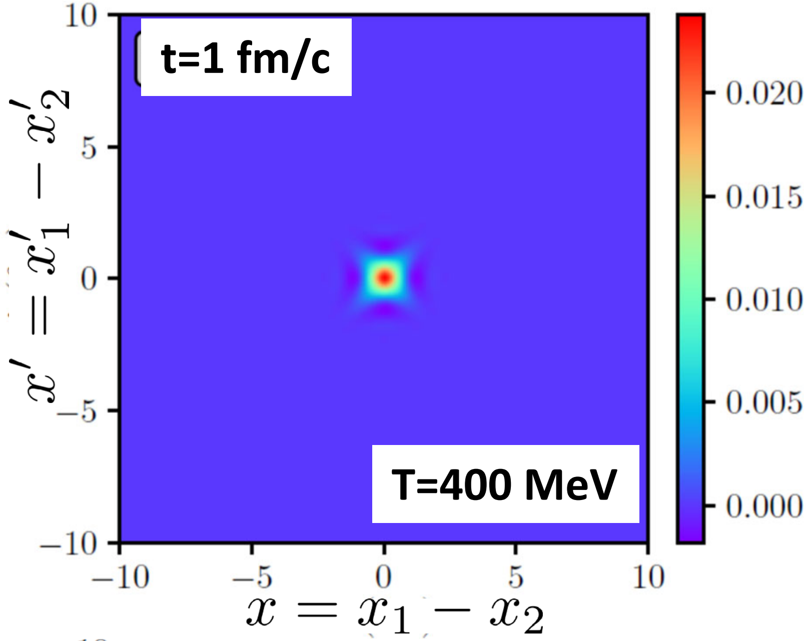
Long wave length (λ >> Δx)

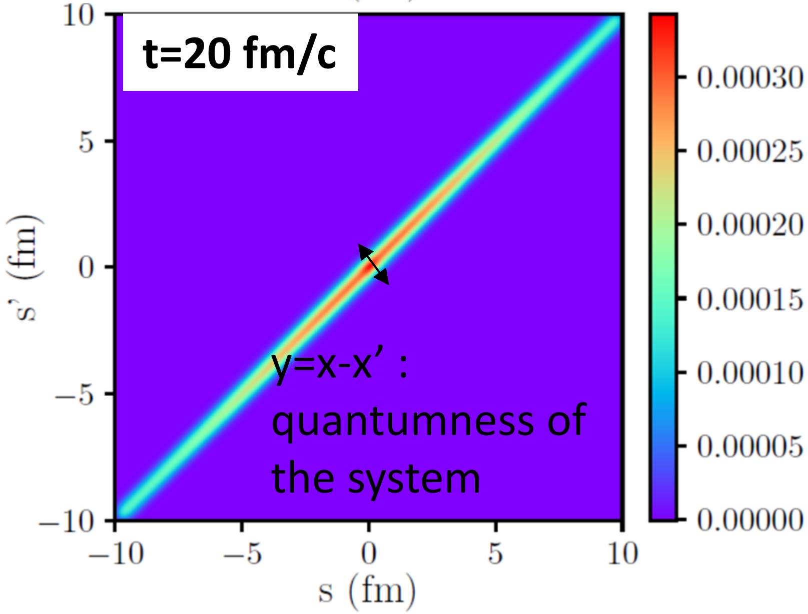
$$F \approx \int dq \rho(q) v(q) q^2 \sigma_{\text{transp}}(q) \times (\mathbf{x} - \mathbf{x}')^2 \approx \kappa(\mathbf{x} - \mathbf{x}')^2$$

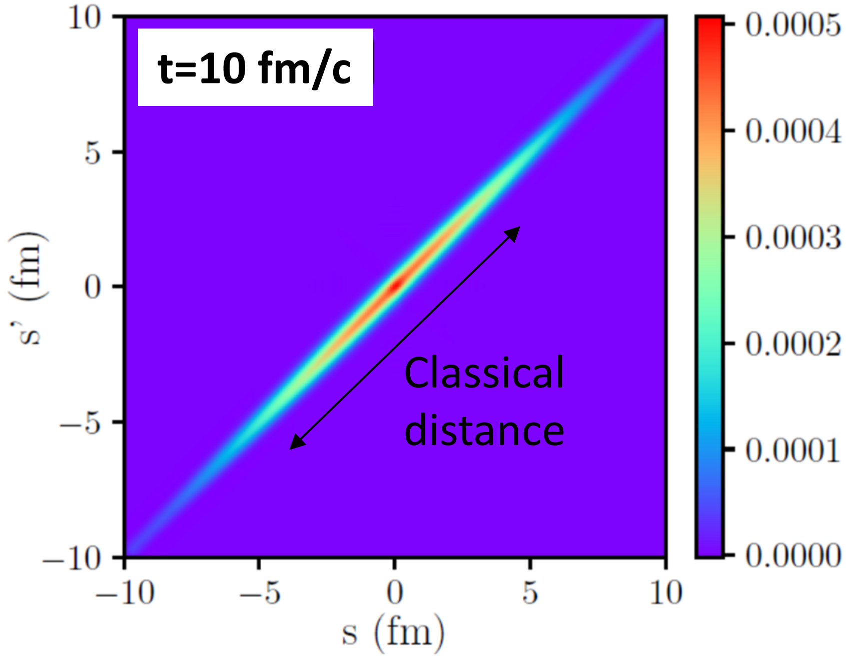
Suppresses coherence at large x-x' : classicalization

For small objects, coherence can be preserved over long times (several “cycles”)

Decoherence from system-env. interaction







$$\rho_S(x,x',t) \sim \rho_S(x,x',0)e^{-\Lambda(x-x')^2 t}$$

- Compactification along the short diagonal = « classicalization »
- $t_d \sim \frac{1}{\kappa(\Delta x)^2} \sim \frac{1}{TM\eta_D(\Delta x)^2}$

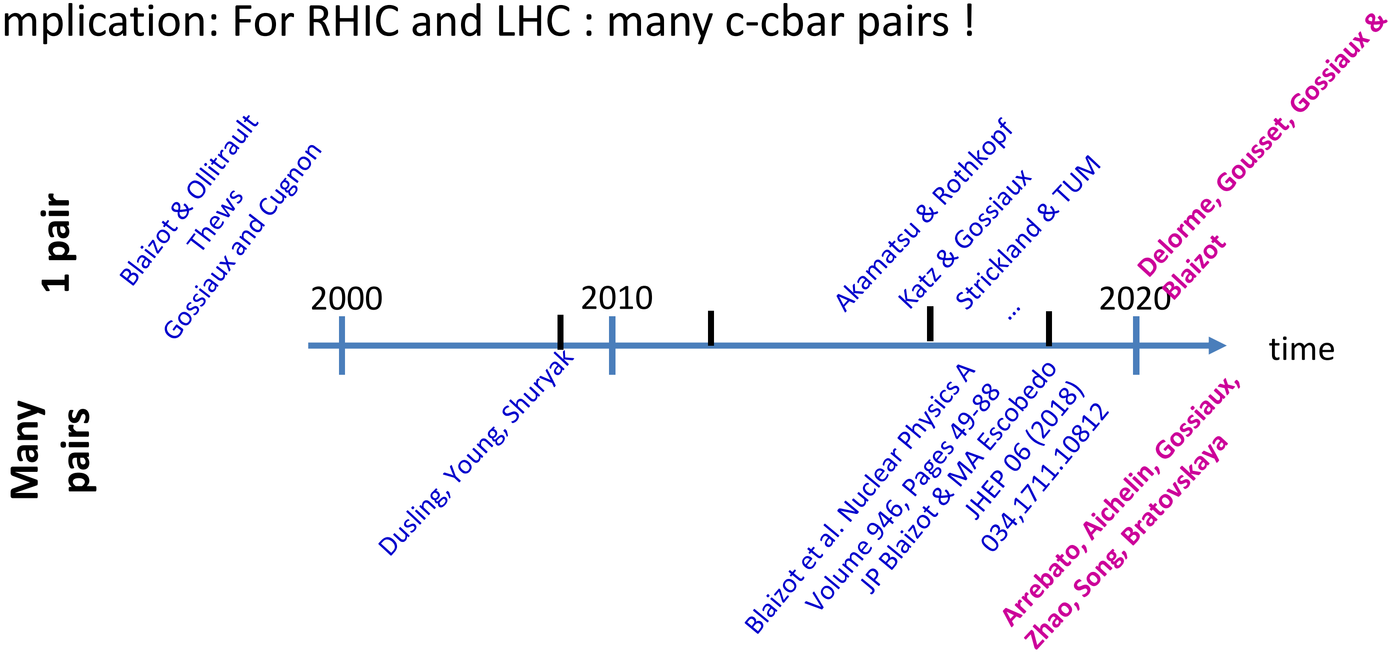
Einstein relation

Single part. relaxation rate

$$t_d \sim \frac{\tau_R^{\text{single}}}{\frac{1}{\lambda_{th}^2}(\Delta x)^2} \sim \tau_R^{\text{single}} \times \left(\frac{\lambda_{\text{th}}}{\Delta x}\right)^2$$

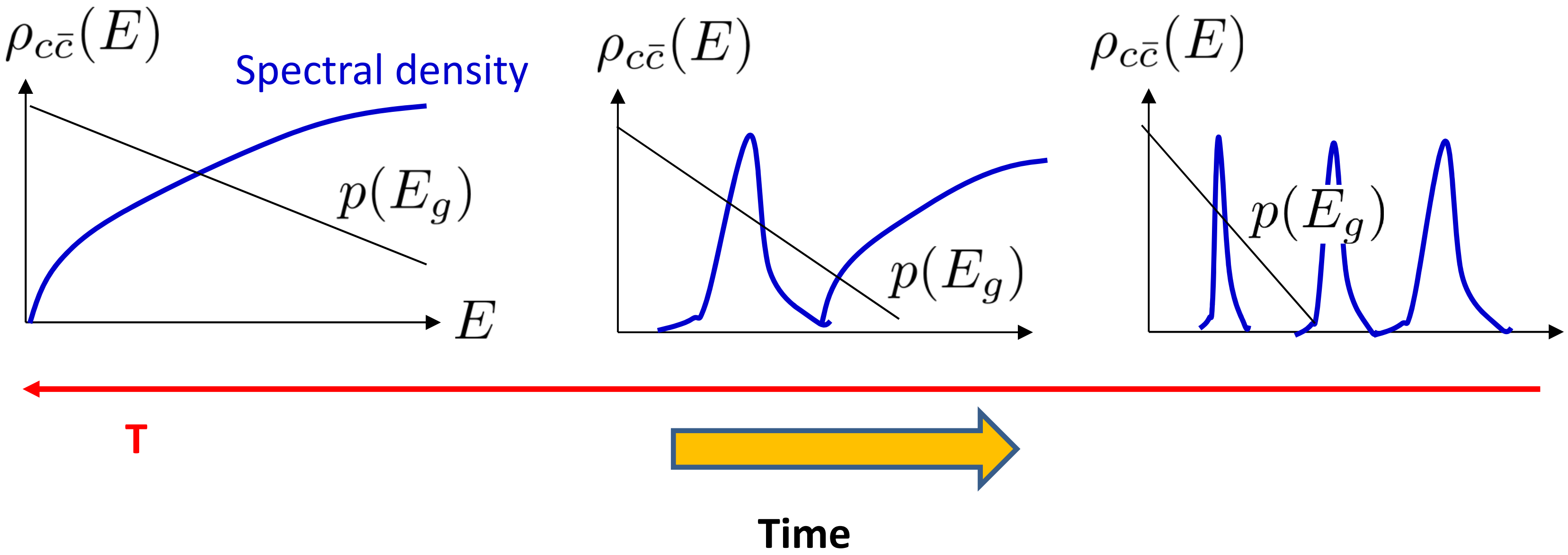
Other motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for **« from small to large »**
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs => Semi-Classical Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

Charmonia in a microscopic theory
Several regimes / effects



Multiple scattering on quasi free states

Gluo-dissociation of well identified levels by scarce "high-energy" gluons (dilute medium => cross section ok)

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Yet, still a need to define the equivalent of a formation – dissociation rate

Two types of dynamical modelling

(and a 3rd class of its own: statistical hadronization)

$T \gg E_{\rm bind}$	$T \sim E_{\rm bind}$	$T \ll E_{\rm bind}$
Quantum Brownian Motion		Quantum Optical Regime

- Correlations growing with cooling QGP
- Best described in position-momentum space
- Time short wrt quantum decoherence time

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt : some limiting cases

?

- Well identified resonances
- Time long enough wrt quantum decoherence time

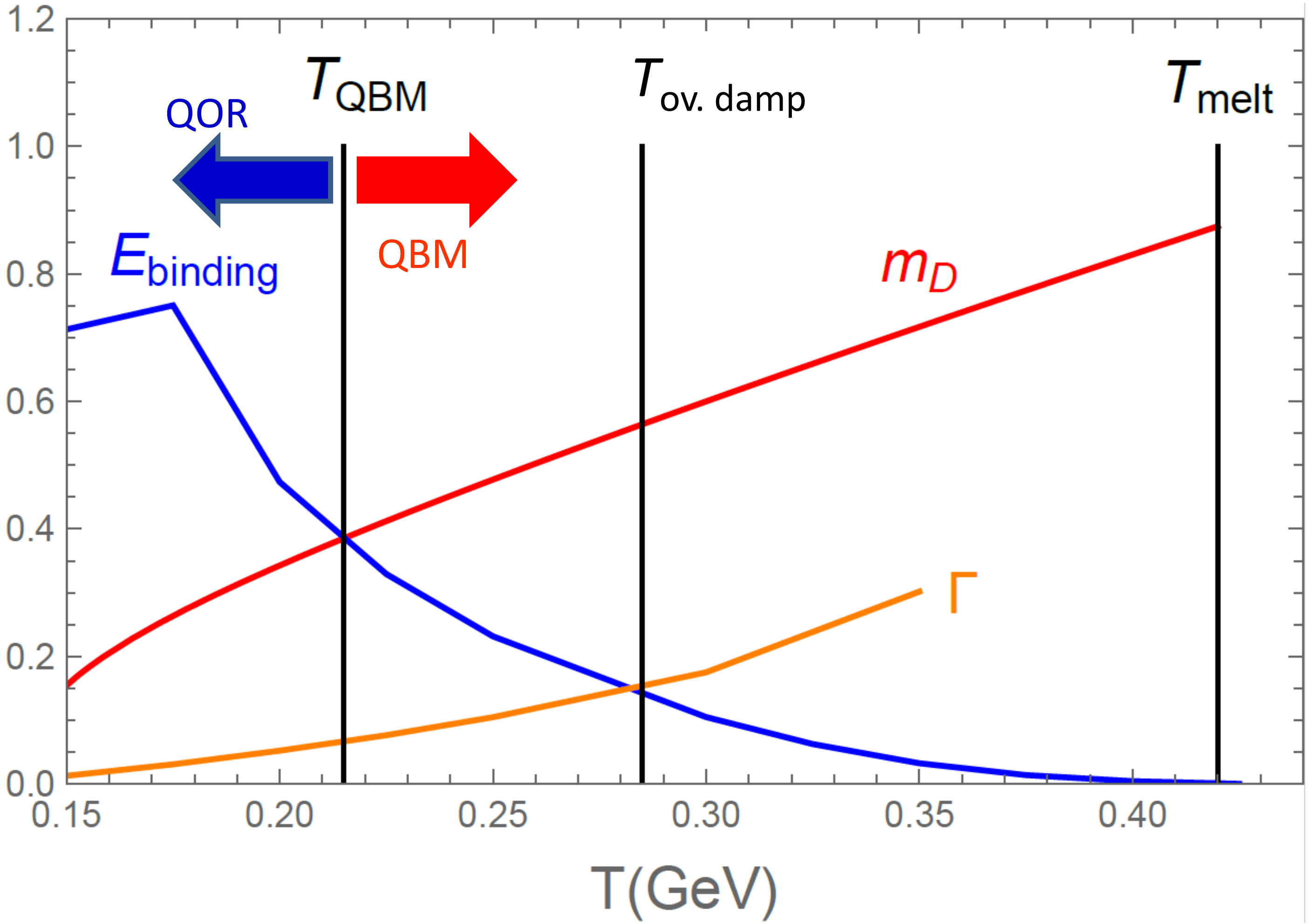
Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : exp(-E_n/T) (theorem)

Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these model is an important prerequisite !!!

Two types of dynamical modelling



Numbers extracted from potential described in Phys. Rev. D 101, 056010 (2020)

Blaizot-Escobedo Quantum Master Equation

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}] \xrightarrow[\text{representation}]{\text{Interaction}} i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

Coulomb gauge

$$\mathcal{D}^I(t) = \mathcal{U}_I(t,t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t,t_0)$$

Free Quark Hamiltonian

Plasma Hamiltonian

Quark-Plasma Interactions…

$$H_1 = -g\int_{\boldsymbol{r}} A_0^a(\boldsymbol{r})\, n^a(\boldsymbol{r})$$

No magnetic term (NR)

color charge density of the heavy particles

Average over plasma d.o.f + rapid environment hypothesis

Generic Linblad – like QME on $\mathcal{D}_Q$

… treated as a perturbation

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^{t}\mathrm{d}t'\int_{\boldsymbol{x}\boldsymbol{x}'}\left([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}')\right.$$
$$\left.+[\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\right)$$

$\Delta^>, \Delta^<$ Time ordered HTL gluon propagators

Blaizot-Escobedo Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2n^a_{\boldsymbol{x}}\mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations => decohence, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q\dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms*** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

Subatech

* As well as another time discretization

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$\mathcal{L}_3$

* As well as another time discretization

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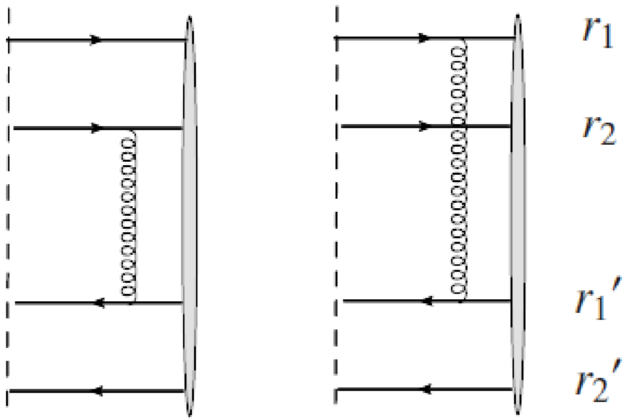
$\mathcal{L}_4$

Application to QED-like and QCD for both cases of 1 body and 2 body densities…

QED-like vs genuine QCD case

Genuine QCD

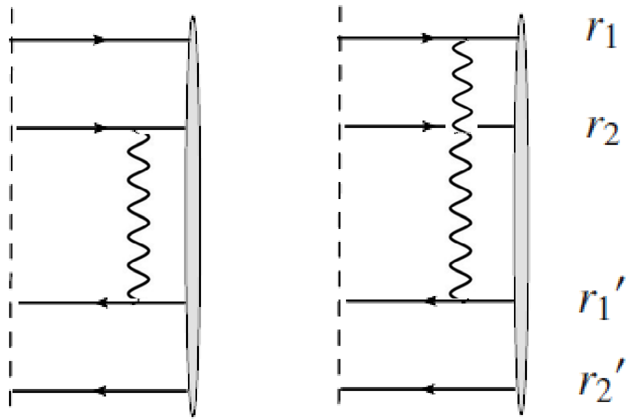


- Scattering from gluons change the color respresentation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- No binding potential in the octet chanel => « large » energy gap

QED-like



- Scattering from do not change the Casimir : s <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \end{pmatrix}$$

- Usual 1S <-> 1P transitions between bound states.

B-E Quantum Master Equation: QED case

- For the relative motion (2 body):

$$\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}_1' - \vec{x}_2'\end{array}\right\} \quad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** (power series in *y* up to 2nd order)

... However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases} \mathcal{L}_0 = \frac{2i\nabla_y\cdot\nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r})+\mathcal{H}(0))\cdot\nabla_{\vec{y}} \end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \to \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

B-E Quantum Master Equation: QCD case

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

octet density matrix

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0\\ \mathcal{D}_8\end{pmatrix}$ Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Example of the $\mathcal{D}_s$ evolution (a semi-classical expansion, i.e po series in y=s-s') :

$$\begin{aligned}(D_s|\mathcal{L}|\mathcal{D}) = &\left(2i\frac{\nabla_r\cdot\nabla_y}{M} + i\frac{\nabla_R\cdot\nabla_Y}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s\\
&-2C_F\Gamma(\boldsymbol{r})(D_s-D_o)\\
&-\frac{C_F}{4}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o\right)\\
&-C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o\\
&+\frac{C_F}{2MT}\left[\nabla^2W(0)-\nabla^2W(\boldsymbol{r})-\boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_r\right](D_s-D_o)\\
&-\frac{C_F}{2MT}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_y\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_y\,D_o\right)\\
&-\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_Y D_o.\end{aligned}$$

Our ongoing projects

Our Goal:

- Explicitly restore the Linbladian form and the positivity of BE equations => term $\mathcal{L}_4$
- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $c\bar{c}$ pair without performing the Semi-Classical approximation:
  - Evolution of the density matrix
  - Evolution of states probabilities over time
  - Singlet-octet transitions
  - Color relaxation time
  - …
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)
- Possibly design improved algorithm for intermediate temperatures

Subatech

Positivity

- Equations for the QED-like plasma in 1D :

$$\frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D} = \frac{i}{M}(\hbar c)^2(\partial_s^2 - \partial_{s'}^2)\mathcal{D} - i[V(s) - V(s')]\mathcal{D}$$
$$+\left[2W(0) - W(s) - W(s') - 2W\left(\tfrac{s-s'}{2}\right) + 2W\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^2}{4MT}\left[2W''(0) - W''(s) - W''(s') - 2W''\left(\tfrac{s-s'}{2}\right) + 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$-\frac{(\hbar c)^2}{4MT}\left[2W'(s)\partial_s + 2W'(s')\partial_{s'} + 2W'\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) - 2W'\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[2W''''(0) + W''''(s) + W''''(s') - 2W''''\left(\tfrac{s-s'}{2}\right) + 2W''''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W'''(s)\partial_s + 4W'''(s')\partial_{s'} - 4W'''\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) + 4W'''\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W''(0)(\partial_s^2 + \partial_{s'}^2) + 4W''(s)\partial_s^2 + 4W''(s')\partial_{s'}^2 + 8W''\left(\tfrac{s-s'}{2}\right)\partial_s\partial_{s'} + 8W''\left(\tfrac{s+s'}{2}\right)\partial_s\partial_{s'}\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}p_{\mathrm{tot}}^2\left[-2W''(0) + W''(s) + W''(s') + 2W''\left(\tfrac{s-s'}{2}\right) - 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$

$\mathcal{L}'_4$

- Indeed subleading in 1/T expansion
- No higher derivatives on D than the 2nd one => still a FP equation in the semi-classical limit.
- Higher derivatives of the imaginary potential W => possible UV divergences => need for some regularization.

Further implementation featu

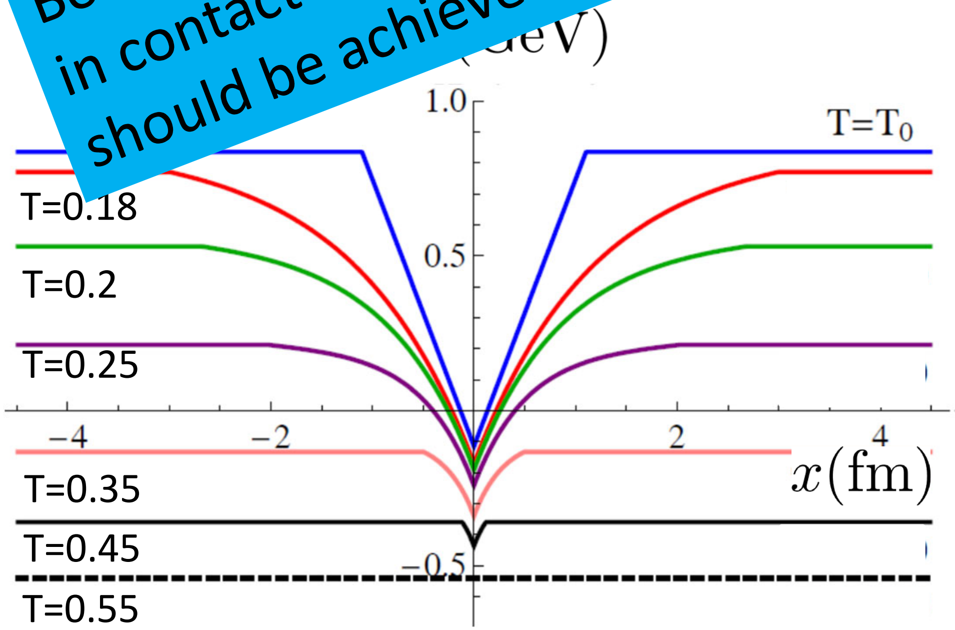
➢ 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and

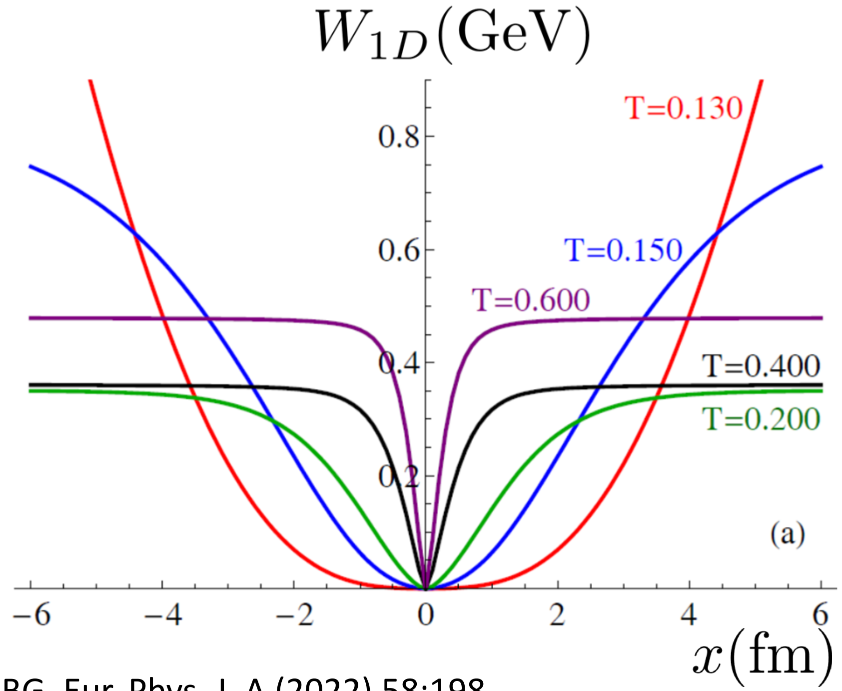
!!! Not the radial decomposition ... bersome

Even states will be c ... d states will be
considered a

Need t ... potential V + i W (based on 3D lQCD
... mass spectra and decay widths)

Both regimes where heavy quarks are close together and far apart in contact with the heat « reservoir » => correct thermalisation should be achieved



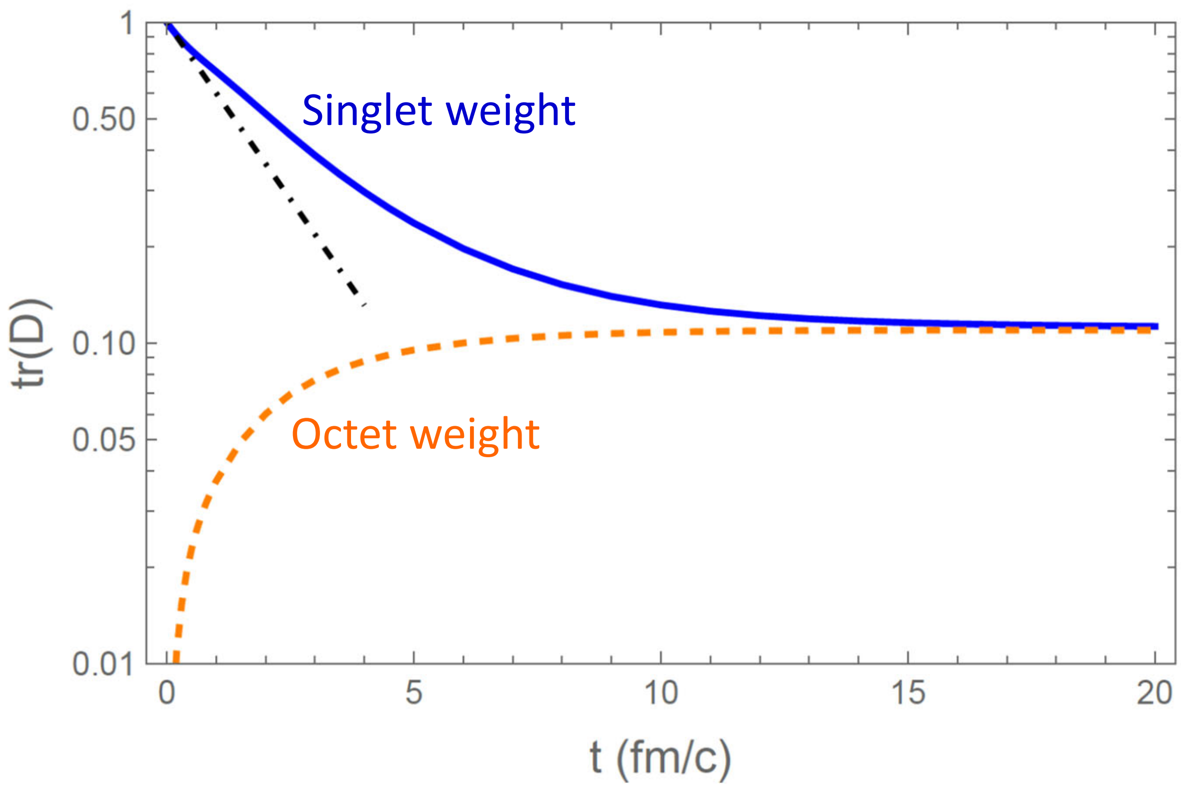


1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

Some selected results for 1 c-cbar system

<u>Color Dynamics : Singlet – octet probabilities:</u>

- Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\rm eq} = D_o^{\rm eq} = \frac{1}{9}$ $\quad (1+8)\times\frac{1}{9}$

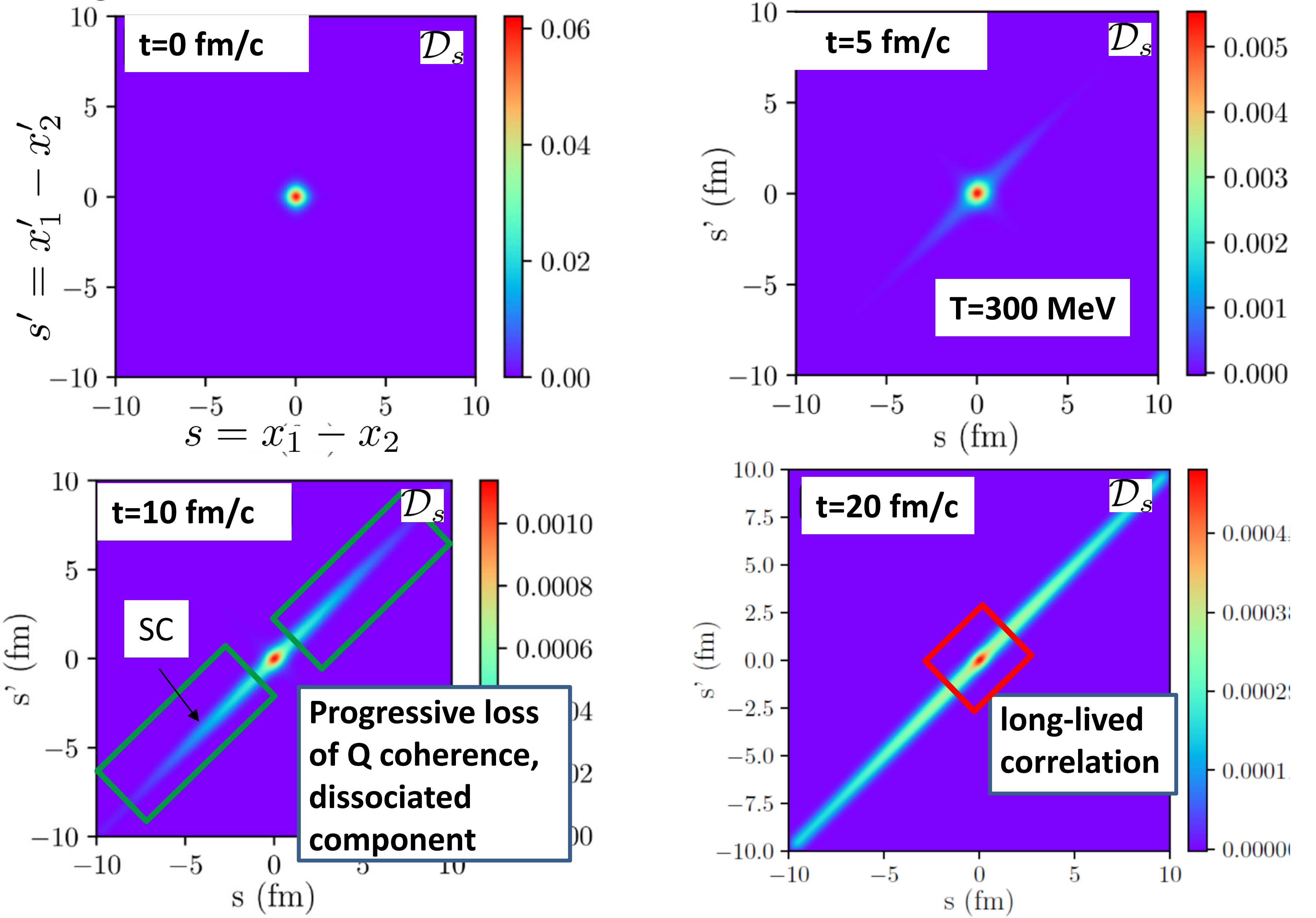


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.
- C-cbar can interact with the surrounding QGP as an octet => energy loss

Subatech

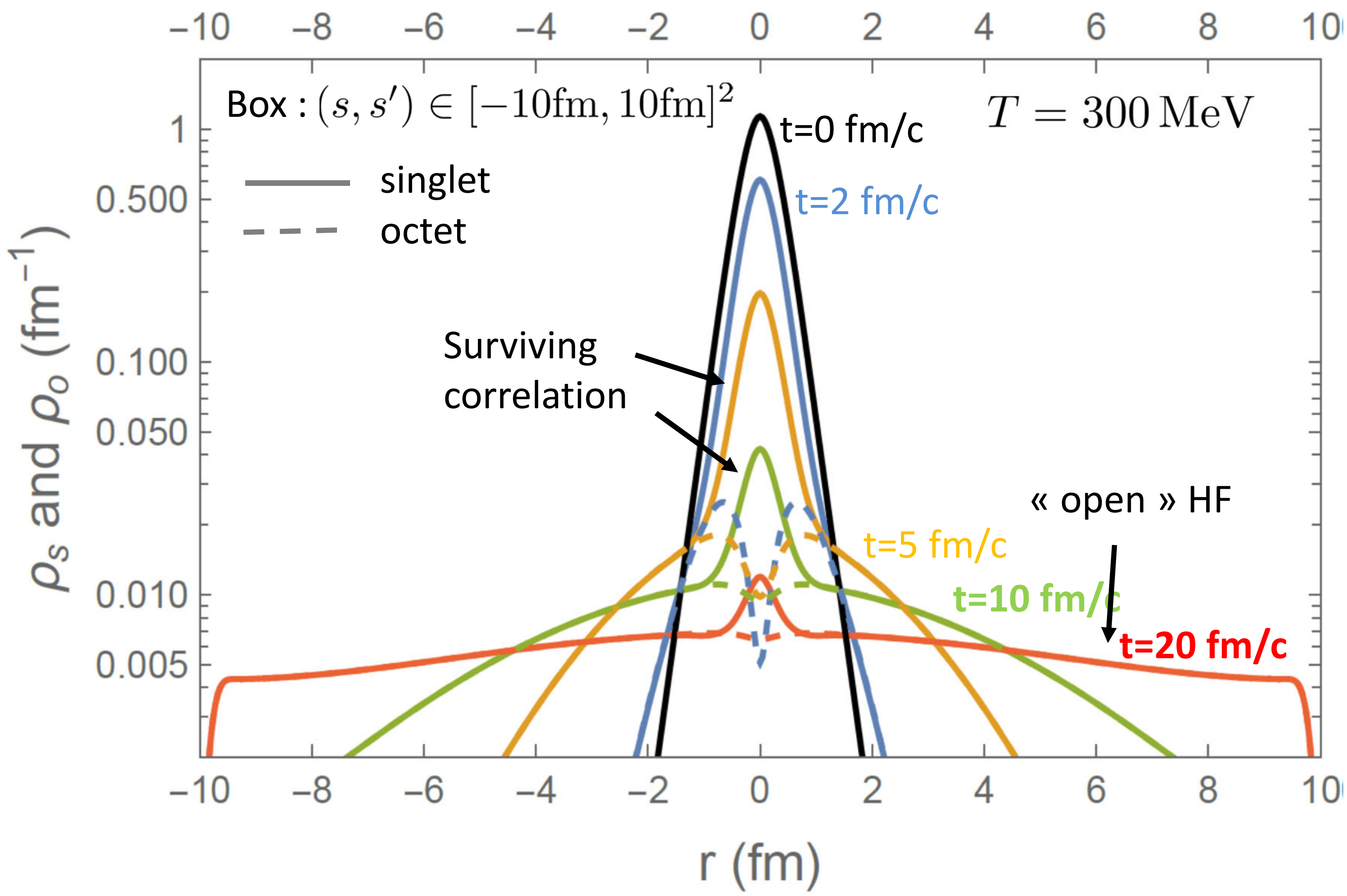
Evolution of the Density matrix

1S singlet initial state:



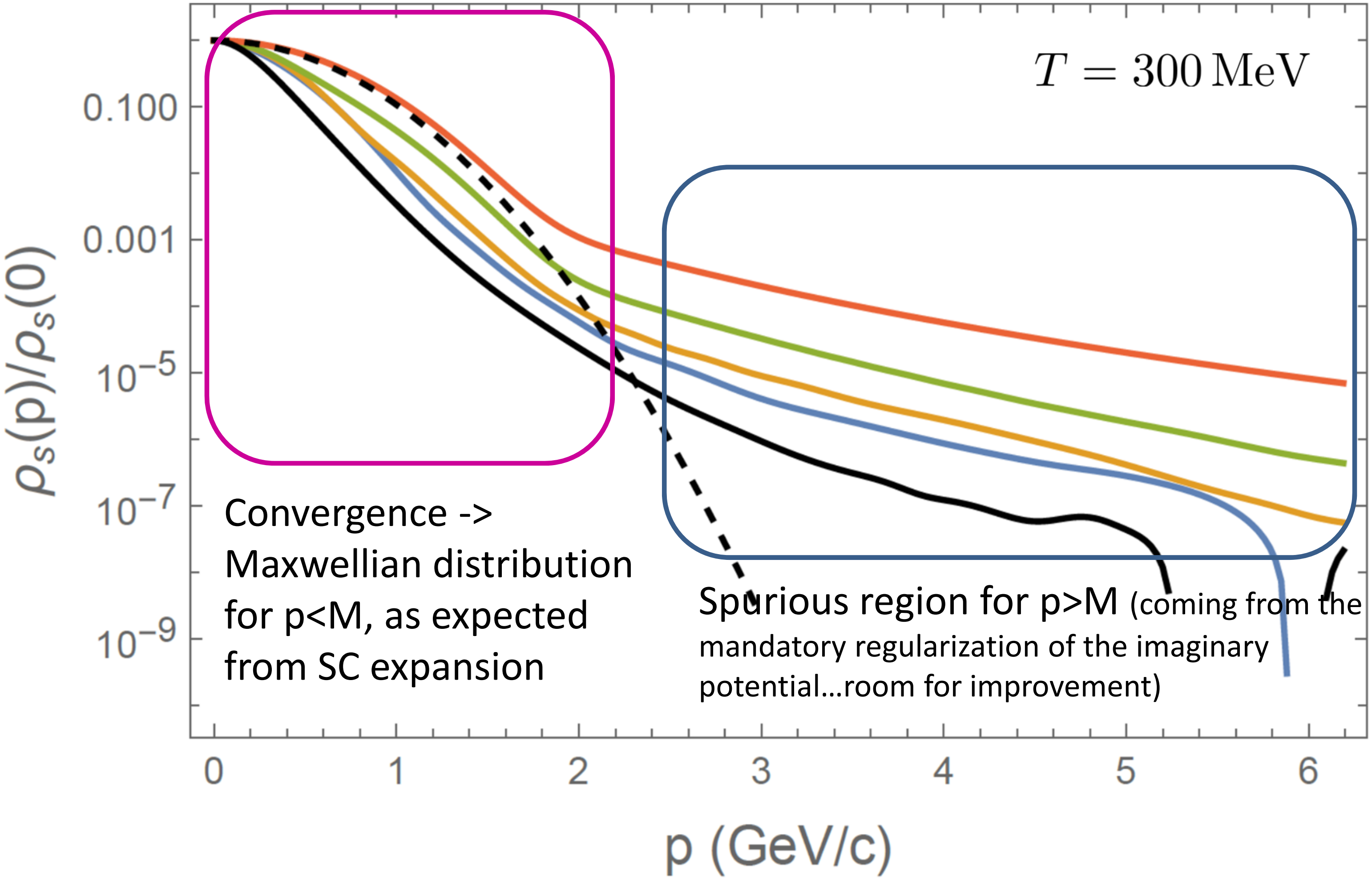
Evolution of the spatial density

1S singlet initial state:



Some c-cbar stay at intermediate distance ("recombination") … remaining peak in the asymptotic distribution

Evolution of the momentum density

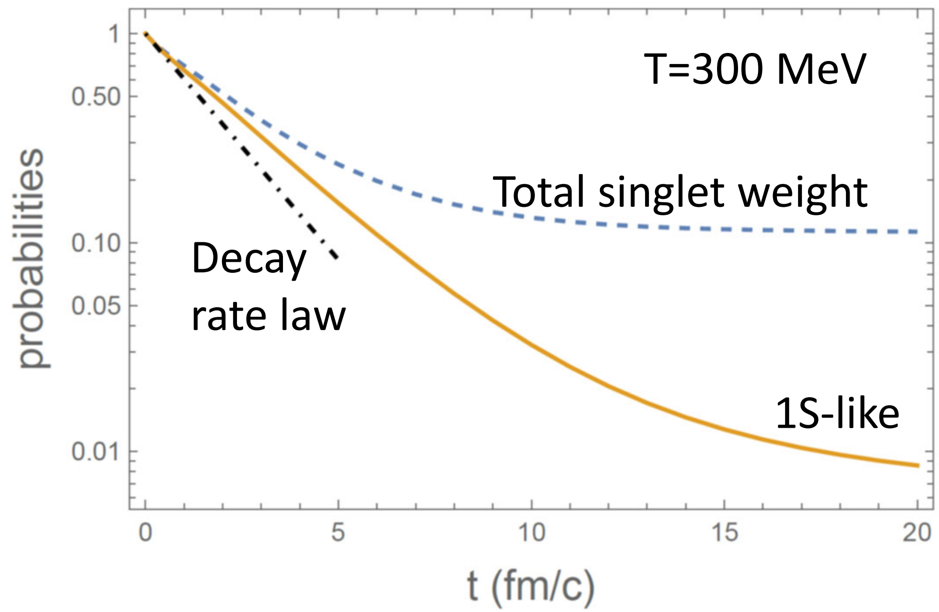


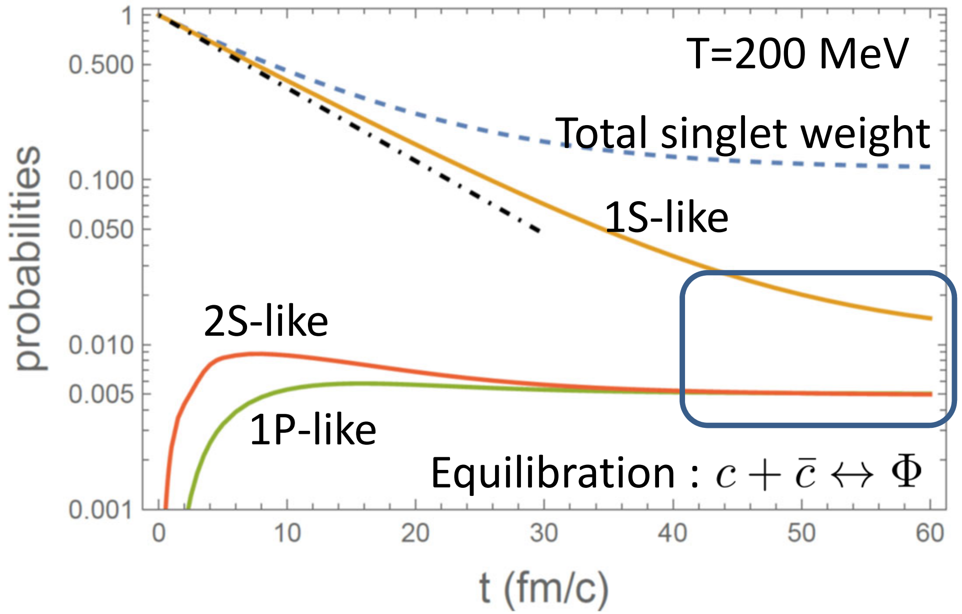
Mostly sensitive to the distribution at large relative distance

Results for projection on local states

« local states » = eigenstates of the screened potential at a given T (<> vacuum states)

$$p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$$



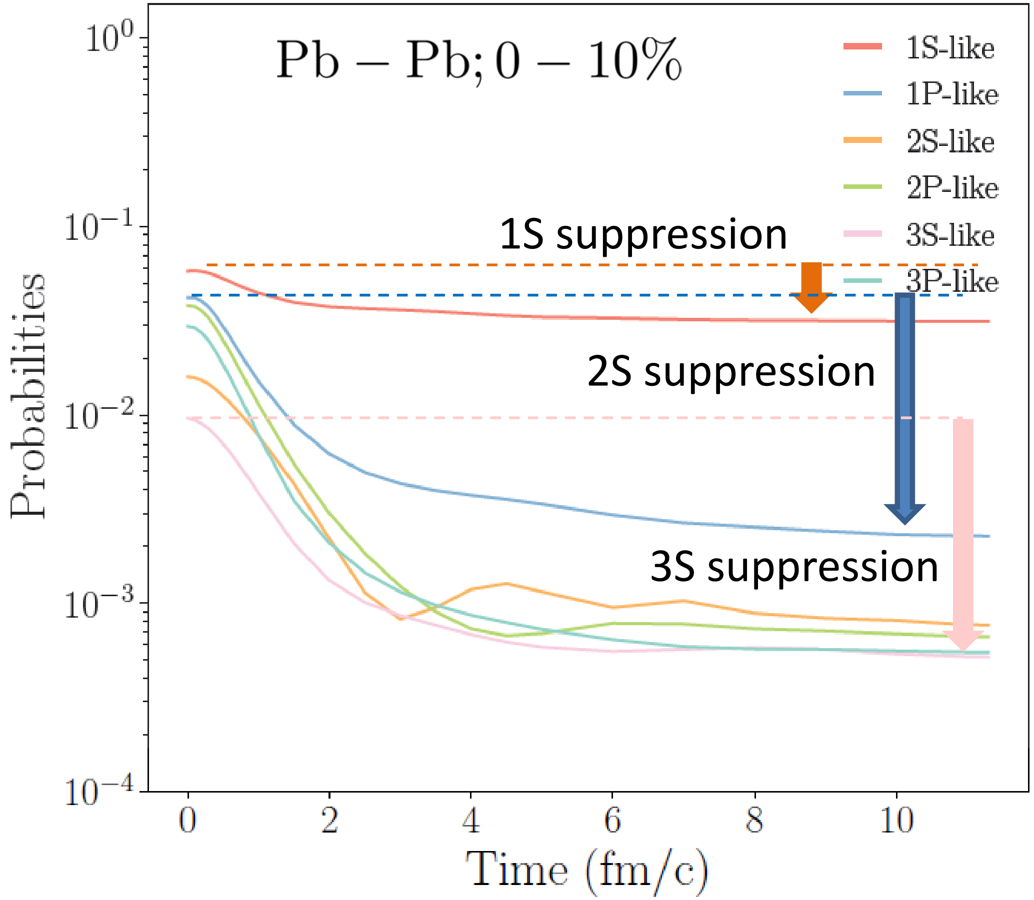


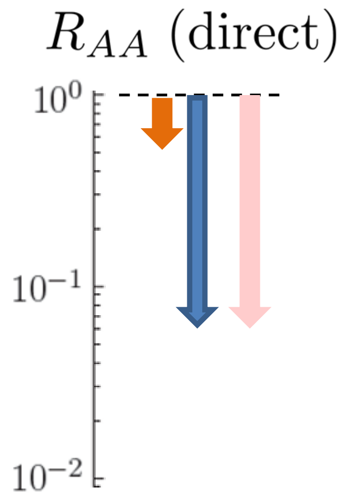
- At small times, $\mathcal{L}_3 \ll \mathcal{L}_2$ fluctuations dominate… higher state repopulation
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states.

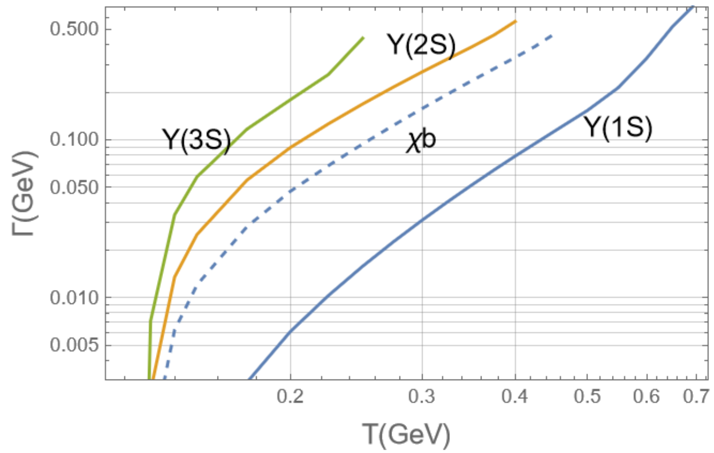
- 1S evolution at small time well described by decay rate law (decay rate can be calculated within the QME)
- 1P and 2S generated from 1S show a more complex behavior, not governed by their own decay rate !!!

Contact with experiment

- Calculation of bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.







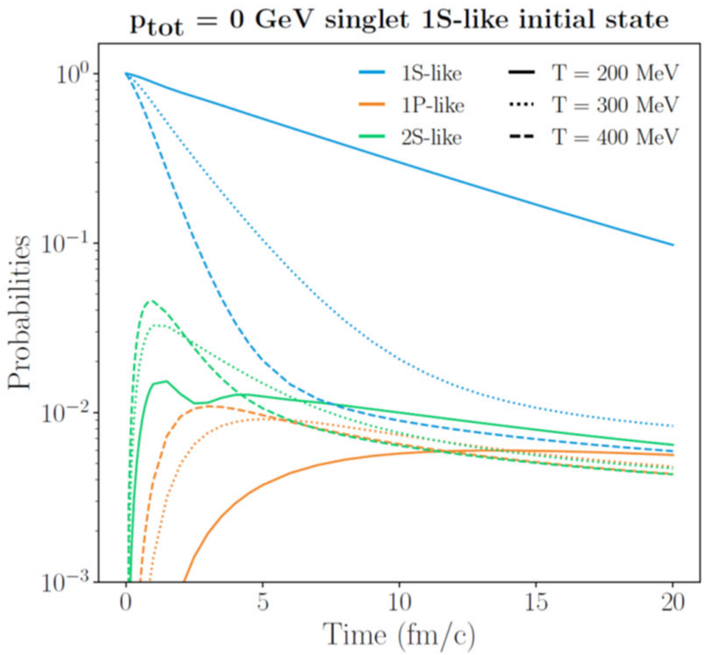
- Similar RAA for Y(3S) and Y(2S) although 3S decay rate >> 2S decay rate
- **See Stephane Delorme's talk at Hard Probe 2023 for more details.**

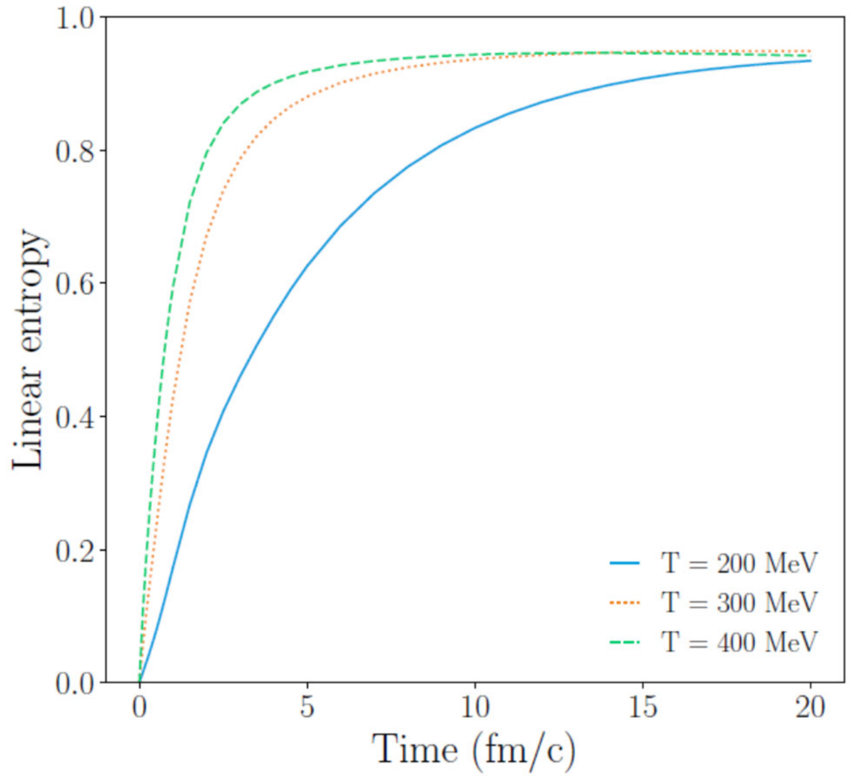
Results for Linear quantum entropy

$$S_L = \mathrm{Tr}\hat{\rho} - \mathrm{Tr}\hat{\rho}^2 = 1 - \mathrm{Tr}\hat{\rho}^2$$

De Boni, J. High Energ. Phys. (2017) 2017: 64

(results for QED like evolution)





- Suppression and decoherence appear to happen on the same time scale…
- … does not seem in favour of applying classical rate equations (to be investigated further)

Conclusions and prospects

- Illustration of a QME solved exactly, with some interesting features and a first (not so bad) contact towards experiment using EPOS4 profiles