Constraining QGP properties using heavy quarks

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Outline

- **Developments of heavy quark models at different momentum scales**
  - Jet energy loss at high p_T
  - Color potential interaction at low to medium p_T
  - Hadronization at low to medium p_T
- **Probing properties of nuclear matter in heavy-ion collisions**
  - Equation of state of QGP
  - Medium geometry and evolution profile of the strong electromagnetic field

High p_T parton-medium interaction

Linear Boltzmann Transport (LBT)

$$p_a \cdot \partial f_a(x_a, p_a) = E_a(\mathcal{C}_a^{\rm el} + \mathcal{C}_a^{\rm inel})$$

High p_T parton-medium interaction

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Elastic energy loss ($ab \rightarrow cd$)

$$\mathscr{C}_a^{\mathrm{el}} = \sum_{b,c,d} \int \prod_{i=b,c,d} \frac{d[p_i]}{2E_a} (\gamma_d f_c f_d - \gamma_b f_a f_b) \cdot (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) \left| \mathscr{M}_{ab \rightarrow cd} \right|^2$$

$2 \rightarrow 2$ scattering matrices

High p_T parton-medium interaction

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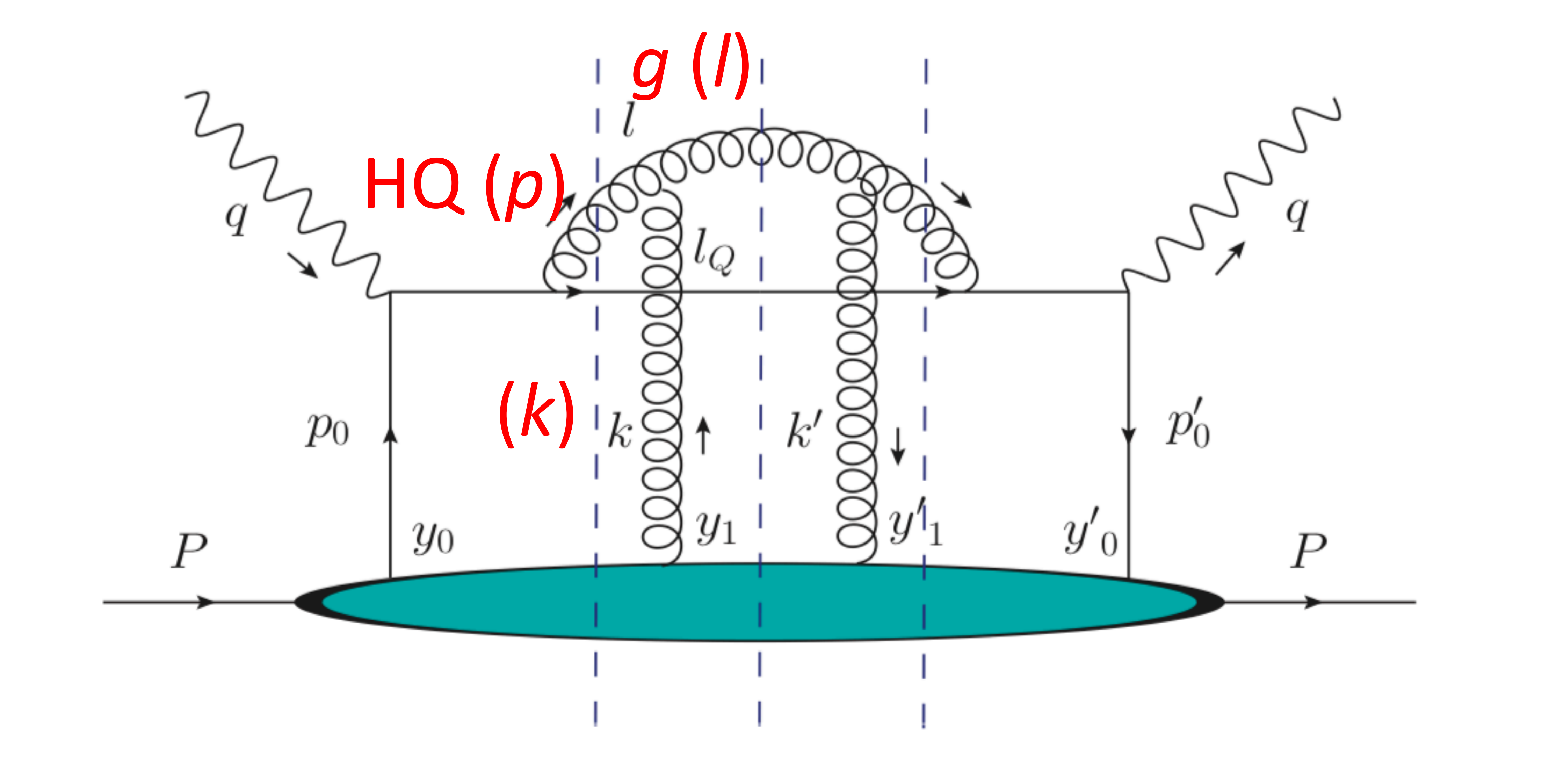
$2 \to 2$ scattering matrices

loss term: **scattering rate**
(for Monte-Carlo simulation)

$$\Gamma_a^{\mathrm{el}}(\mathbf{p}_a, T) = \sum_{b,c,d} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} d[p_i] f_b \cdot (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) \left| \mathscr{M}_{ab\to cd} \right|^2$$

Inelastic energy loss



[Majumder PRD 85 (2012); Zhang, Wang and Wang, PRL 93 (2004)]

- **Higher-twist formalism**: collinear expansion ($\langle k_\perp^2 \rangle \ll l_\perp^2 \ll Q^2$)

$$\frac{d\Gamma_a^{\mathrm{inel}}}{dz dl_\perp^2} = \frac{dN_g}{dz dl_\perp^2 dt} = \frac{6\alpha_s P(z) l_\perp^4 \hat{q}}{\pi (l_\perp^2 + z^2 M^2)^4} \sin^2\left(\frac{t - t_i}{2\tau_f}\right)$$

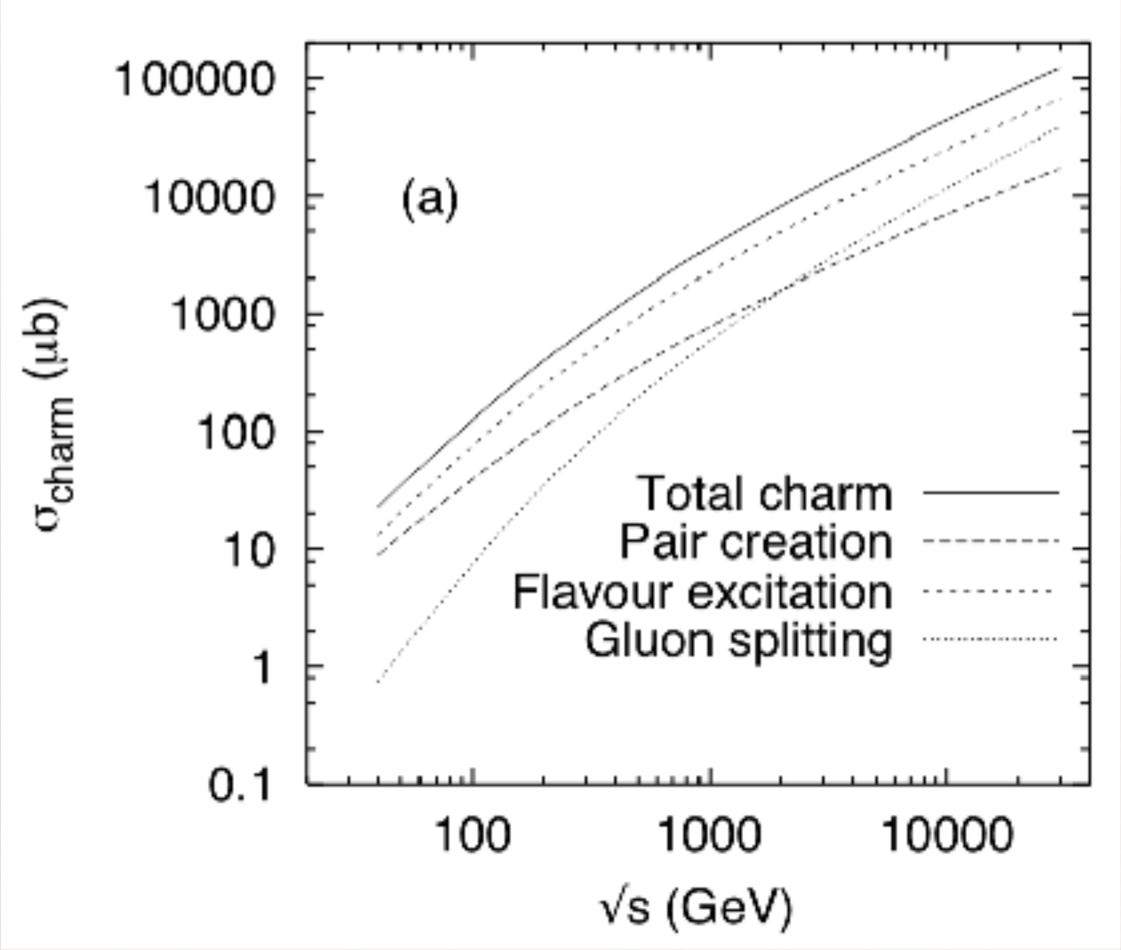
- Medium information absorbed in $\hat{q} \equiv d\langle p_\perp^2 \rangle / dt$

Flavor hierarchy of jet quenching

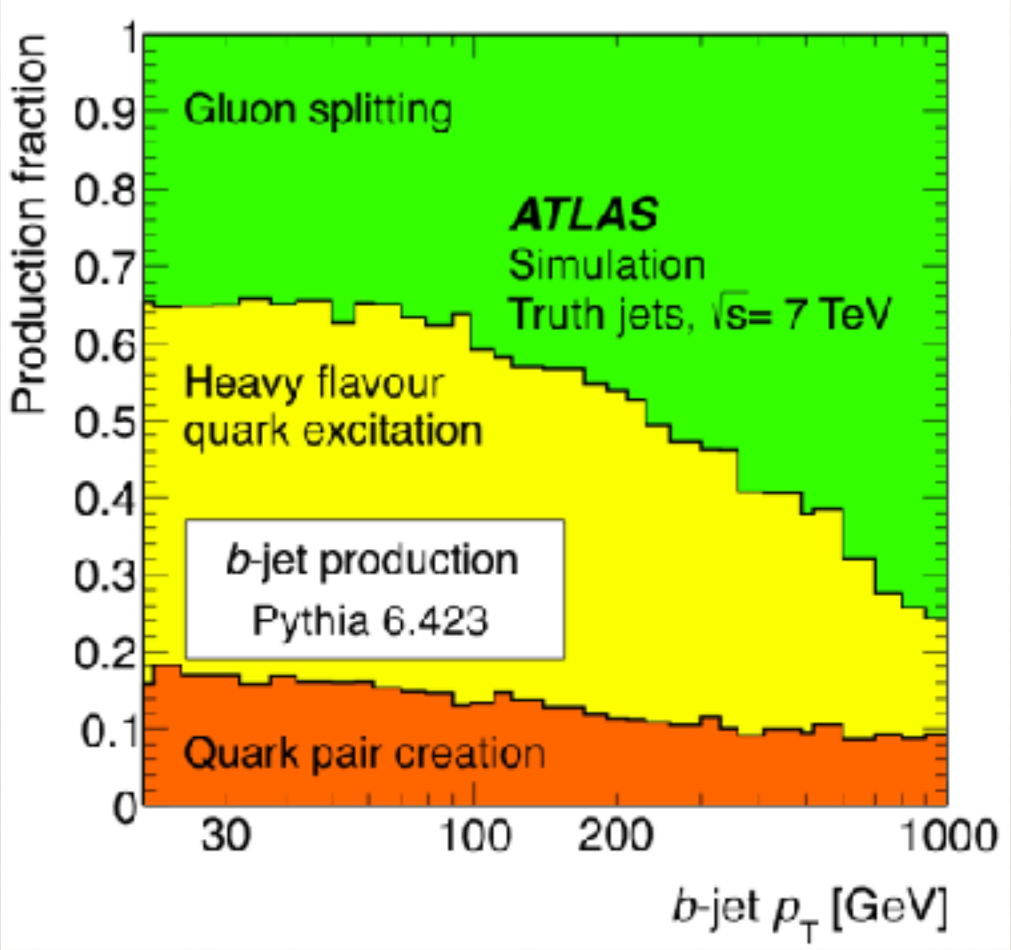
Clean perturbative framework is sufficient for describing the flavor hierarchy at high p_T (> 8 GeV)

[Xing, SC, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

NLO contribution to HQ production in Pythia simulation (gluon splitting)



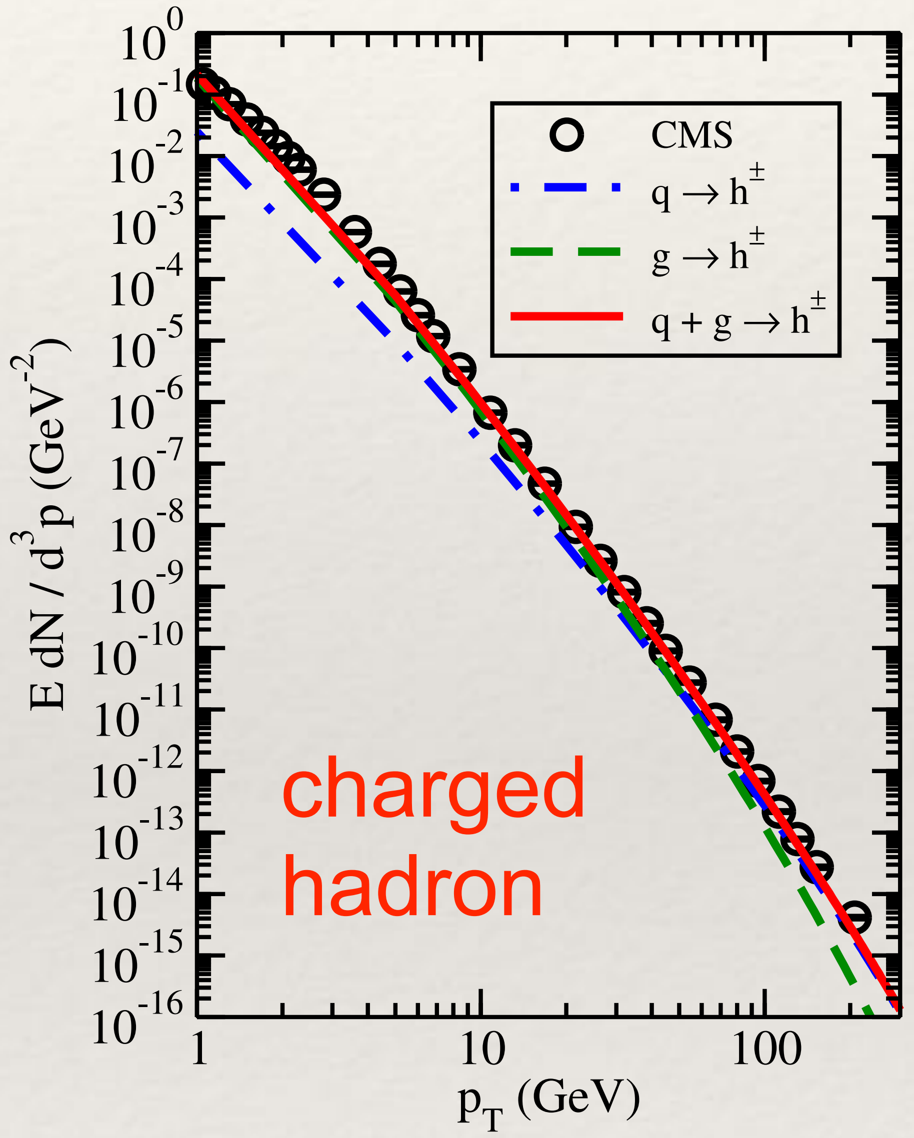
[Norrbin and Sjostrand, EPJC 17 (2000)]

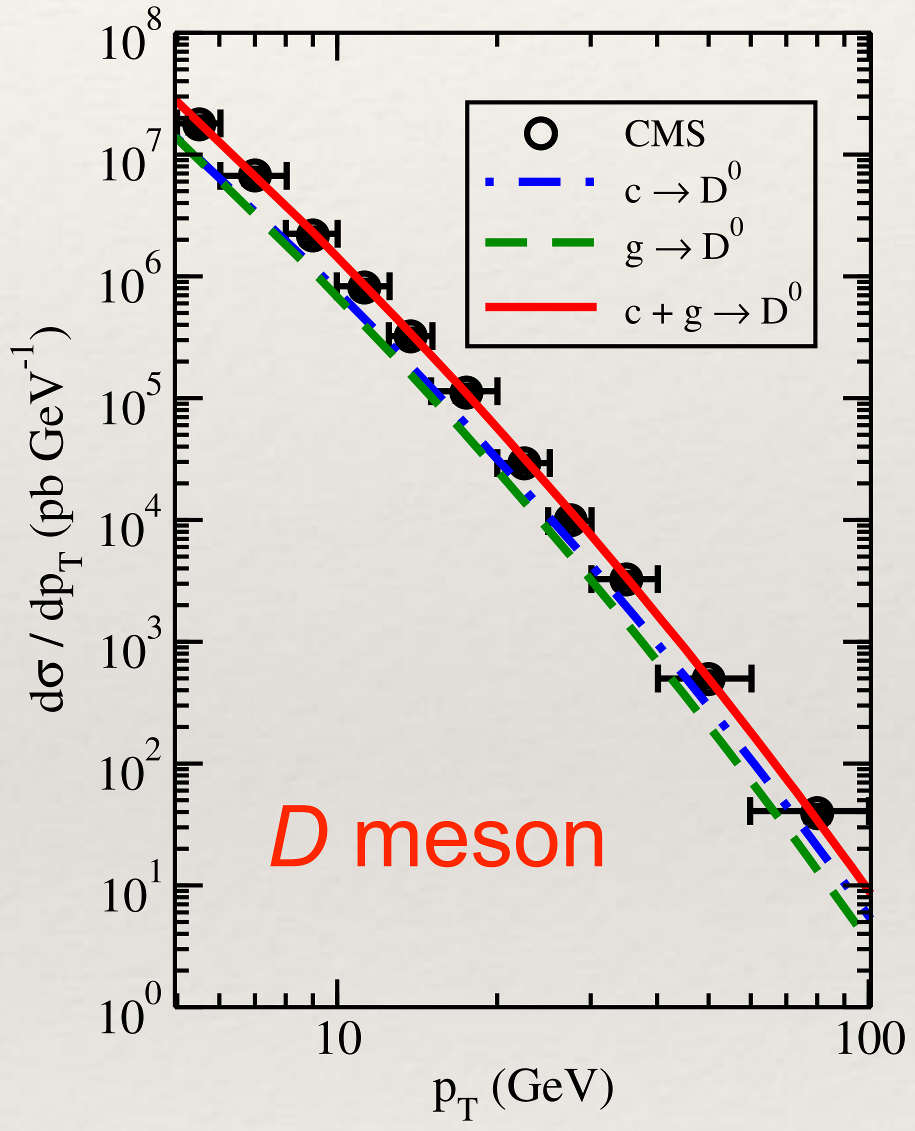


[ATLAS, EPJC 73 (2013)]

- NLO contribution increases with $\sqrt{s}$
- NLO contribution increases with *b*-jet p_T

Light vs. heavy hadron spectra within NLO production + fragmentation framework





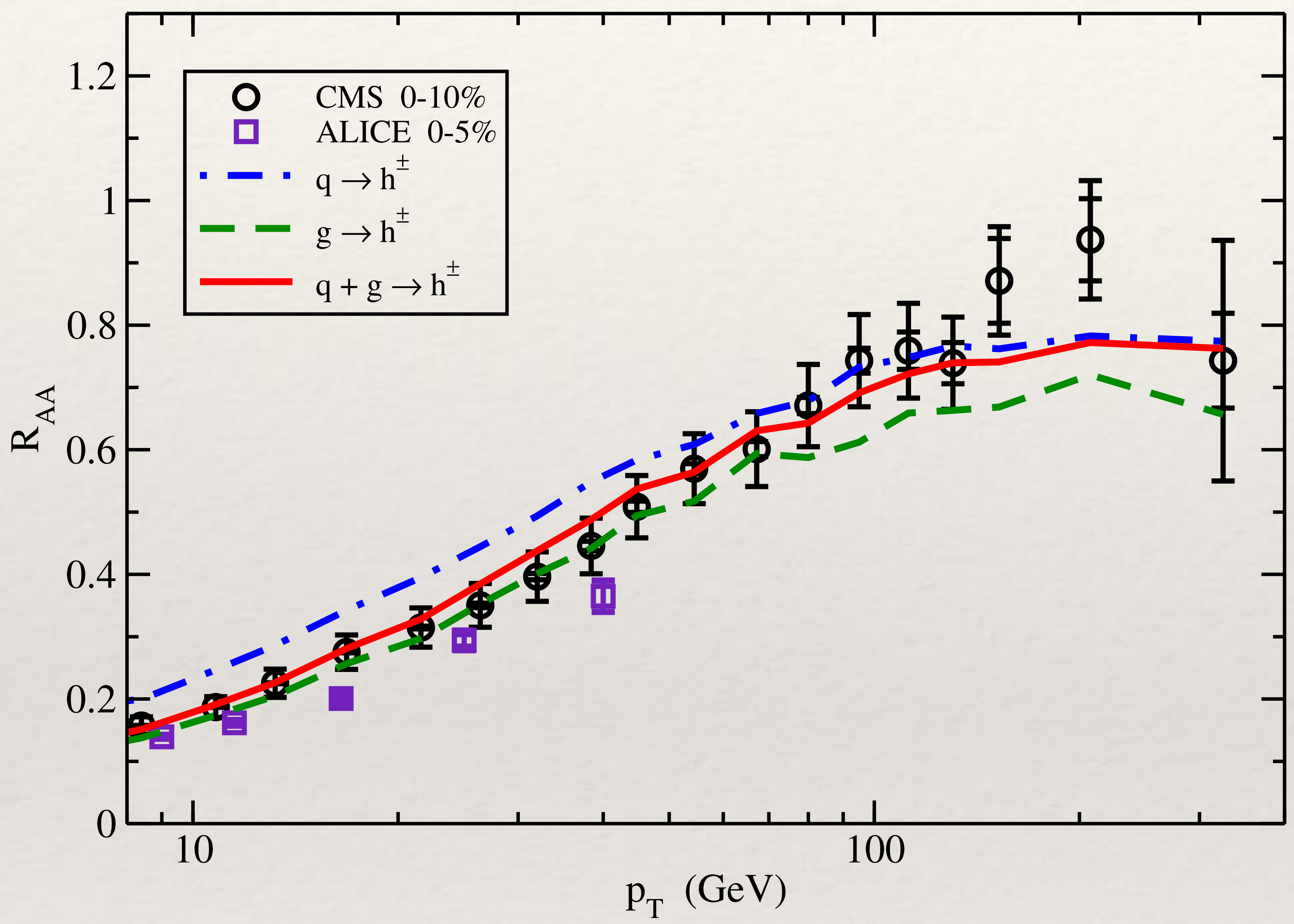
Gluon fragmentation

- dominates $h^{\pm}$ production up to 50 GeV
- contributes to over 40% *D* up to 100 GeV

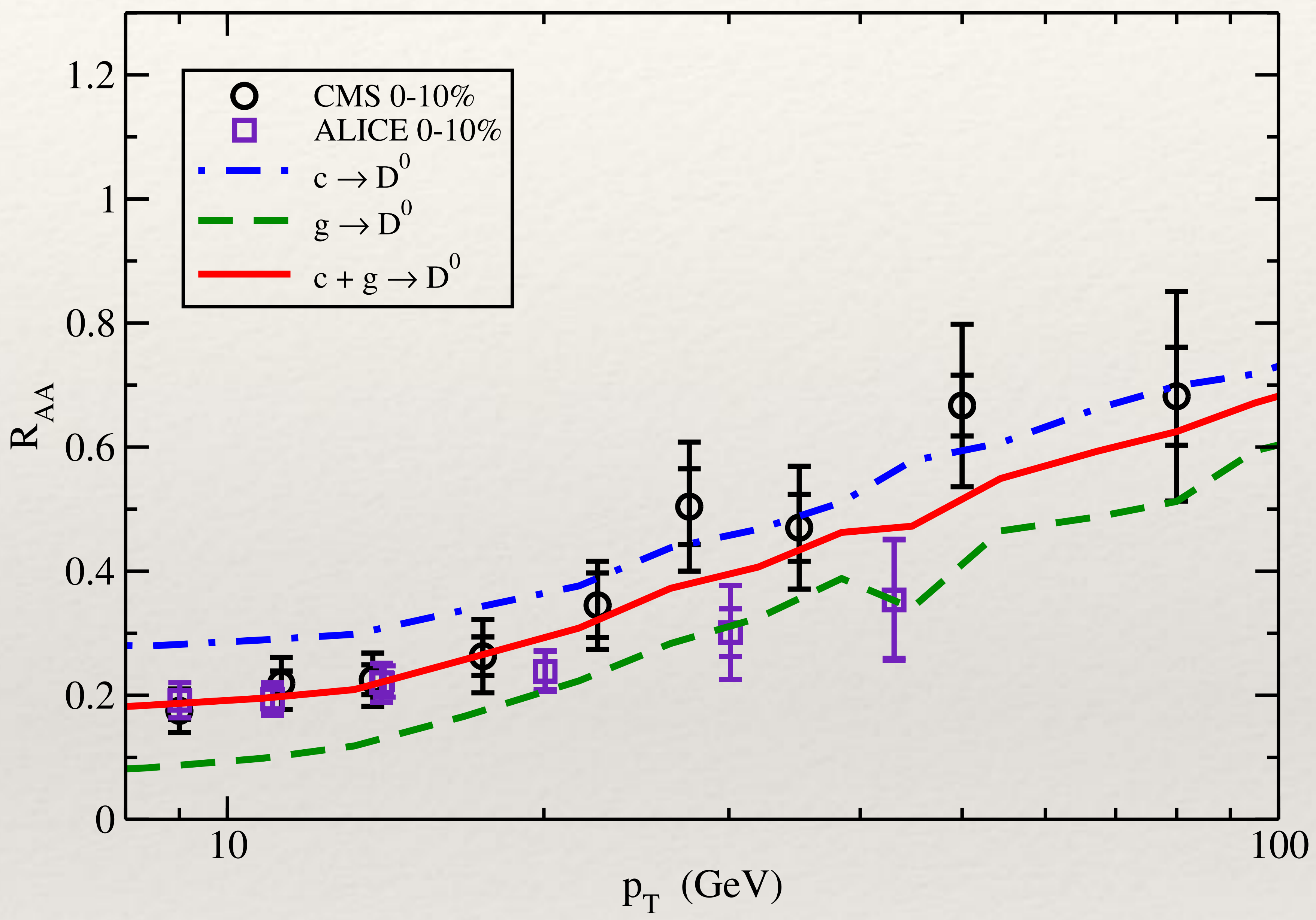
Flavor hierarchy of jet quenching

NLO initial production and fragmentation + Boltzmann transport (elastic and inelastic energy loss) + hydrodynamic medium for QGP

charged hadron



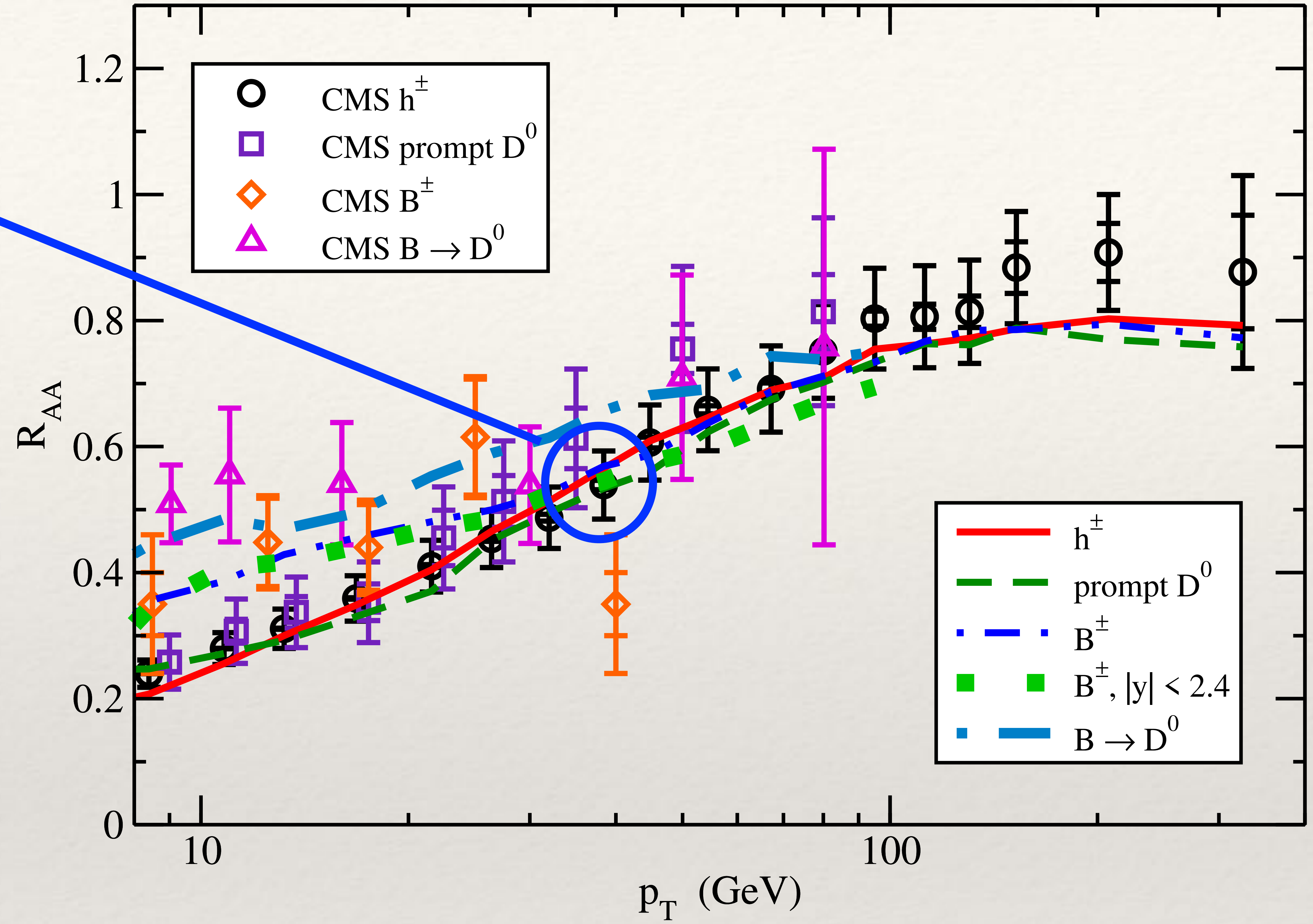
D meson



- *g*-initiated *h* & *D* R_{AA} < *q*-initiated *h* & *D* R_{AA} => $\boldsymbol{\Delta E_g > \Delta E_{q/c}}$
- Although R_{AA} (*c*->*D*) > R_{AA} (*q*->*h*), R_{AA} (*g*->*D*) < R_{AA} (*g*->*h*) due to different fragmentation functions => $\boldsymbol{R_{AA}(h) \approx R_{AA}(D)}$

Flavor hierarchy of jet quenching

Merging of *D* and *B* R_{AA} at $p_T \sim 40$ GeV



[Xing, SC, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

- A simultaneous description of charged hadron, *D* meson, *B* meson, *B*-decay *D* meson R_{AA}'s starting from $p_T \sim 8$ GeV
- Predict R_{AA} separation between *B* and *h* / *D* below 40 GeV, but similar values above – **wait for confirmation from future precision measurement**

Low to medium p_T — effects of non-perturbative interaction

➢ **Parametrization of the heavy-quark-QGP interaction potential:**

$$V(r,T) = -\boxed{\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r}} - \boxed{\frac{\sigma}{m_s} e^{-m_s r}}$$

Yukawa (color coulomb) String

in which $m_d = a + b * T$ and $m_s = \sqrt{a_s + b_s * T}$ are the respective screening masses, α_s and σ are the respective Yukawa and confining interaction strength.

➢ **By Fourier transformation,**

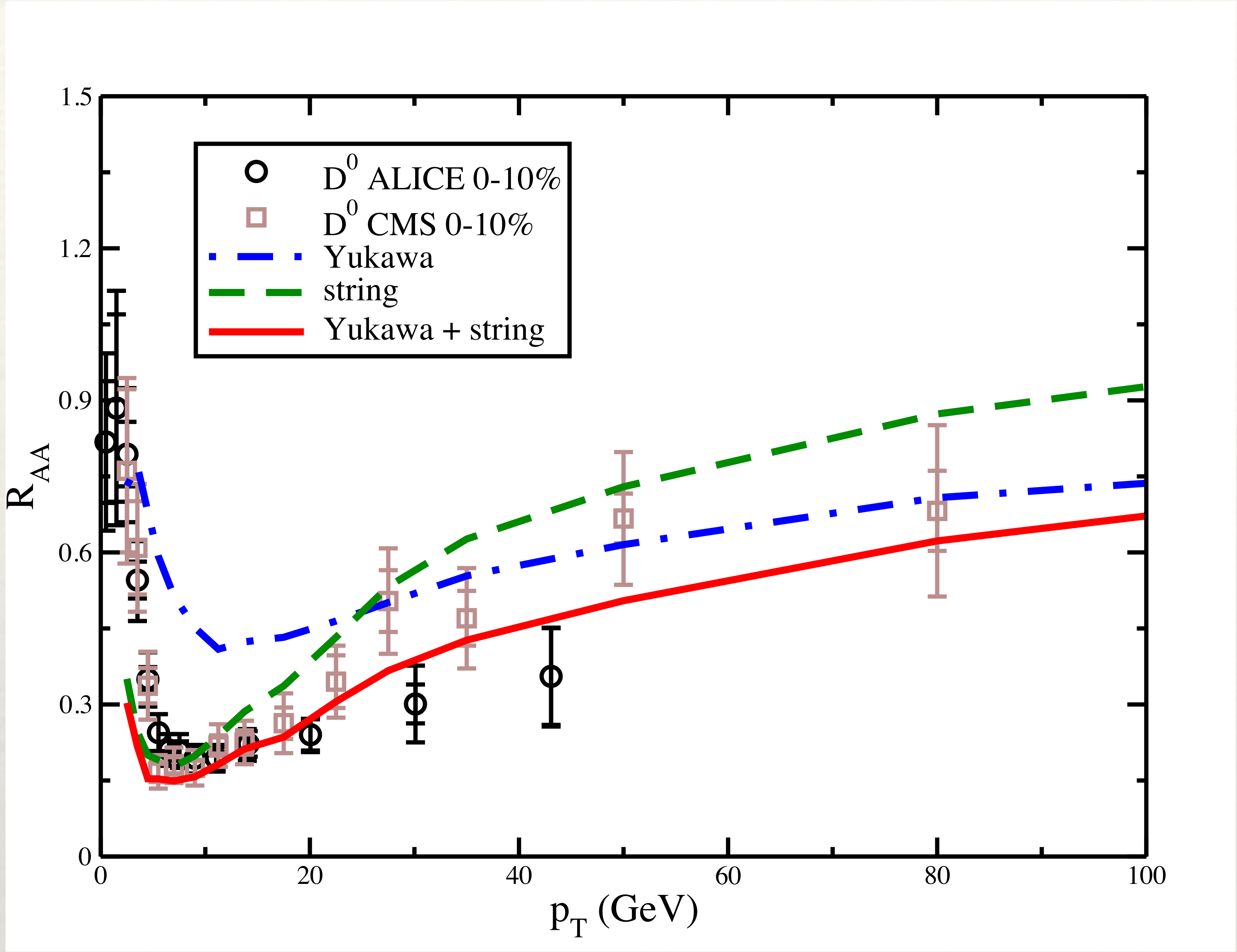
$$V(\vec{q},T) = -\boxed{\frac{4\pi\alpha_s C_F}{m_d^2 + |\vec{q}|^2}} - \boxed{\frac{8\pi\sigma}{\left(m_s^2 + |\vec{q}|^2\right)^2}}$$

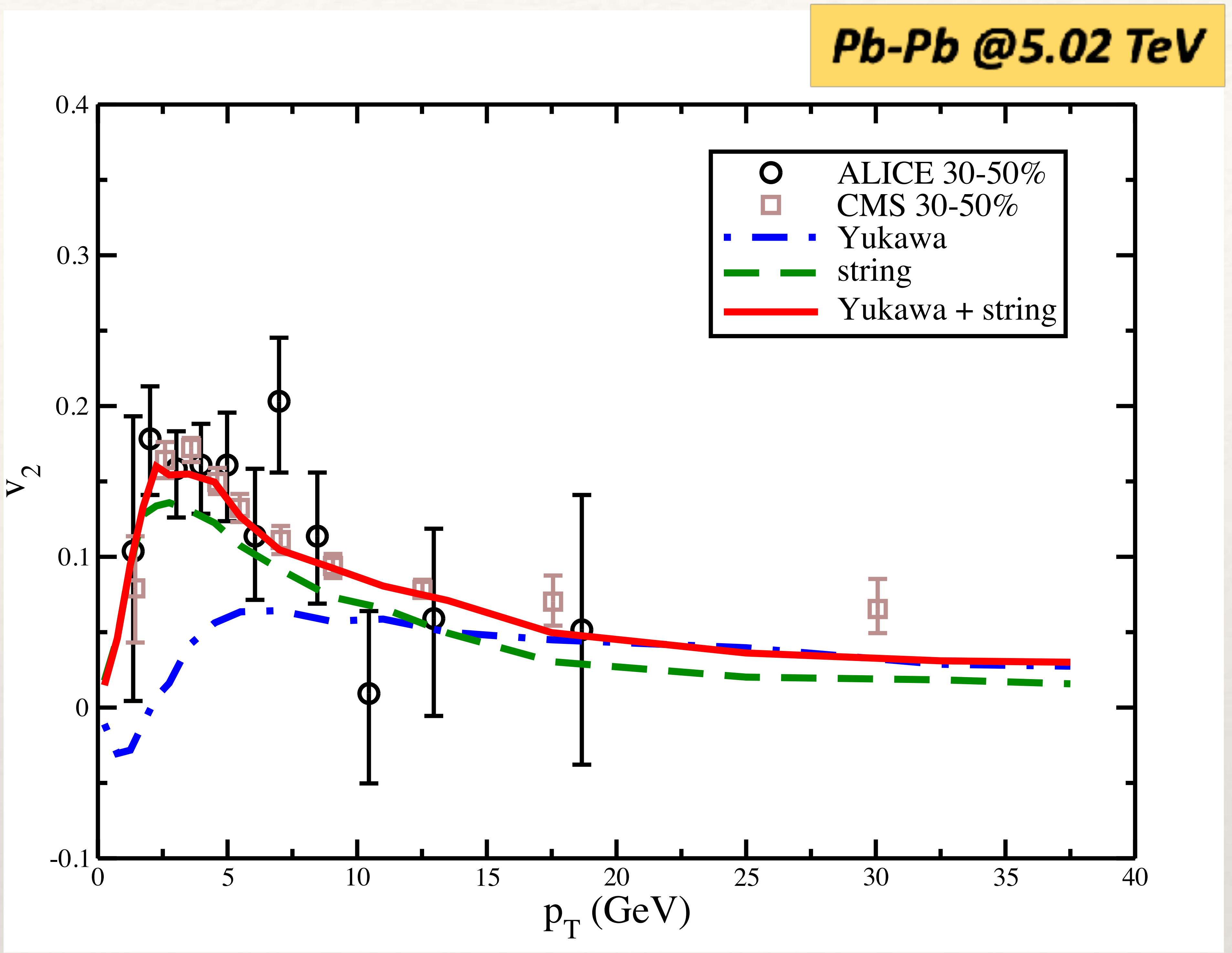
➢ **For $Qq \longrightarrow Qq$ process, we express the scattering amplitude with effective potential propagator,**

Riek and Rapp, Phys. Rev. C 82 (2010) 035201

$$iM = iM_C + iM_S = \bar{u}\gamma^{\mu}uV_c\bar{u}\gamma^{\nu}u + \bar{u}uV_S\bar{u}u$$

R_{AA} and v_2 of D mesons at LHC

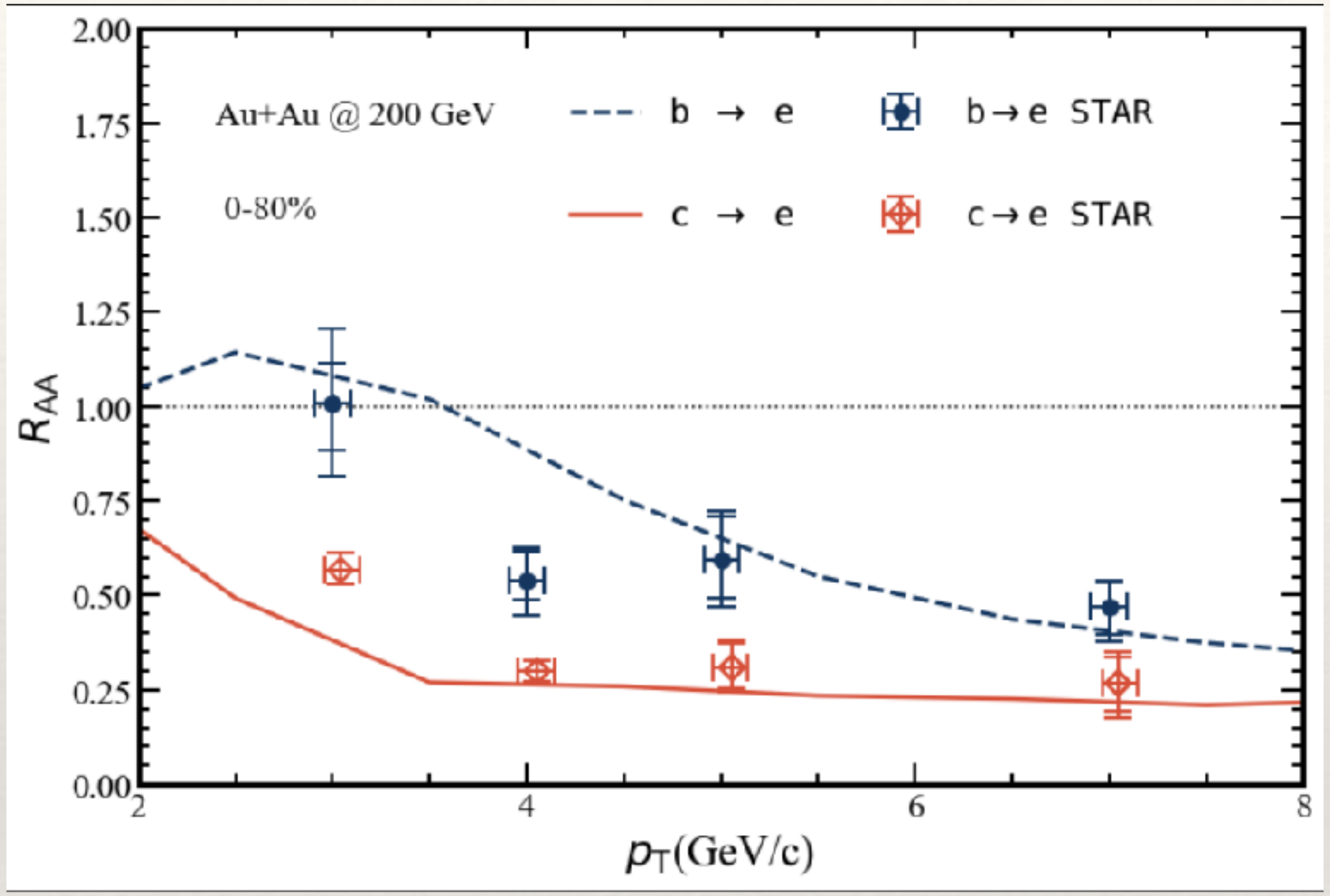


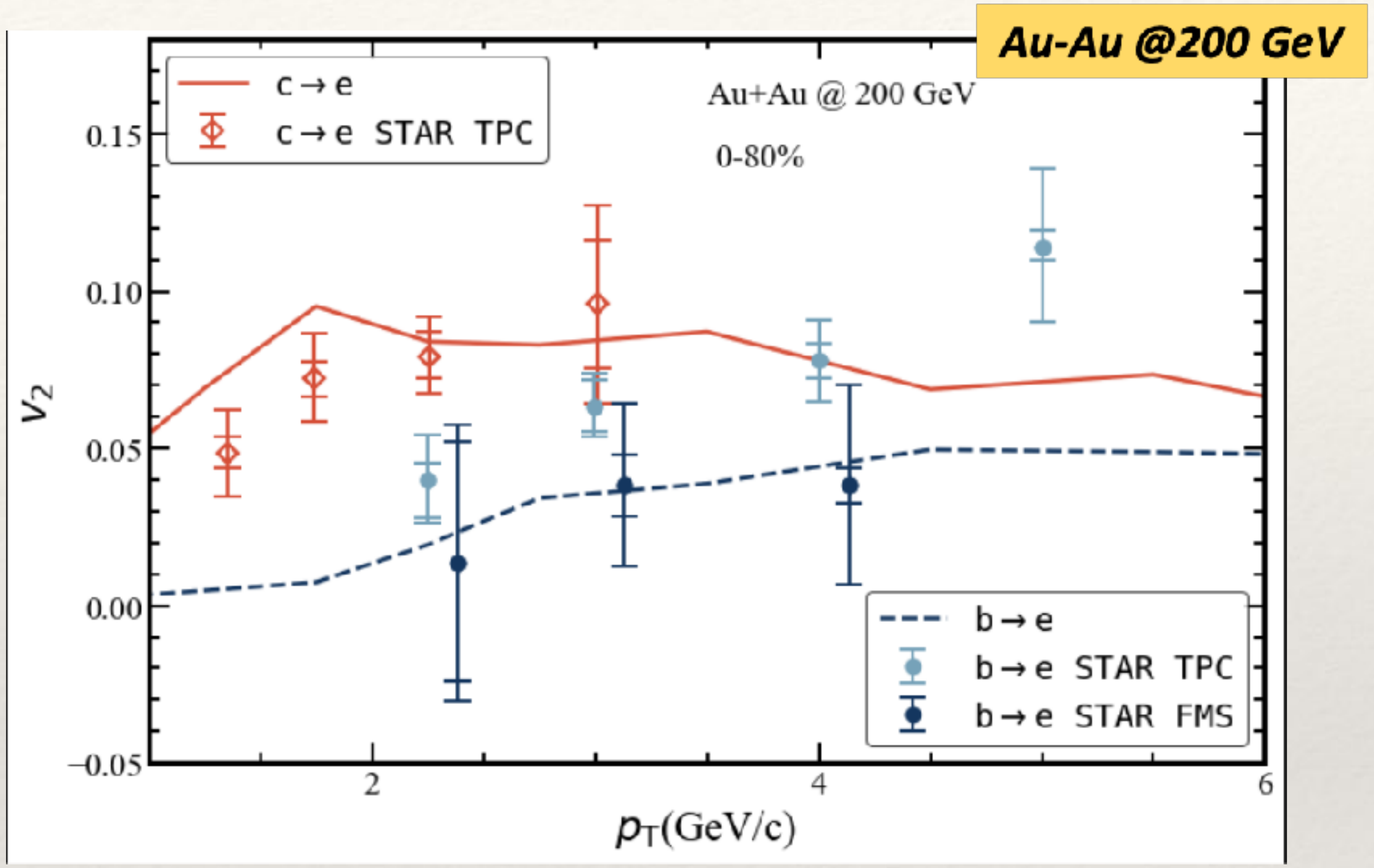


Xing, Qin, SC, Phys. Lett. B 838 (2023) 137733

- At high p_T, the Yukawa interaction dominates heavy-quark-medium interaction
- At low to intermediate p_T, the string interaction dominates, stronger contribution at later evolution stage (near T_c)

R_{AA} and v_2 of heavy flavor decayed electrons at RHIC



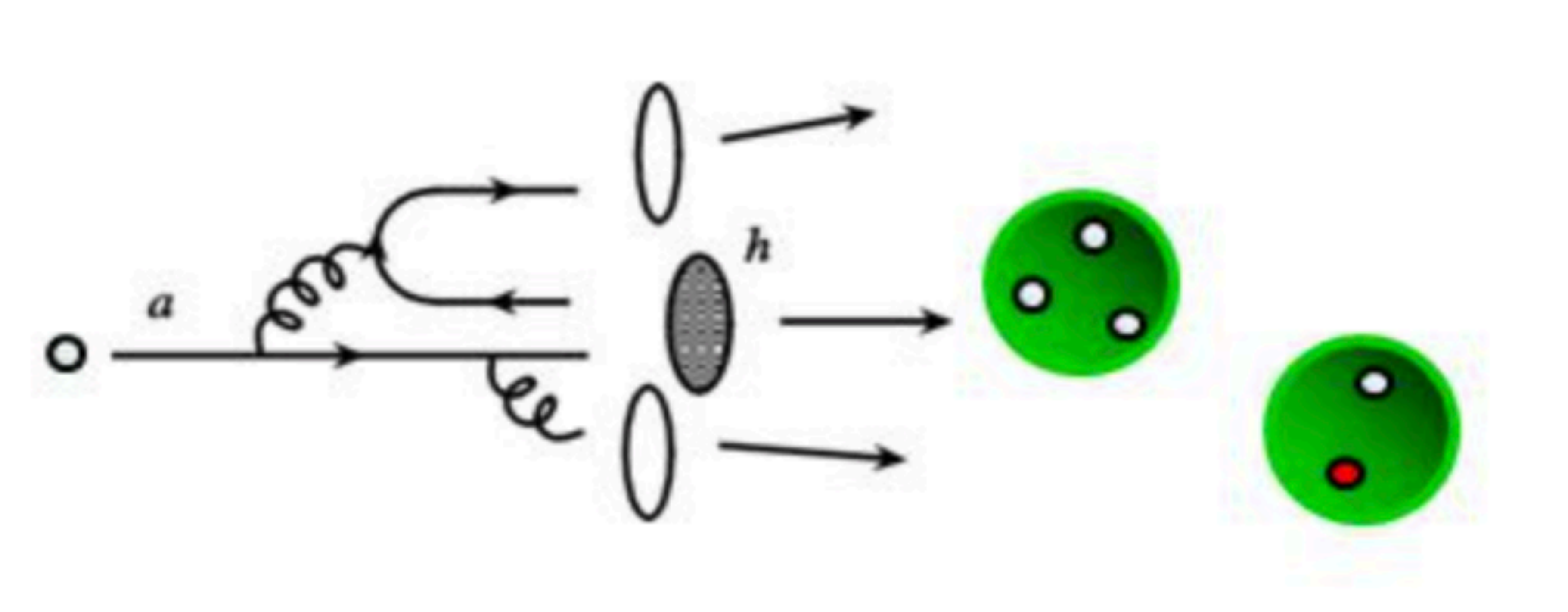


- Combination of short-range Yukawa and long-range string interactions provide a reasonable description of the *D* meson R_{AA} and v_2

Medium to low p_T hadrons — hadronization

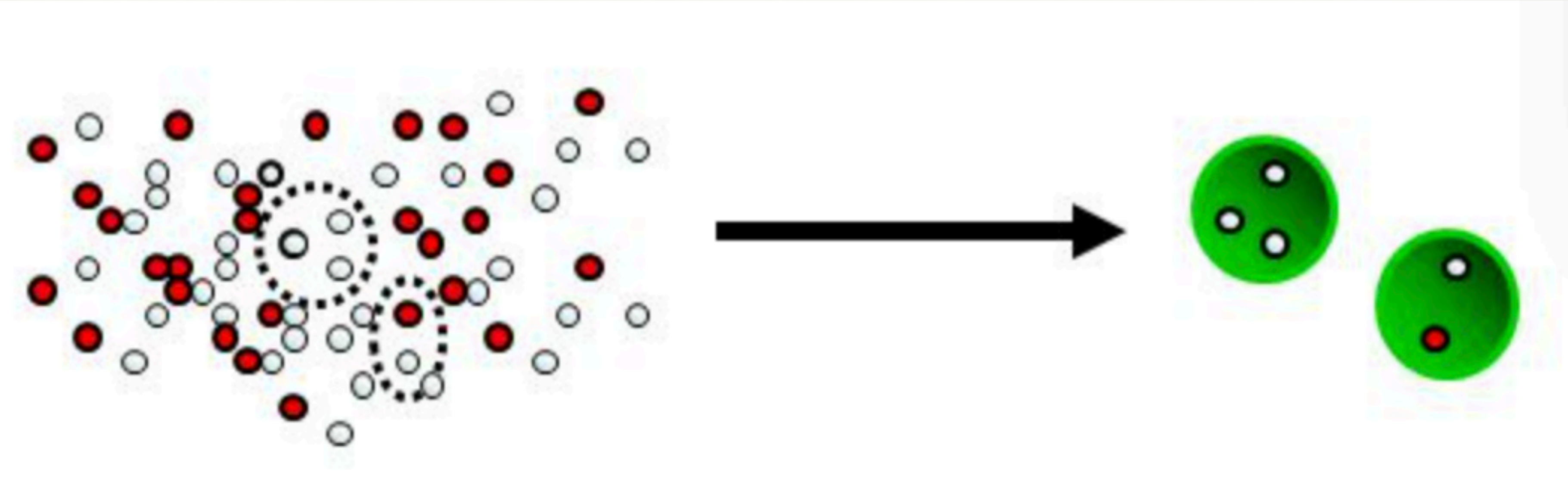
***Fragmentation*:**

High momentum heavy quarks are more likely to fragment into hadrons
[Peterson, FONLL, NLO, Pythia, etc.]



***Coalescence* (*recombination*):**

Low momentum heavy quarks are more likely to combine with thermal partons into hadrons



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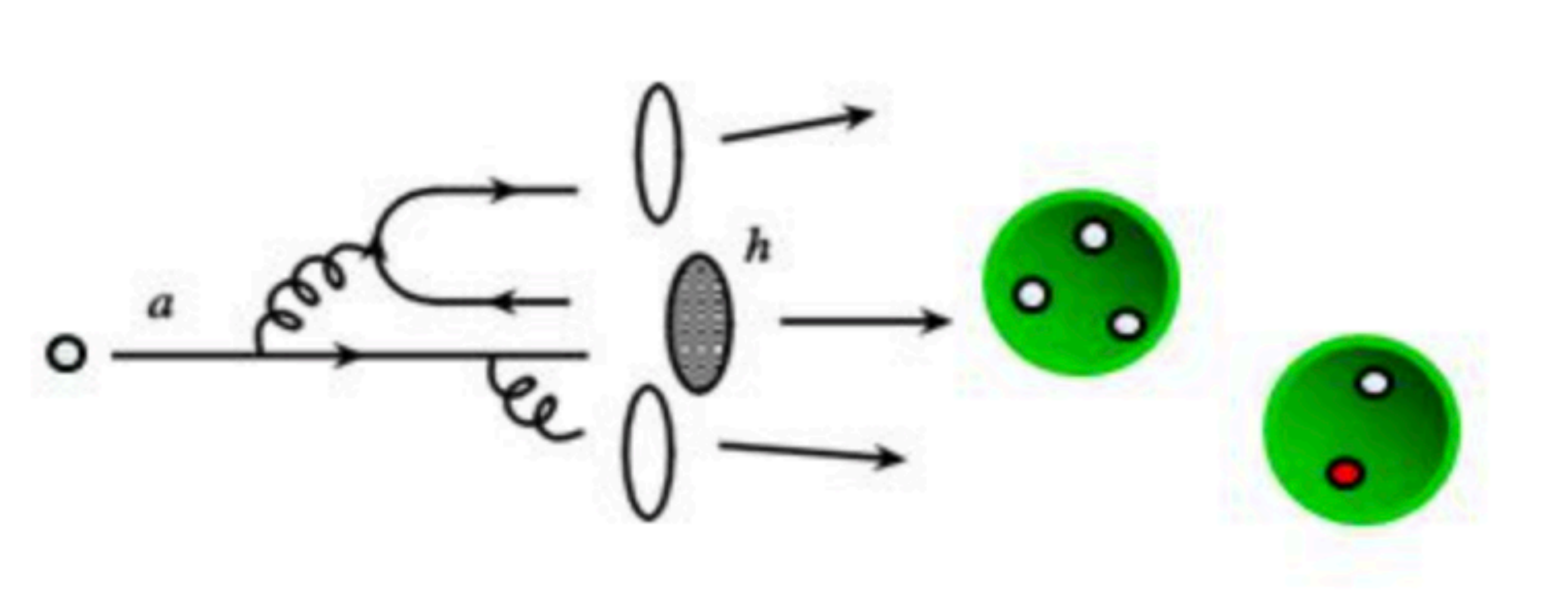

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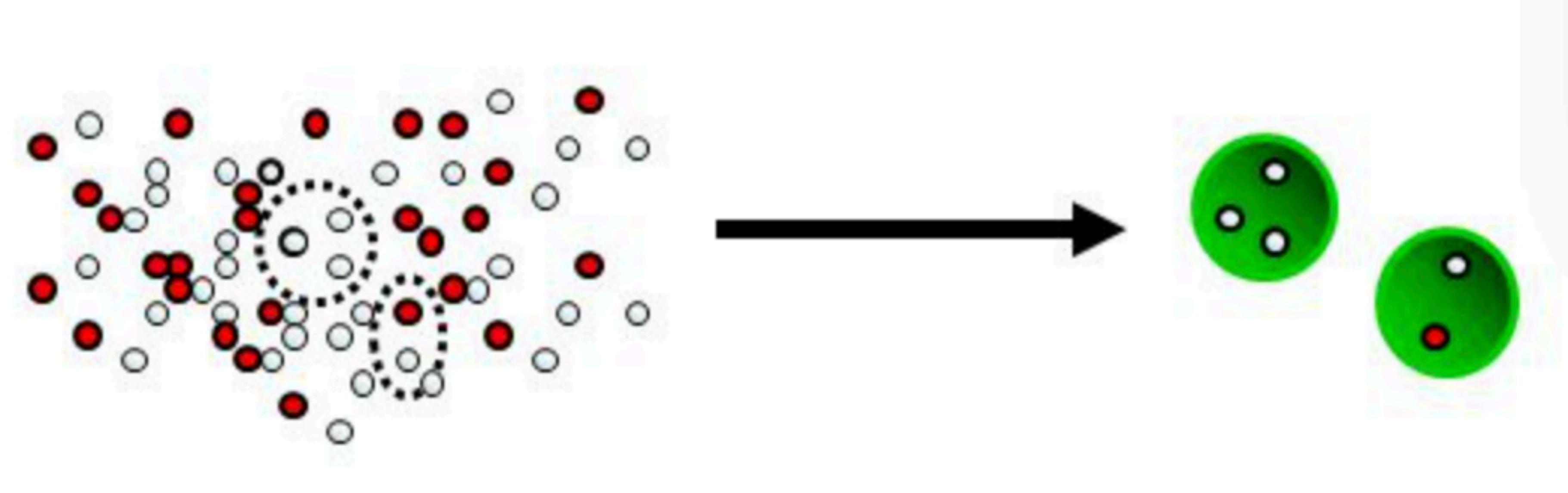
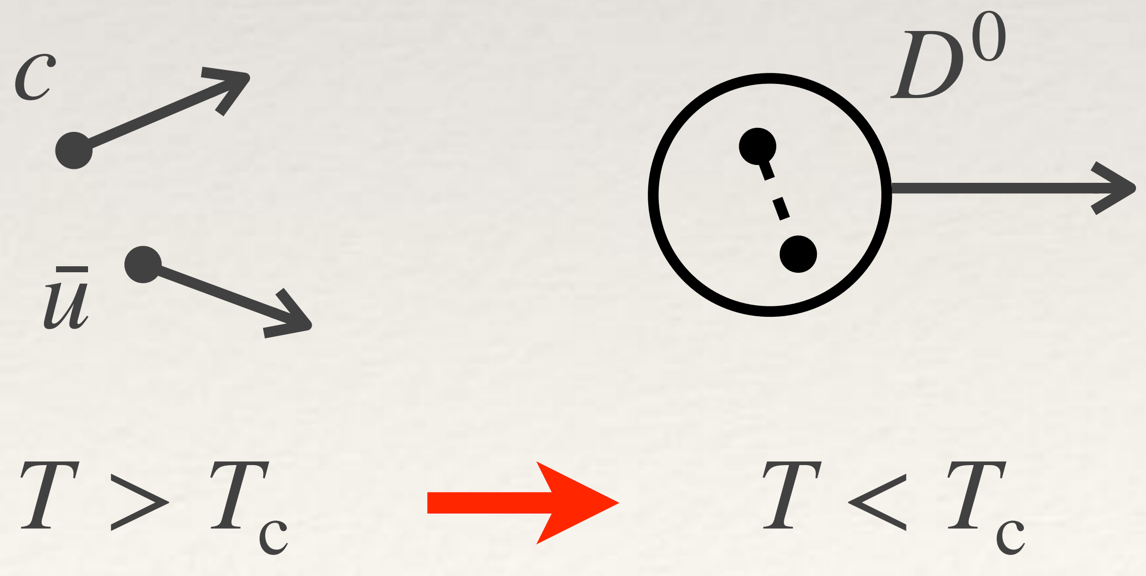


- Sudden approximation: $|q, g\rangle \rightarrow |h\rangle$ as T drops across T_c
- Probability: wave function projection $W_M \equiv |\langle M|q_1, q_2\rangle|^2$



Coalescence model

- Example: 2-body system for meson formation

$$W(\vec{r},\vec{k}) \equiv |\langle M|q_1,q_2\rangle|^2 = g_M \int d^3r' e^{-i\vec{k}\cdot\vec{r}'} \phi_M(\vec{r}+\vec{r}'/2)\phi_M^*(\vec{r}-\vec{r}'/2)$$

g_M: ratio of spin-color degeneracy between meson and quark states

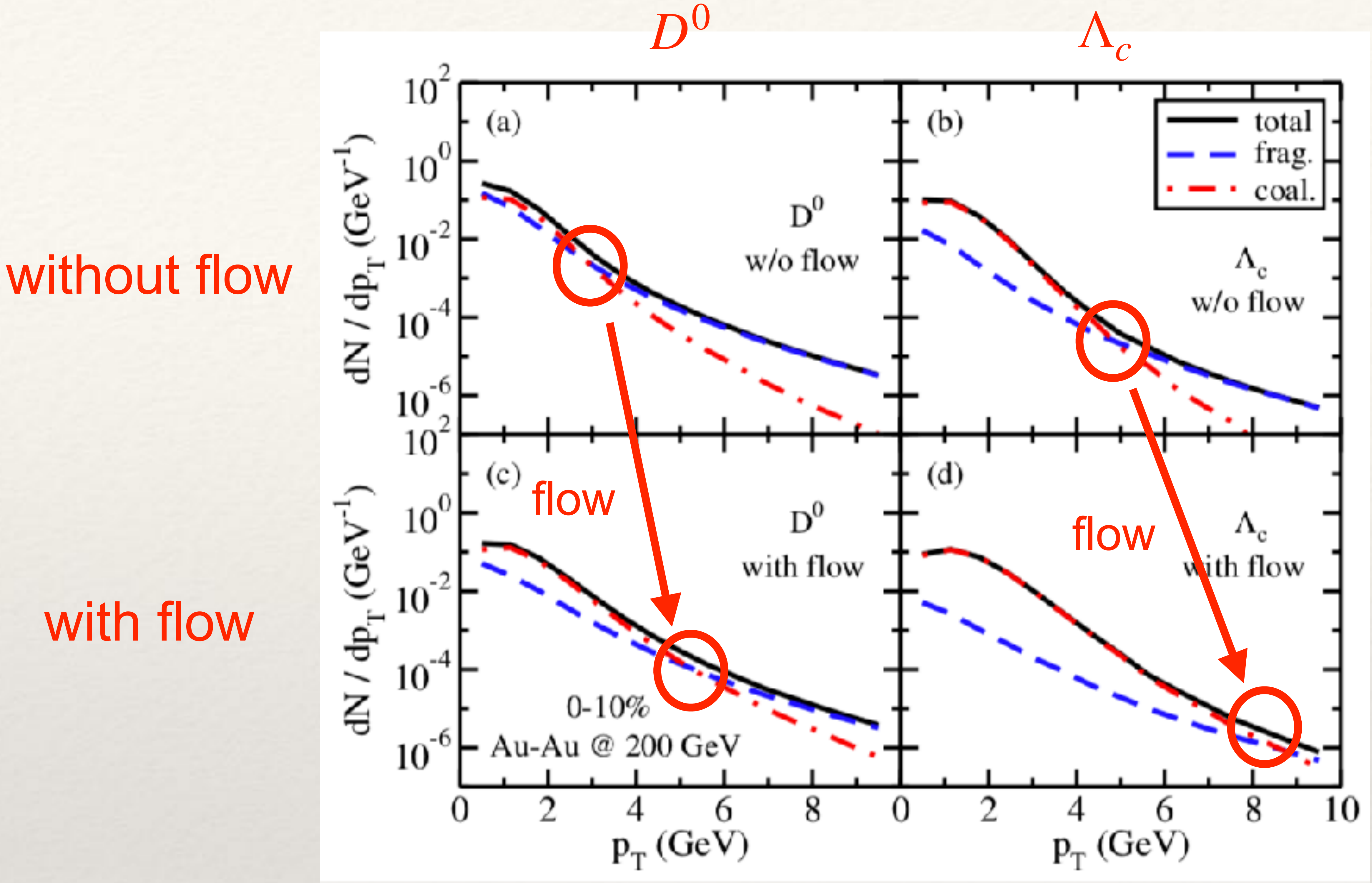
ϕ_M: meson wavefunction (S.H.O. approximation with a frequency parameter ω)

$$\vec{r} = \vec{r}_1' - \vec{r}_2' \qquad \vec{k} = \frac{1}{E_1' + E_2'}(E_2'\vec{p}_1' - E_1'\vec{p}_2')$$ (r' and p' defined in the meson rest frame)

- Momentum space Wigner function (after averaging over position space) for s and p wave ϕ_M:

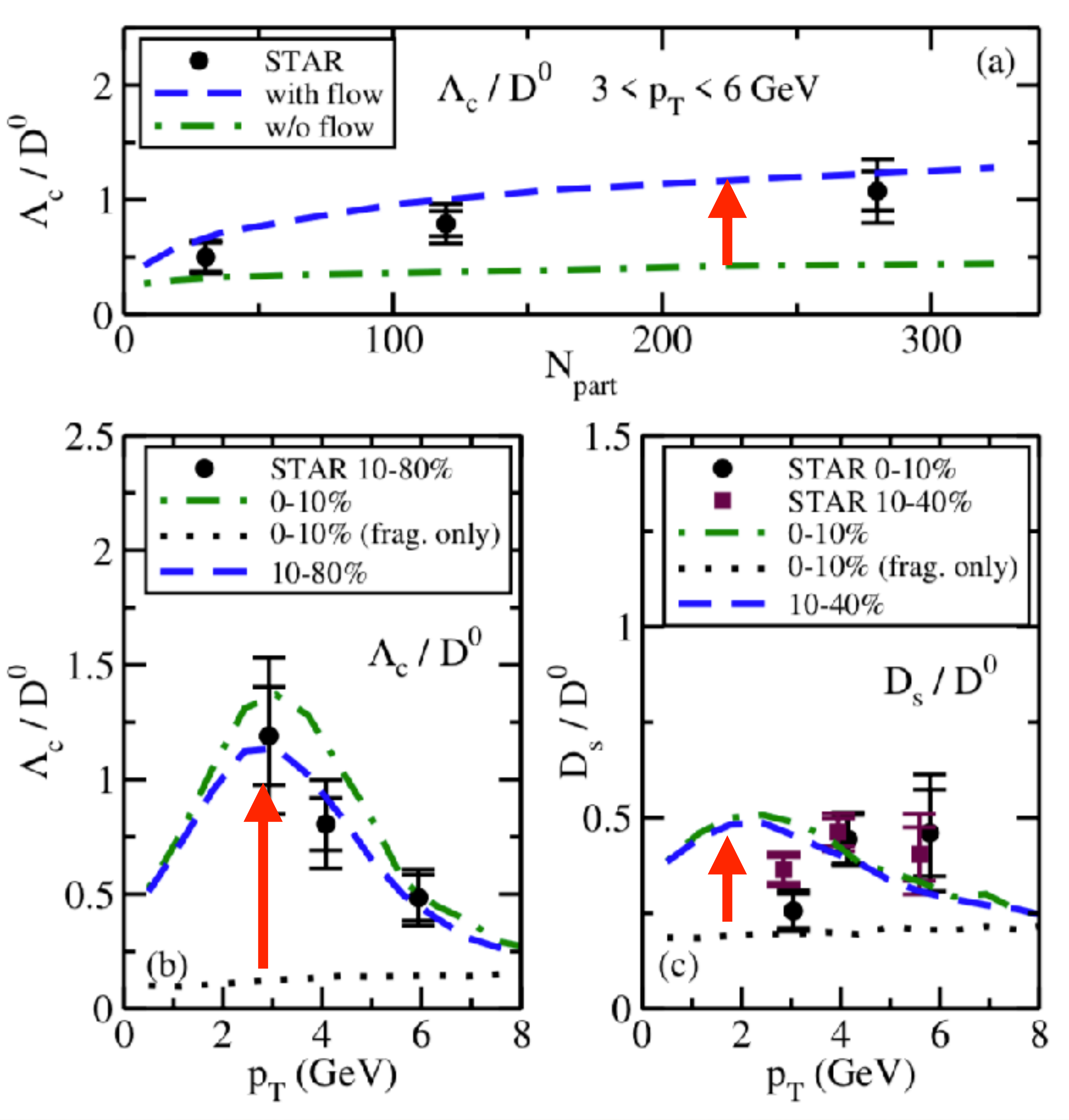
$$W_s = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2 k^2} \qquad W_p = g_M \frac{(2\sqrt{\pi}\sigma)^3}{V} \frac{2}{3}\sigma^2 k^2 e^{-\sigma^2 k^2}$$ ($\sigma = 1/\sqrt{\mu\omega}$, μ: reduced mass)

Charmed hadron spectra: QGP flow effect



- Coalescence dominates Λ_c production over a wider p_{T} region than D^0
- The QGP radial flow significantly enhances the coalescence contribution
- The inaccuracy of default Pythia fragmentation in pp should have minor effects on AA results, could be improved later (color reconnection [Velasquez et. al., PRL 111 (2013)], or coalescence in pp [Song, Li, Shao, EPJC 78 (2018); Minissale, Plumari, Greco, PLB 821 (2021)])

Charmed hadron chemistry at RHIC

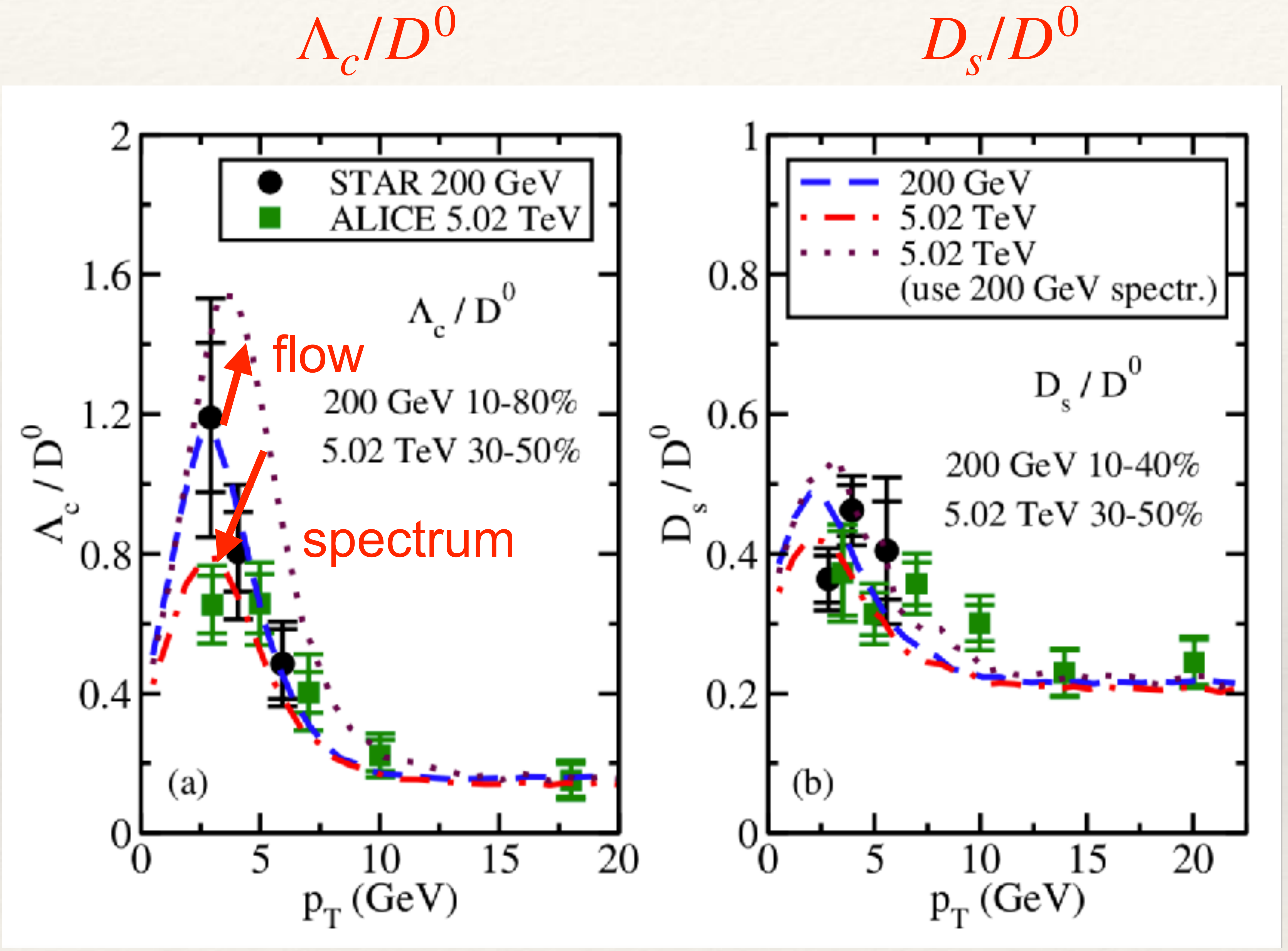


effects of the QGP flow

effects of coalescence

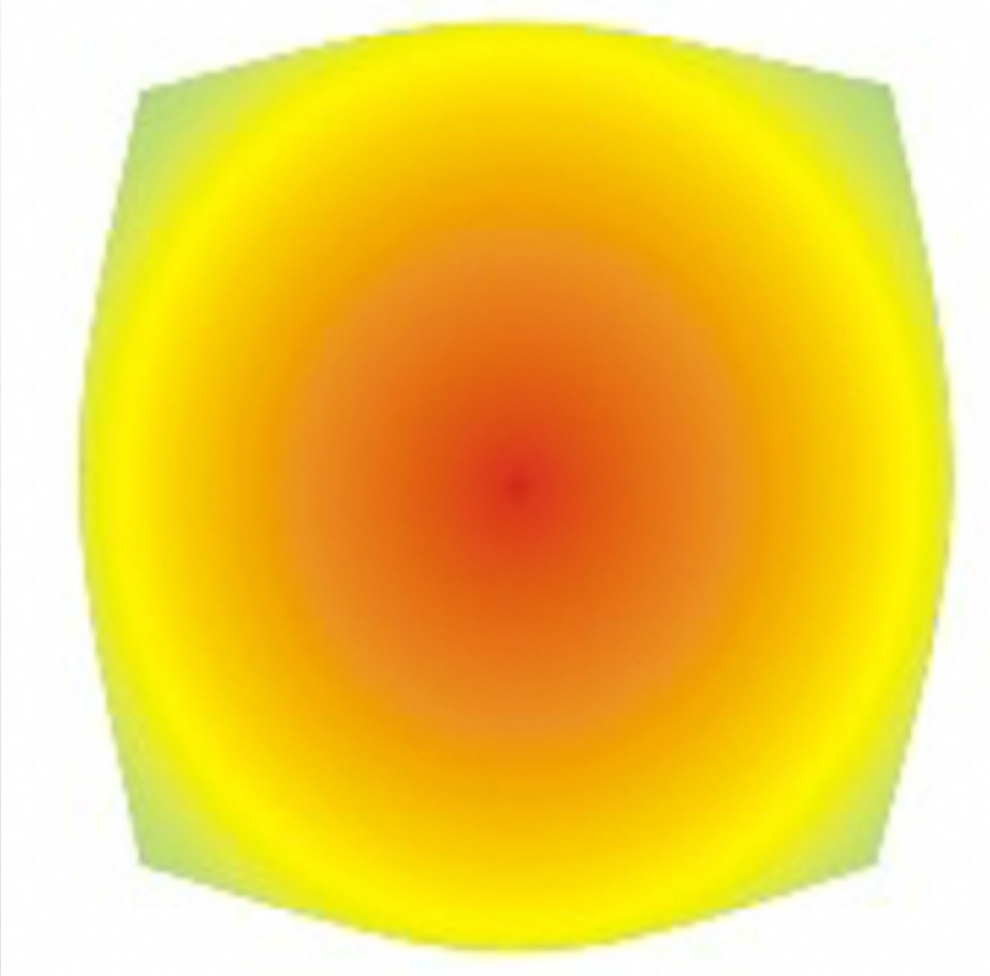
- (a) Stronger QGP flow boost on heavier hadrons => increasing Λ_c/D^0 with N_{part}
- (b) Coalescence significantly increases Λ_c/D^0, larger value in more central collisions (stronger QGP flow)
- (c) Enhanced D_s/D^0 due to strangeness enhancement in QGP and larger D_s mass than D^0

RHIC vs. LHC



- IF charm quarks have the same initial spectrum at RHIC and LHC, Λ_c/D^0 would be larger at LHC than RHIC due to the flow effect
- The harder initial charm quark spectra at LHC reduces Λ_c/D^0
- Similar theoretical prediction on D_s/D^0

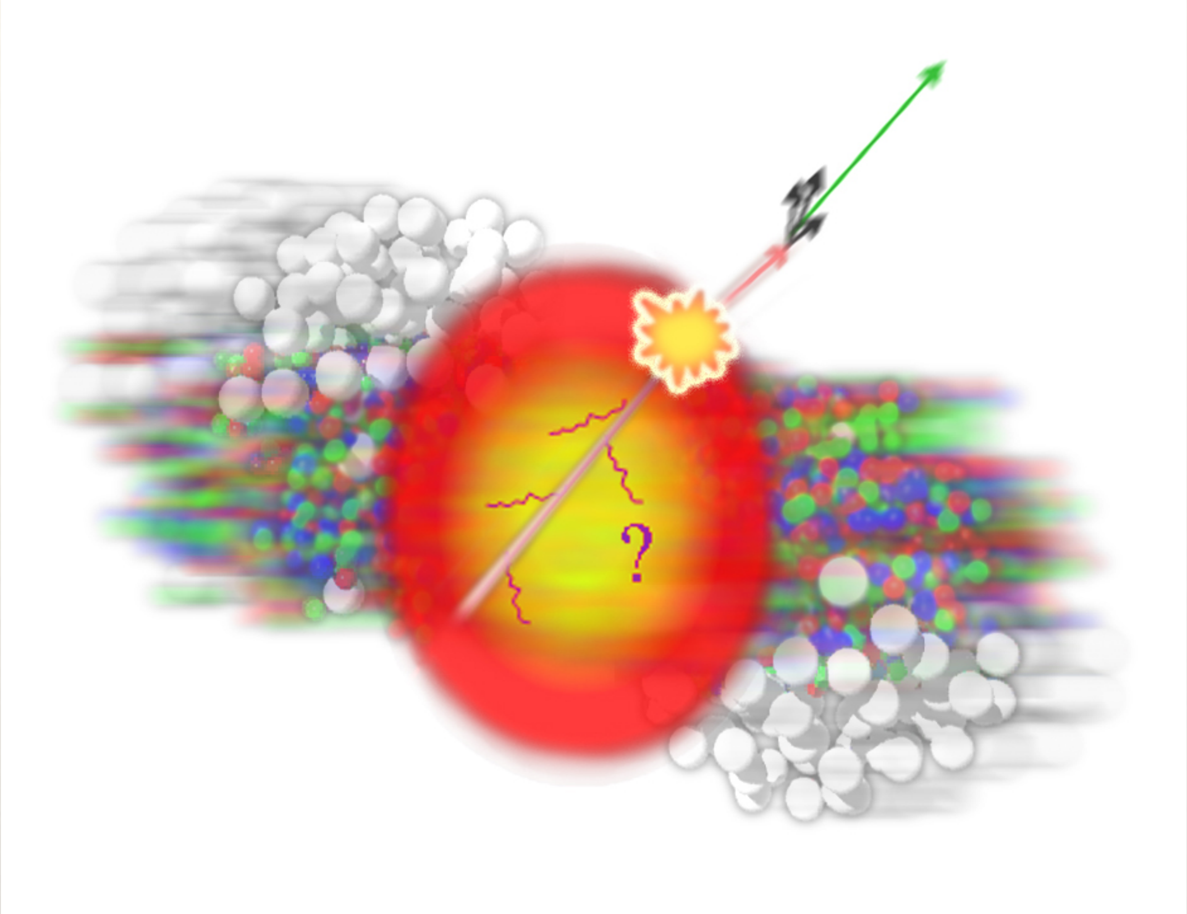
Probing the EoS of QGP



QGP

Usual conduct: fix QGP properties using soft hadron observables and study nuclear modification on hard particles

Inverse question: can we probe QGP properties using hard particle observables?



Hard probes through QGP

F.-L. Liu, W.-J. Xing, X.-Y. Wu, G-Y. Qin, SC, X.-N. Wang, Eur. Phys. J. C 82 (2022) 4, 350

F.-L. Liu, X.-Y. Wu, SC, G-Y. Qin, X.-N. Wang, arXiv:2304.08787

Connection between transport and EoS

Transport

$$p_a \cdot \partial f_a(x_a, p_a) = E_a(\mathscr{C}_a^{\rm el} + \mathscr{C}_a^{\rm inel})$$

Thermal mass of partons

$$m_g^2 = \frac{1}{6} g^2 \left[(N_c + \frac{1}{2} n_f) T^2 + \frac{N_c}{2\pi^2} \Sigma_q \mu_q^2 \right]$$

$$m_{u,d}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left[T^2 + \frac{\mu_{u,d}^2}{\pi^2} \right]$$

$$m_s^2 - m_{0s}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left[T^2 + \frac{\mu_s^2}{\pi^2} \right]$$

Strong coupling strength

$$g(E, T)$$

Equation of state

$$P_{qp}(m_u, m_d, ..., T) = \Sigma_{i=u,d,s,g} d_i \int \frac{d^3p}{(2\pi)^3} \frac{|\vec{p}^2|}{3E_i(p)} f_i(p) - B(T)$$
$$= \Sigma_i P_{kin}^i(m_i, T) - B(T)$$

$$\epsilon = T dP(T)/dT - P(T), \qquad s = (\epsilon + P)/T$$

Strategy:

Fit g from comparing transport model to data

Calculate EoS from g

Parametrization and Bayesian analysis

Strong coupling strength

Interaction between thermal partons (thermal scale):

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\frac{(aT/T_c + b)^2}{1+ce^{-d(T/T_c)^2}}\right]}$$

Interaction with hard partons (parton energy scale):

$$g^2(E) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[(AE/T_c + B)^2\right]}$$

Parameters: $\theta = (a, b, c, d, A, B)$

Bayes Theorem

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\text{data}|\boldsymbol{\theta})P(\boldsymbol{\theta})$$

posterior distribution

prior distribution

model-to-data comparison

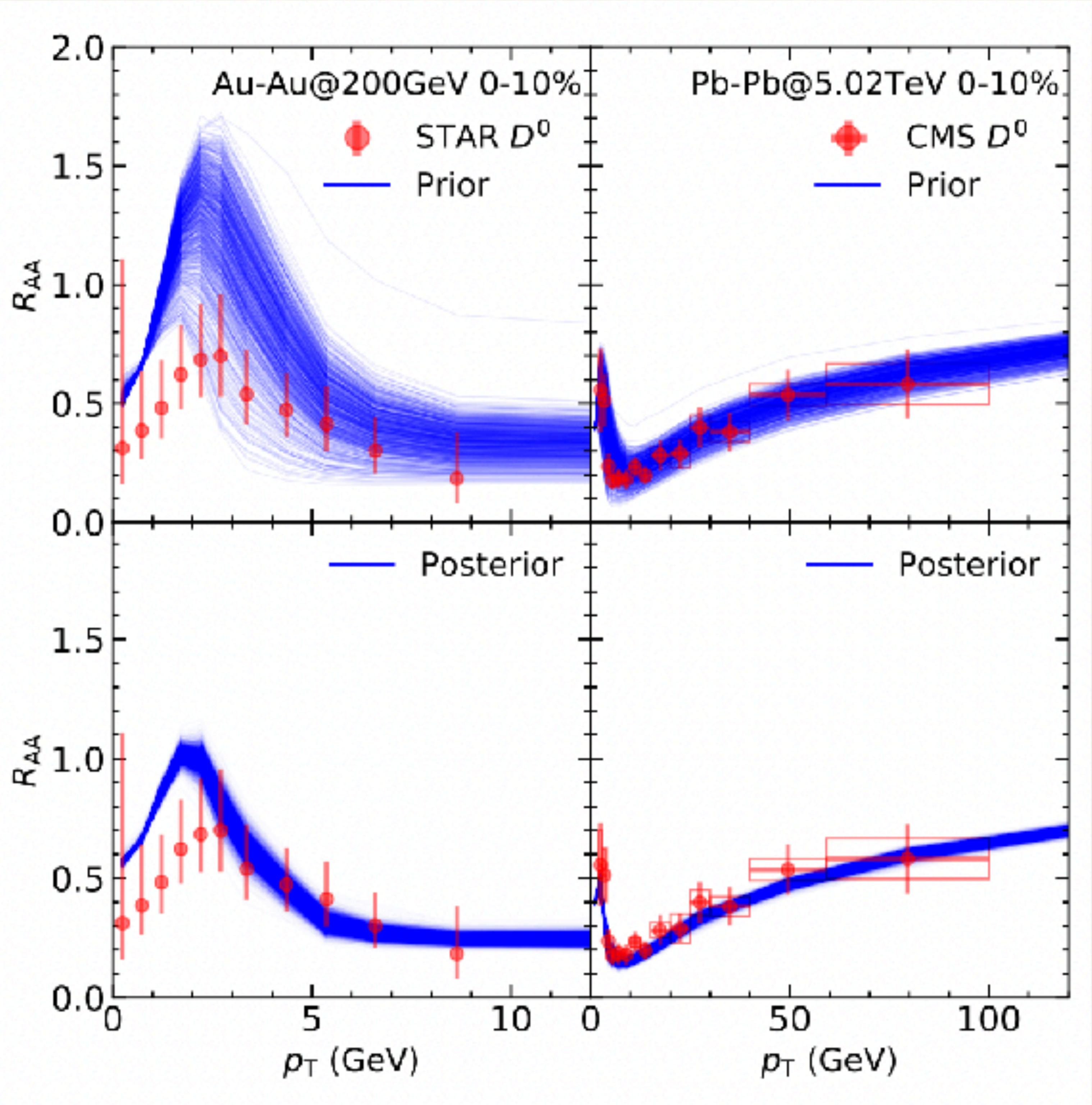
$$P(\text{data}|\boldsymbol{\theta}) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{[y_i(\boldsymbol{\theta}) - y_i^{\exp}]^2}{2\sigma_i^2}}$$

Model calibration and parameter extraction

Calibration against observables

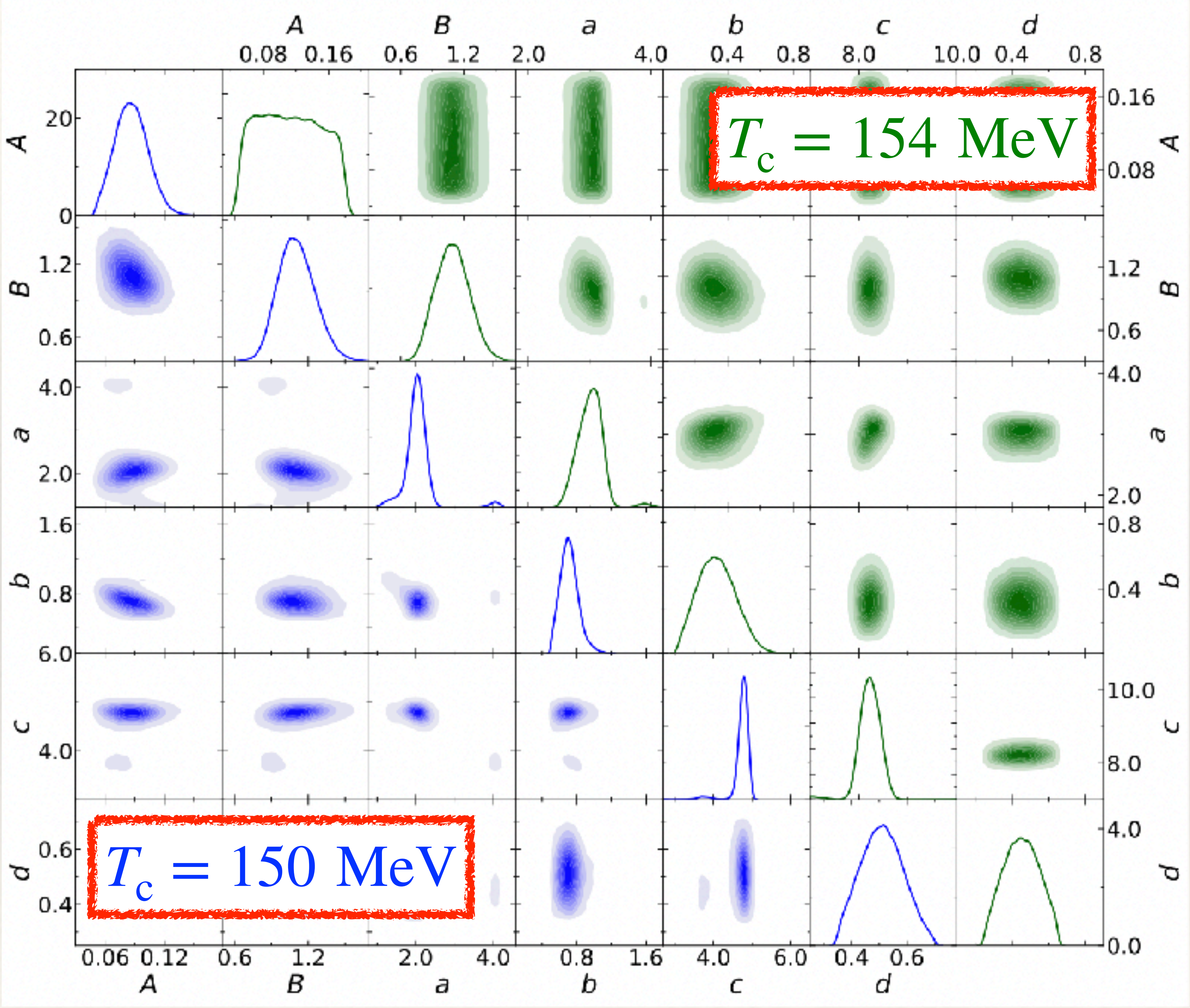
prior

posterior



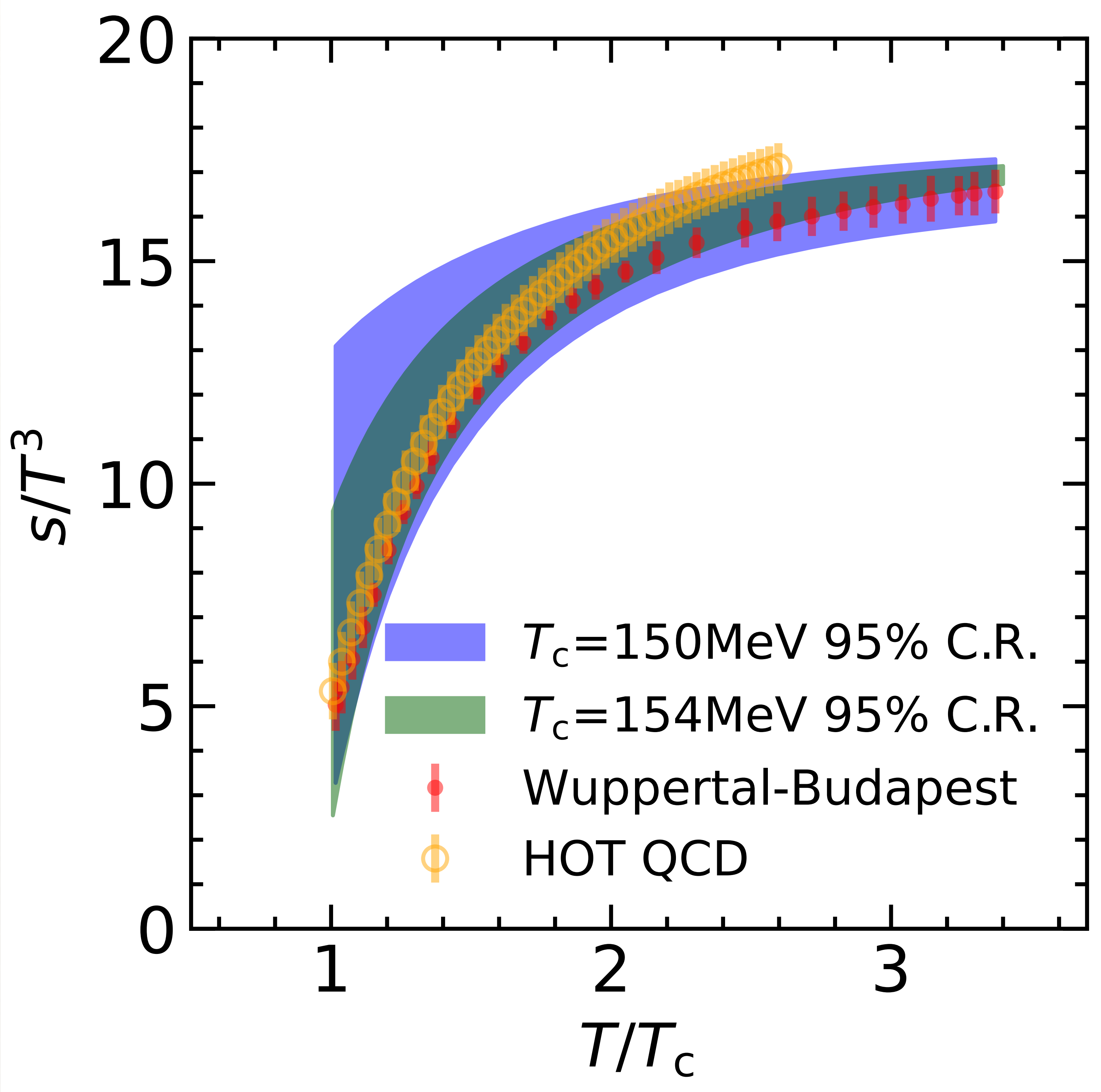
(Two examples from many observables)

Extraction of model parameters

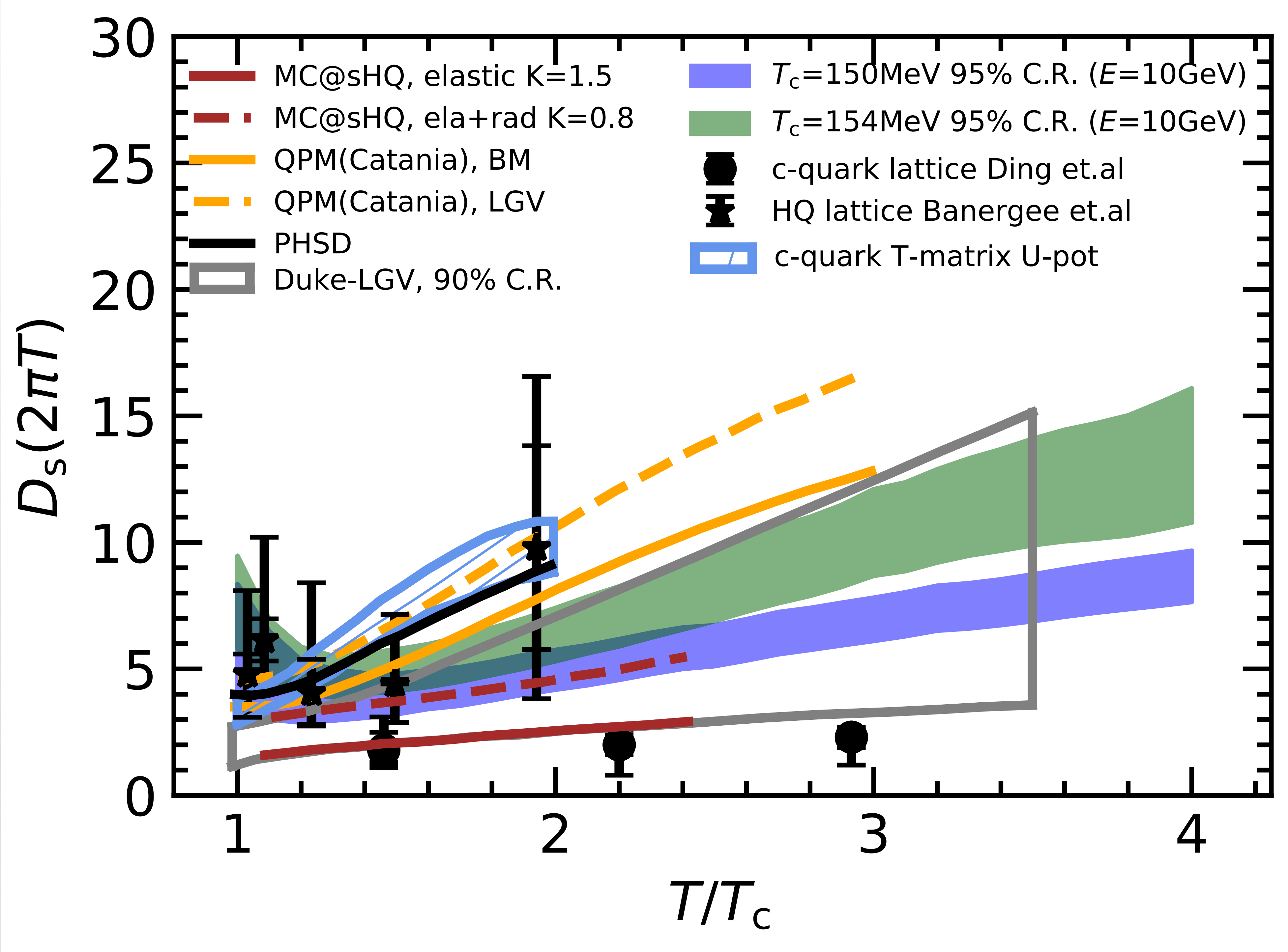


EoS of QGP and diffusion coefficient of heavy quarks

Equation of state

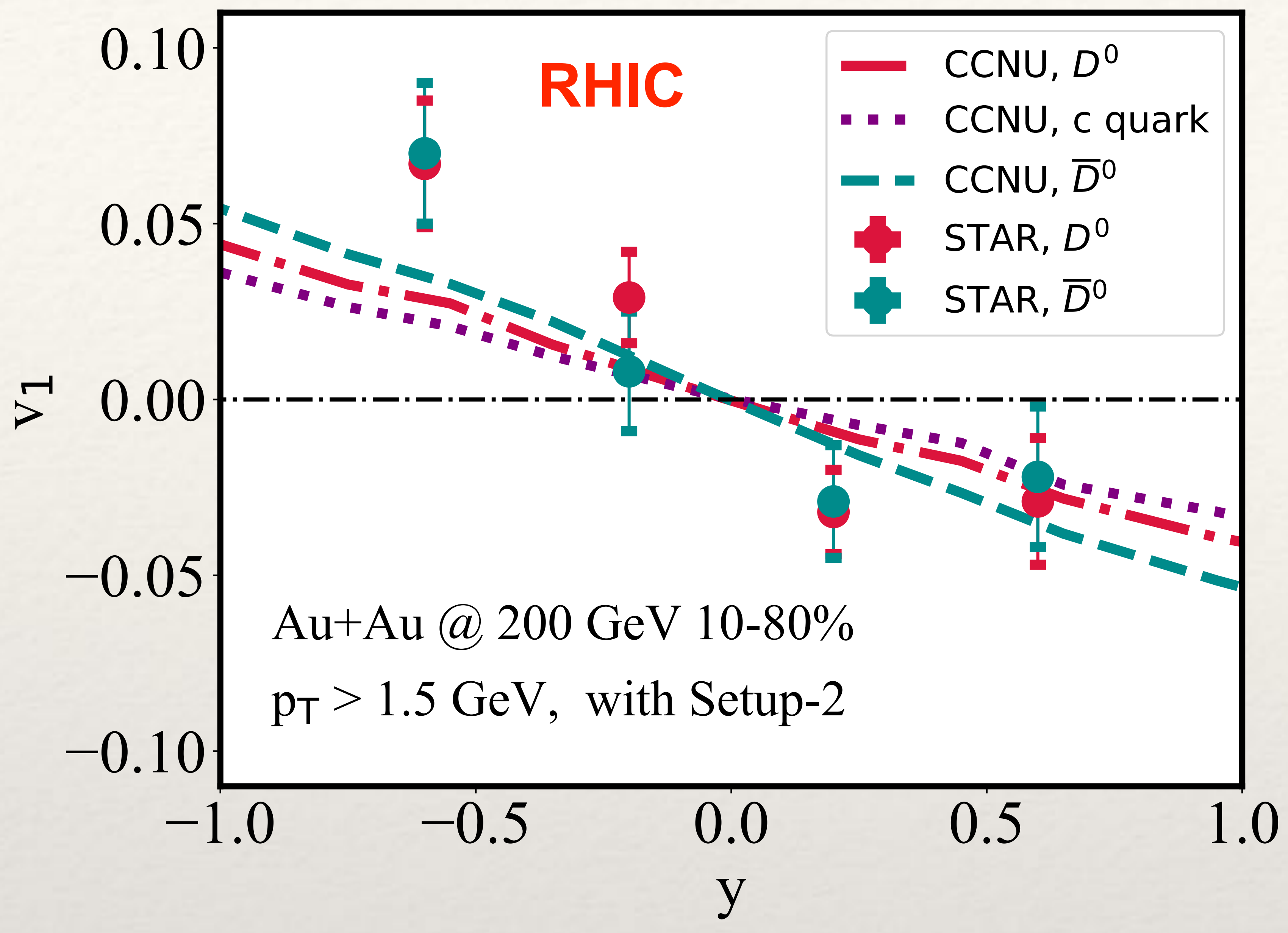


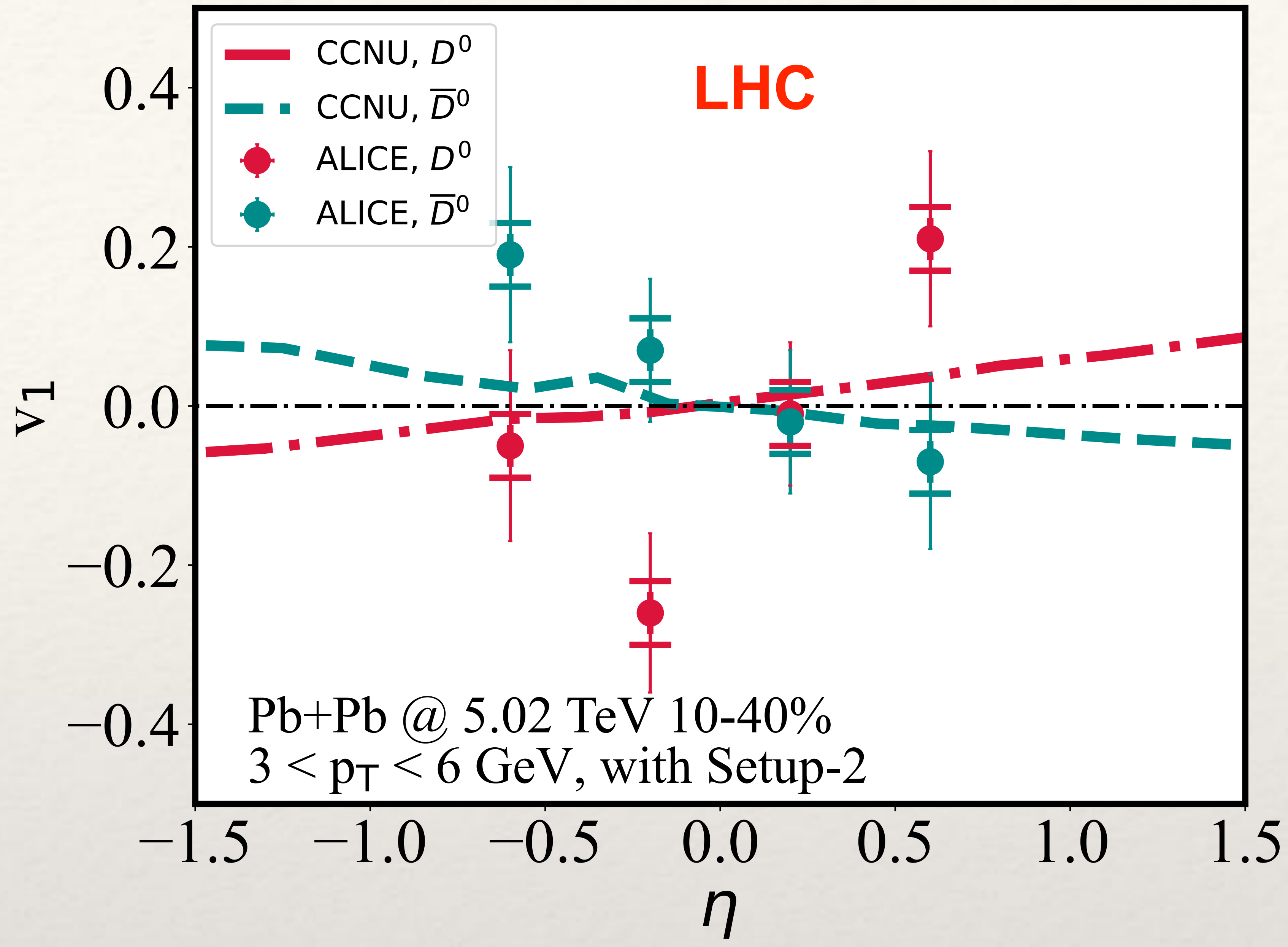
Diffusion coefficient



- Agreement with the lattice data
- Simultaneous constraint on QGP properties and transport properties of hard probes

Probing medium geometry and *E&M* field with heavy flavor v_1





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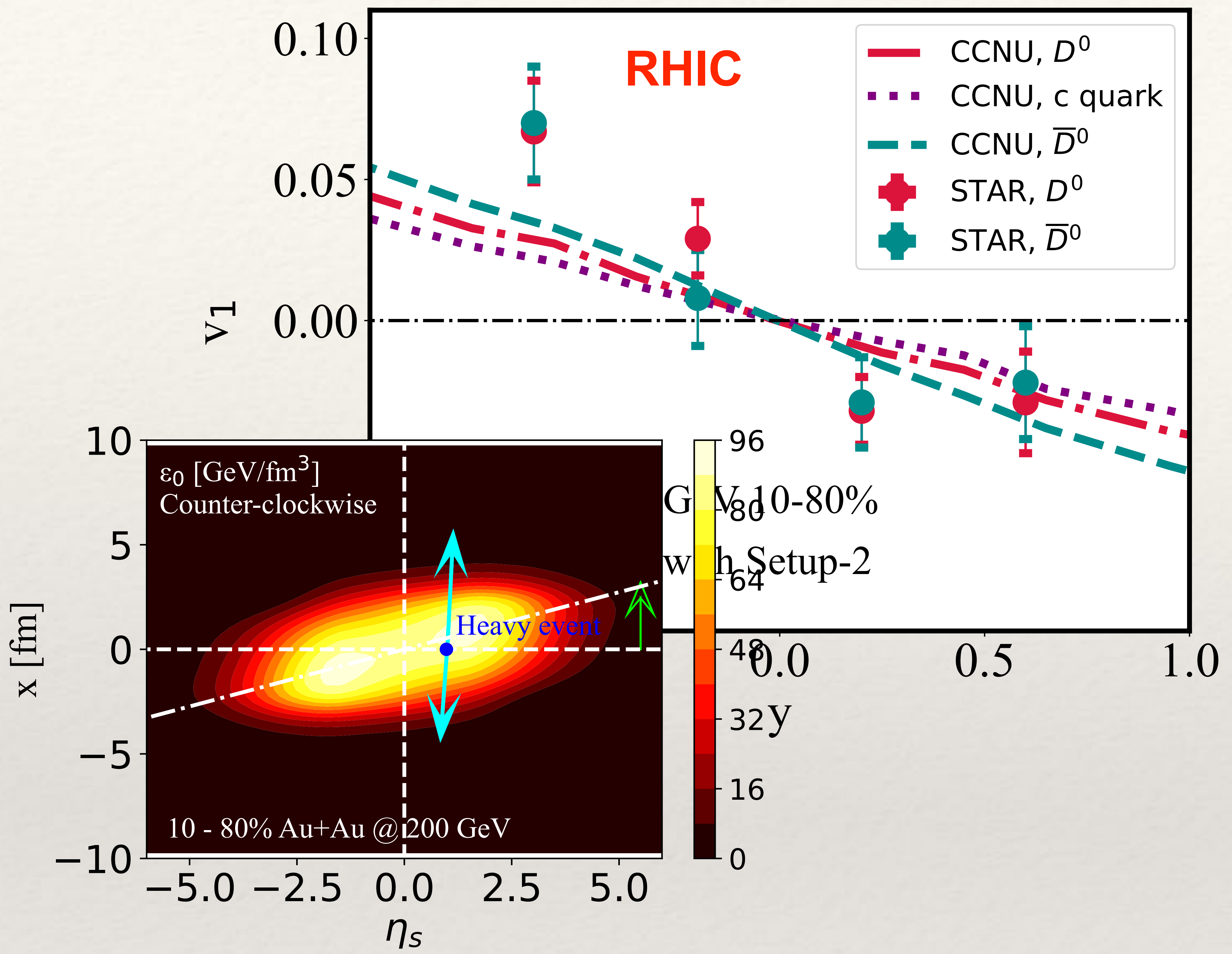
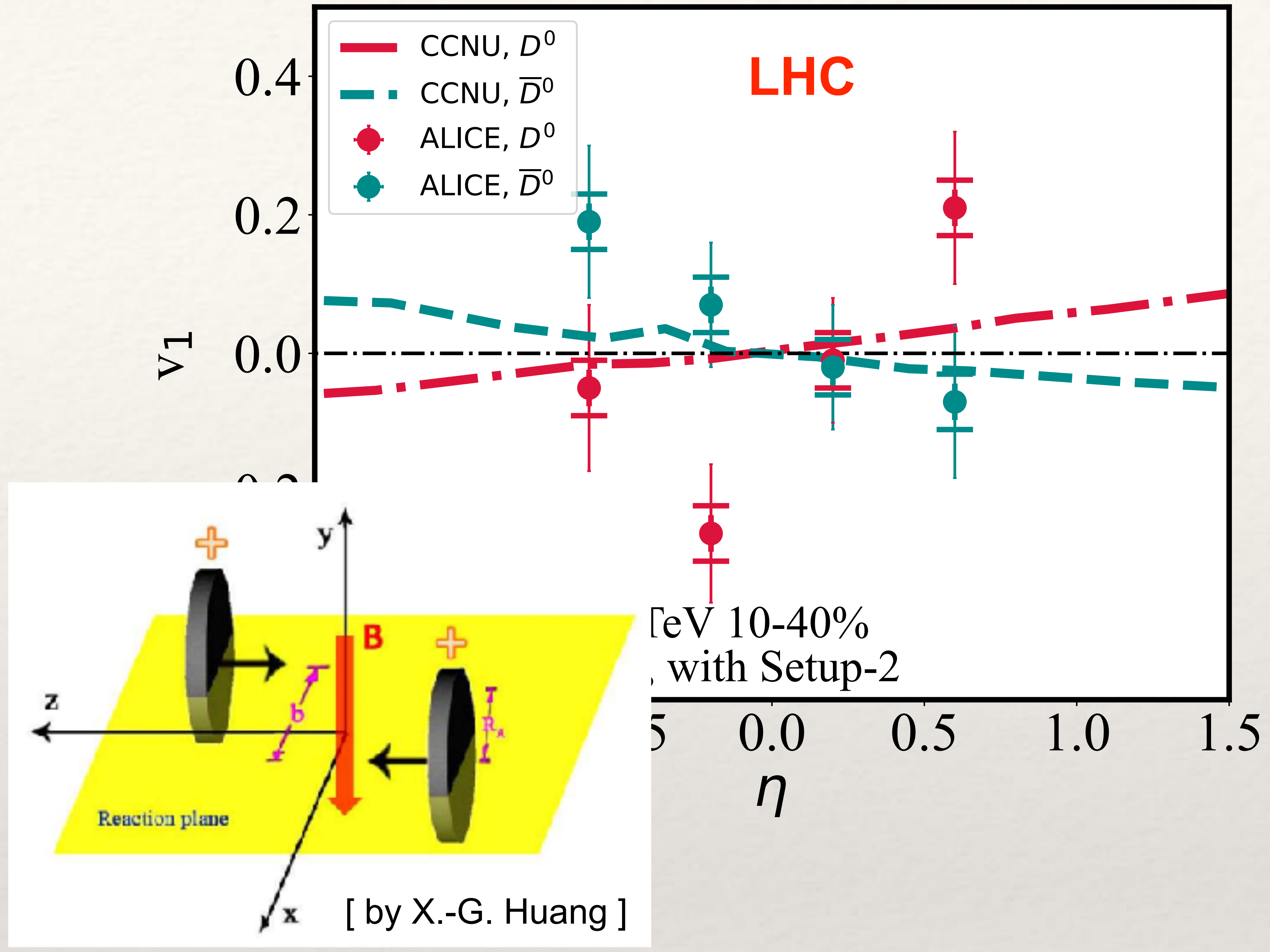
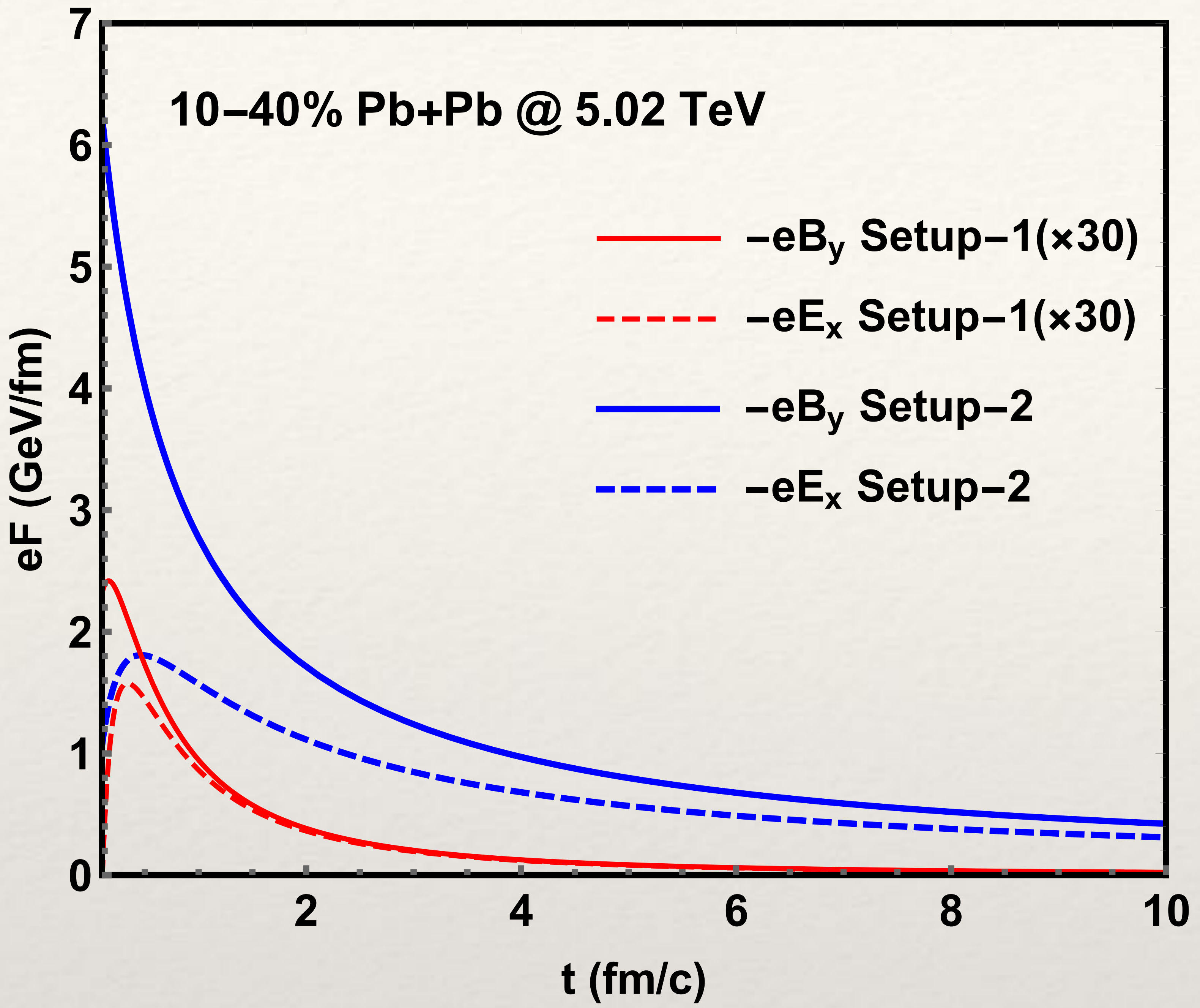


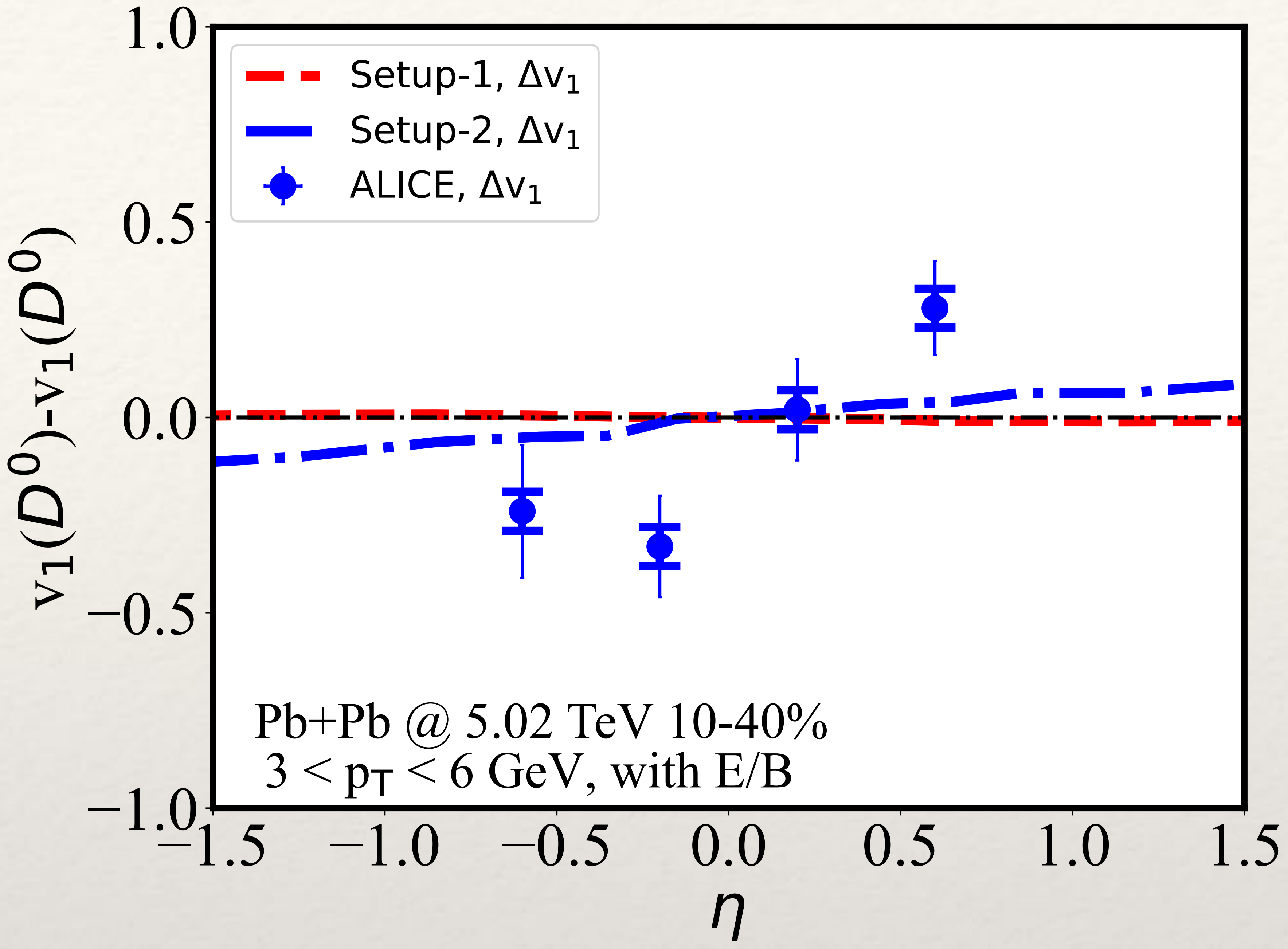


- Tilted geometry w.r.t. the beam direction dominates at the RHIC energy
- Strong E&M field dominates at the LHC energy

[Jiang, SC, Xing, Wu, Yang, Zhang, Phys. Rev. C 105 (2022) 5, 054907]

Probing evolution profiles of the *E&M* field





- Compare two model calculations of *E&M* field
  - Setup 1: Direct solution of Maxwell equation with constant electric conductivity $\sigma = 0.023\ \mathrm{fm}^{-1}$
  - Setup 2: Model $B_y(\tau) \sim B_y^{\mathrm{vac}}(0)/(1+\tau/\tau_B)$, then solve E_x from B_y with Maxwell equation
- Δv_1 data favor larger magnitude of B_y than E_x → guide improvement for *E&M* calculation

Summary

Heavy-quark-QGP interaction at different p_T and in different collision systems

- pQCD is sufficient to describe flavor hierarchy of jet quenching above 8 GeV
- Color potential interaction improves model calculation at low to medium p_T
- Coalescence + fragmentation hadronization is crucial for understanding hadron chemistry at low to medium p_T
- Heavy flavor observables can be used to constrain the EoS of QGP
- Heavy quark v_1 probes medium deformation at RHIC, while *E&M* field at LHC