State space of a black hole and soft hair

B. Cvetković* and D. Simić†
Institute of Physics, University of Belgrade, P. O. Box 57, 11001 Belgrade, Serbia

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In this article, we reflect on a problem of a black hole entropy. The main point of the article is that the black hole horizon should be treated as a boundary as well as the boundary at infinity. To make things more concrete, we apply the general ideas to the extremal Oliva-Tempo-Troncoso black hole and construct the corresponding Hilbert space using near-horizon hair. After creating the state space by using the proposed construction, we identify the natural candidates for the microstates responsible for the black hole entropy. The correct value of the black hole entropy is reproduced by counting the number of distinct microstates and applying the Boltzmann formula.

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I. INTRODUCTION

The origin of a black hole entropy is one of the most important open problems in physics. There are many approaches to the problem of black hole entropy, most of which rely heavily on the algebra of asymptotic symmetries [1–9].

The recognition that symmetries near the horizon play an important role in understanding black holes has long been known [9,10]. This idea is further refined for the case of extremal black holes and goes by the name Kerr/CFT (conformal field theory) [8]. New insight, that appeared a few years ago, is that black holes have soft hair [4]; the first specific realization of this idea can be found in Refs. [1–3], in which soft hair microstates, known as fluffs, are constructed.

There is also an approach which suggests that near-horizon Virasoro algebra and 2D CFT are underlining the dynamics of four-dimensional black holes [5,6] similar to asymptotic Virasoro in the three-dimensional case.

These approaches have in common that they all are, in the end, trying to give a better understanding of a black hole entropy. The (definite) solution of the black hole entropy problem is a construction of the Hilbert state space of a black hole. When the Hilbert state space is at our disposal, we can count the number of microstates which correspond to the same macrostate, and after using the Boltzmann formula, we will derive black hole entropy. Because full treatment requires formulation of consistent theory of quantum gravity, we can only hope to obtain semiclassical understanding of microstates. This would be very valuable because it will answer two important questions. The first question can be formulated as follows: is there such thing as quantum gravity? The second question is as follows: are there any microstates underlying black hole entropy, or is something fundamentally wrong with our current understanding of black holes? Indirect affirmation to both of these questions is already obtained through AdS/CFT. Nonetheless, indirect verifications via AdS/CFT cannot substitute direct construction and insights it, possibly, provides. This is, exactly, the problem we will (try to) address in this work.

In the following section, which contains the main ideas of the paper, we will motivate why a black hole horizon should be treated as a true boundary and some related questions. In the next section, we will review the necessary results, which will be used later. After that, we consider the algebras of asymptotic and near-horizon symmetries. The next section is devoted to construction of a state space of an extremal Oliva-Tempo-Troncoso (OTT) black hole, using results of the previous sections and deriving entropy by counting the microstates. In the end, we summarize the results of the paper.

II. TWO BOUNDARIES

In this section, we motivate why we have to acknowledge the black hole horizon as the real boundary and obtain some general conclusions about the factorization of phase space. We will focus on black hole created by collapse of matter, as it is expected to be the only physically realistic situation.

The black hole is distinguished by the presence of the event horizon, which divides space-time into two parts, the interior and exterior of the black hole, the horizon being the dividing surface. Classically, nothing cannot escape from the interior, which means that the event horizon is a null surface. In the standard coordinates, the horizon is located in $r = r_0$, and the fact that horizon is a null surface with normal

$$n^\mu = g^{r\mu}, \tag{2.1}$$

*cbranislav@ipb.ac.rs
†dsimic@ipb.ac.rs

where $g^{\mu\nu}$ is the inverse metric, gives the constraint on the metric

$$n_\mu n^\mu = g^{rr}(r = r_0) = 0. \tag{2.2}$$

The black hole is created the moment enough matter is inside of the area $r < r_0$. We will focus on pure gravity outside of the horizon; consequently, all matter must be inside of the horizon. The important point is that we know nothing about the distribution of matter inside of the black hole. The standard black hole solutions assume that all mater is compressed in $r = 0$, which is the origin of singularity. Singularities are physically unappealing, but, as stated, their origin lies in the additional assumption that all the matter is compressed into a point. This assumption, implicitly, assumes quite a lot about the short-distance behavior of all interactions including gravity. As is well known, short-distance behavior is governed by quantum gravity, about which we know very little. In light of this, it is better to not get into the details of matter distribution inside of the black hole; it is questionable if this is even possible. This discussion suggests that we should treat the event horizon as a boundary, at least until a satisfactory theory of quantum gravity is established. Immediate implication is that, besides boundary conditions at infinity, we need to specify them, as well, at the horizon.

Now, the question arises as to what features boundary conditions on the horizon must have. We will answer this question in the Hamiltonian formulation. Basic operation in the Hamiltonian formalism is the Poisson bracket

$$[A, Q]; \tag{2.3}$$

as usual, this contains the implicit assumption that functional derivatives of variables are well defined. The procedure for improving variables so that they have well-defined functional derivatives in the case of only a boundary at infinity is very well understood. This procedure consists of adding surface terms which lead to well-defined functional derivatives.

When we introduce a horizon, we have two boundaries. By locality, we can divide all variables into in and out, describing dynamics inside and outside of the black hole, respectively,

$$Q_{\text{full}} = Q_{\text{in}} + Q_{\text{out}}. \tag{2.4}$$

This separation of variables, though, might seem arbitrary but in fact is quite natural. Namely, values of in variables Q_{in} are inaccessible to us due to the presence of a horizon which hides the interior of a black hole. One can object that Q_{full} is found as a solution of (an adequate) (system of) equation(s), and knowing it, we automatically know the Q_{in}. This once again steps into the problem, which we stressed at the beginning, of the matter content and distribution inside of a black hole. With this in mind, we come to the conclusion that for the effective description of a black hole it is very natural to divide variables in the aforementioned manner.

When we focus solely on in variables, the procedure is no different than in the case of only a boundary at infinity; for the sake of completeness, we give the analysis of this case. It is possible that upon functional differentiation of Q_{in} a nonzero surface term, at the horizon, arises and we have to improve it, after which we obtain

$$\tilde{Q}_{\text{in}} = Q_{\text{in}} + \Gamma_{r\to r_0}, \tag{2.5}$$

where $\Gamma_{r\to r_0}$ is a surface term defined at $r = r_0$, which is finite and must assure that variation

$$\delta\tilde{Q}_{\text{in}} \tag{2.6}$$

has no surface term; this is the so-called integrability condition.

The analysis on out space is a bit different due to the presence of two boundaries. At infinity, we come to the same conclusion as in the case of in space. Additionally, functional derivatives can give surface terms at the horizon, and we must add a surface term at the horizon which cancels it. Consequently, we obtain

$$\tilde{Q}_{\text{out}} = Q_{\text{out}} + \Gamma_{r\to\infty} - \Gamma_{r\to r_0}; \tag{2.7}$$

note that

$$\Gamma_{r\to r_0} \tag{2.8}$$

is the same surface term as the one needed for improving Q_{in} but contributes with the opposite sign due to opposite orientation of the boundary at the horizon. Alternatively, the reason for this is easy to see if we look at $Q_{\text{full}} = Q_{\text{in}} + Q_{\text{out}}$ and note that full variables see only one boundary at infinity,

$$\tilde{Q}_{\text{full}} = \tilde{Q}_{\text{out}} + \tilde{Q}_{\text{in}} = Q_{\text{full}} + \Gamma_{r\to\infty}, \tag{2.9}$$

which is a very well-known result in the Hamiltonian approach to conserved charges [11], so surface terms at horizon from $\tilde{Q}_{\text{in}}$ and $\tilde{Q}_{\text{out}}$ must cancel each other.

In the rest of the paper we will be concerned with the case in which Q is a generator of symmetry, in this case we can extract even more information about the structure of surface terms.

The general structure of generators of symmetry is that they are a combination of first-class constraints plus a surface term (charge). Because in quantum, as well as in classical, state space all the constraints must hold, we conclude that the generator of symmetry reduces to a surface charge.

For calculating $\Gamma_{r\to\infty}$, we only need boundary conditions and dynamics at infinity, while for determining $\Gamma_{r\to r_0}$, we need boundary conditions and dynamics in the vicinity out of the horizon. This means that this analysis is insensitive to the concrete matter inside of the black hole. This is a known property of the black hole entropy, and it is tempting to conjecture that this kind of analysis will capture all the aspects relevant for its explanation. In turn, this would mean that we are able to determine semiclassical degrees of freedom of the black hole responsible for the appearance of entropy.

On the quantum level, this discussion, superficially, seems to imply the separation of full Hilbert state space $\mathcal{H}^{\text{full}}$ into a tensor product of in and out space,

$$\mathcal{H}^{\text{full}} = \mathcal{H}^{\text{in}} \otimes \mathcal{H}^{\text{out}}. \tag{2.10}$$

The truth is that we need to take care of the condition that charges are continual,

$$\tilde{Q}_{\text{in}} = \tilde{Q}_{\text{full}} - \tilde{Q}_{\text{out}}; \tag{2.11}$$

consequently, the Hilbert space is a tensor product of in and out space modulo the previous constraint

$$\mathcal{H}^{\text{full}} = \mathcal{H}^{\text{in}} \otimes \mathcal{H}^{\text{out}}/\text{Constraint}(\text{Continuity of charges}). \tag{2.12}$$

Now, a few words about the general properties of this construction are in order. In state space, where all constraints hold, we are left only with surface charges. As a manifestation of locality, spacelike separated operators commute, and we have that near-horizon $Q_{nh} = \Gamma_{r\to r_0}$ and asymptotic $Q_{as} = \Gamma_{r\to\infty}$ charges commute,

$$[Q_{nh}, Q_{as}] = 0. \tag{2.13}$$

Asymptotic symmetries are transformations that change boundary data and lead to different field configurations. They act on whole space-time, not only near infinity. The asymptotic form of symmetry is investigated at infinitesimal level; this way, we obtain algebra. For example, in 3D gravity, which is the most investigated and best understood example, with Brown-Henneaux boundary conditions, asymptotic symmetry is Virasoro algebra [12].

Near-horizon symmetries are transformations that make changes near the horizon. As stated, they should contain asymptotic algebra, possibly with different central charges. There can also be additional symmetry not seen from the perspective of infinity. This means that these additional symmetries are small gauge transformations from the infinity viewpoint. The important thing to stress is that at infinity asymptotic conditions capture many different field configurations, while the asymptotic conditions near the horizon describe only black hole with different matter distributions behind the horizon.

The observer very far away from the black hole is well approximated by the observer at infinity, and he will only observe asymptotic charges $\Gamma_{r\to\infty}$. Because at the semiclassical level equations of motion hold, it is to be expected that charges at infinity and near the horizon are not unrelated. The idea for microstates is as follows. Acting with near-horizon algebra, we produce an observable change at infinity which can be measured. But there is more than one transformation that we can apply that leads to the same charges at infinity. Counting different ways W to obtain the same asymptotic charges, we should be able to reproduce black hole entropy using Boltzmann relation

$$S = \ln W. \tag{2.14}$$

This procedure can be consistently applied only for semiclassically well-defined objects; otherwise, we would need quantum gravity, which implies that all measurable nonzero charges are much larger than $\hbar$. This idea identifies black hole microstates as different geometries which differ from each other by small gauge transformations from the infinity perspective but a physical one from the near-horizon viewpoint. This states, in nature, are soft hair on a black hole because a generator which should generate them at infinity is zero.

To extract some general conclusions about what properties of the state space constructed from near-horizon symmetry to expect, we review some known approaches for deriving black hole entropy and draw some conclusion from their success.

Euclidean calculation of black hole entropy in a nutshell is as follows. Cut out the interior from the spacetime and Wick rotate the time outside a black hole and compactify it on the circle of radius $\frac{\beta}{2\pi}$. The partition function calculated using this space-time is identified with appropriate thermodynamical potential from which the entropy is derived. This approach yields viable entropy in all known examples.

Euclidean calculation suggests that the interior of a black hole is not important, at least for the semiclassical properties. This means that $\mathcal{H}^{\text{out}}$ contains all the information about the black hole. From this, we expect an isomorphism between the full state space $\mathcal{H}^{\text{full}}$ and the state space outside of the black hole $\mathcal{H}^{\text{out}}$.

Cardy formula calculation of a black hole entropy is specific for three dimensions only. For three-dimensional asymptotically anti-de Sitter space-times, the algebra of asymptotic symmetries is Virasoro with central charges $c^{\pm}$. Asymptotic charges act in whole Hilbert state space, implying that quantum gravity is 2D CFT. The high-energy, $E \gg 1$, density of states $\rho(E)$ can be calculated using modular invariance, which, after using the Boltzmann formula, leads to the entropy formula

$$S = 2\pi\sqrt{\frac{c^- L_0^-}{6}} + 2\pi\sqrt{\frac{c^+ L_0^+}{6}}. \tag{2.15}$$

Using the Cardy formula, we reproduce black hole entropy in all known cases.

The success of the Cardy formula implies that semiclassical entropy is due to high-energy and angular momentum, if nonzero, states. This is to be expected, because for quantum correction to be negligible, meaning that semiclassical gravity is applicable, the same requirements are needed.

III. REVIEW OF THE NECESSARY RESULTS

In this short section, we review the basic results about the OTT black hole.

A. OTT black hole

The stationary OTT black hole [13], an exact solution of Bergshoeff-Hohm-Townsend gravity [14] and 3D PGT [15], is a three-parameter solution defined by the metric

$$ds^2 = N^2dt^2 - F^{-2}dr^2 - r^2(d\varphi + N_\varphi dt)^2, \tag{3.1a}$$

where

$$\begin{aligned} F &= \frac{H}{r}\sqrt{\frac{H^2}{\ell^2} + \frac{b}{2}H(1+\eta) + \frac{b^2\ell^2}{16}(1-\eta)^2 - \mu\eta}, \\ N &= AF, \qquad A = 1 + \frac{b\ell^2}{4H}(1-\eta), \\ N_\varphi &= \frac{\ell}{2r^2}\sqrt{1-\eta^2}(\mu - bH), \\ H &= \sqrt{r^2 - \frac{\mu\ell^2}{2}(1-\eta) - \frac{b^2\ell^4}{16}(1-\eta)^2}. \end{aligned} \tag{3.1b}$$

The roots of $N = 0$ are

$$r_\pm = \ell\sqrt{\frac{1+\eta}{2}}\left(-\frac{b\ell}{2}\sqrt{\eta} \pm \sqrt{\mu + \frac{b^2\ell^2}{4}}\right).$$

As we already stated, the metric (3.1) depends on three free parameters, μ, b, and η. For $\eta = 1$, the stationary OTT black hole reduces to the static solution, while for $b = 0$, it reduces to the rotating Bañados-Teitelboim-Zanelli (BTZ) black hole with parameters (m, j), defined by $4Gm \coloneqq \mu$ and $4Gj \coloneqq \mu\ell\sqrt{1-\eta^2}$.

The conserved charges of the rotating black hole take the following form:

$$E = \frac{1}{4G}\left(\mu + \frac{1}{4}b^2\ell^2\right), \tag{3.2a}$$

$$J = \ell\sqrt{1-\eta^2}E. \tag{3.2b}$$

B. Near-horizon of extremal OTT and near-horizon algebra

The extremal limit of a stationary OTT black hole can be achieved in two different ways as shown in Ref. [16]:

(1) The first way imposes the requirement $4\mu + b^2\ell^2 = 0$. This leads to a vanishing of both conserved charges, and consequently the asymptotic symmetry trivializes as we showed in Ref. [15].

(2) The second way to obtain an extremal black hole is to set $\eta = 0$, which is equivalent to the requirement that the angular momentum takes the maximal possible value. This corresponds to the usual procedure for the Kerr black hole.

In the latter case, after imposing consistent asymptotic conditions [15], we get that the Poisson bracket algebra of the well-defined canonical generators takes the form of the semidirect sum of centrally extended Kac-Moody and Virasoro algebra without central extension,

$$\begin{aligned} \{L_m, L_n\} &= -i(m-n)L_{m+n}, \\ \{L_m, J_n\} &= inJ_{m+n}, \\ \{J_m, J_n\} &= -i\kappa m\delta_{m+n,0}, \end{aligned} \tag{3.3}$$

where the central charge is given by

$$\kappa = \frac{\ell}{G}. \tag{3.4}$$

IV. SYMMETRY ALGEBRAS

In this section, we will review some results about asymptotic and near-horizon symmetry algebras of an extremal OTT black hole.

A. Algebra of asymptotic symmetries and its reduction on space of extremal geometries

We start with the quick review of the results obtained in Ref. [15], which are necessary for further analysis of this paper. The authors analyzed the asymptotic symmetry of an OTT black hole and derived the form of asymptotic symmetry

$$\xi^t = \ell T + \frac{\ell^5}{2r^2}\partial_t^2 T + \mathcal{O}(r^{-3}), \quad \xi^\varphi = S - \frac{\ell^2}{2r^2}\partial\varphi^2 S + \mathcal{O}(r^{-3}), \tag{4.1}$$

$$\xi^r = -\ell r\partial_t T + \mathcal{O}(1), \tag{4.2}$$

where functions T and S are subject to the constraints

$$T^\pm = T \pm S \qquad \partial_\mp T^\pm = 0, \tag{4.3}$$

meaning that functions satisfy $T^\pm = T^\pm(x^\pm)$ with $x^\pm = \frac{t}{\ell} \pm \varphi$. Also, they constructed the generator of the symmetry in the framework of Poincaré gauge theory, which is given by

$$\tilde{G} = G + \Gamma, \tag{4.4}$$

$$\Gamma = \int_0^{2\pi} (\xi^t \mathcal{E} + \xi^\varphi \mathcal{J}). \tag{4.5}$$

After passing on the Fourier mode of $T^\pm$, it is obtained that Fourier modes of Γ, denoted with $L^\pm$, satisfy the commutation relations of Virasoro algebra with central charges

$$c^+ = c^- = \frac{3\ell}{G}. \tag{4.6}$$

Virasoro algebra is an asymptotic symmetry of an arbitrary geometry with given asymptotic behavior. But we want to describe an extremal OTT black hole and need to further specify boundary conditions which will lead to asymptotic algebra of extremal geometries.

The condition for obtaining an extremal rotating black hole is the equality of energy and angular momentum

$$\ell E = J; \tag{4.7}$$

this is the constraint which we will impose on the general asymptotic algebra of charge. This constraint is realized for all extremal geometries if and only if the generators satisfy the same relation

$$\ell \mathcal{E} = \mathcal{J}; \tag{4.8}$$

this bring us to the conclusion

$$\Gamma(\xi) = \int_0^{2\pi} d\varphi \mathcal{J}\left(\frac{\xi^t}{\ell} + \xi^\varphi\right) = \int_0^{2\pi} d\phi \mathcal{J} \xi^\phi[\phi], \tag{4.9}$$

where

$$\phi = \varphi + \frac{t}{\ell}. \tag{4.10}$$

Consequently, charges are of the form

$$Q[\xi^\phi] = \int_0^{2\pi} d\phi \mathcal{J} \xi^\phi[\phi]. \tag{4.11}$$

This means that asymptotic symmetry of extremal geometries is not full Virasoro algebra but only chiral part L_n^+, which are Fourier modes of $Q[\xi^\phi]$ defined as $L_n^+ = Q[e^{(in\phi)}]$, with commutation relations

$$[L_n^+, L_m^+] = (n-m)L_{n+m}^+ + \frac{c}{12} n^3 \delta_{n,-m}, \tag{4.12}$$

where

$$c = \frac{3\ell}{G}, \tag{4.13}$$

while

$$L_n^- = 0. \tag{4.14}$$

B. Near-horizon symmetry algebra

From original coordinates (r, t, φ) to near-horizon extremal geometry (NHEG) (ρ, τ, ϕ) we pass after change of coordinates

$$t = \tau/\epsilon^2, \qquad r = r_0 + \epsilon\rho, \qquad \varphi = \phi - \Omega_H \frac{\tau}{\epsilon^2} = \phi - \frac{\tau}{\ell\epsilon^2}, \tag{4.15}$$

and taking limit $\epsilon \to 0$; see Ref. [16].

Asymptotic symmetry of NHEG of an extremal OTT is studied in Ref. [16], and the following symmetry is derived:

$$\xi^\tau = T(\tau), \qquad \xi^\rho = \rho U(\phi), \qquad \xi^\phi = S(\phi). \tag{4.16}$$

Further construction of the generator revealed that ξ^τ is pure gauge, so we will treat it as zero from now on.

Charges in NHEK are given by

$$Q[\xi^\tau] = 0, \tag{4.17}$$

$$Q[\xi^\rho] = -8a_0 \int_0^{2\pi} U(\phi) e^1_\phi, \tag{4.18}$$

$$Q[\xi^\phi] = -4a_0 \int_0^{2\pi} S(\phi) \omega^i_\phi e_{i\phi}. \tag{4.19}$$

C. Vacuum

Space-time which belongs to the allowed field configurations, is a solution of a field equations and with minimal energy is vacuum of a theory. Bearing in mind that we are interested in state space of the black hole, the first guess in three space-time dimensions would be a massless BTZ black hole. Because a massless BTZ does not have a horizon, it is hard to make sense of a near-horizon limit. For this reason, another more appropriate candidate for a black hole vacuum is a massless OTT, which possesses a horizon even in a massless case because of the presence of a hair parameter.

The metric of a massless OTT is

$$ds^2 = \frac{(r-r_0)^2}{\ell^2} dt^2 - \frac{\ell^2}{(r-r_0)^2} dr^2 - r^2 d\varphi^2. \tag{4.20}$$

The energy and angular momentum of this solution are zero,

$$E = 0, \qquad J = 0, \tag{4.21}$$

as are all other Virasoro charges

$$L_n^+ = L_n^- = 0. \tag{4.22}$$

When the near-horizon limit is taken,

$$t = \tau/\epsilon, \qquad r = r_0 + \epsilon\sqrt{\rho}, \qquad \varphi = \phi - \Omega_H \frac{\tau}{\epsilon} = \phi, \tag{4.23}$$

after the redefinition of τ and ℓ, we obtain

$$ds^2 = \frac{\rho}{\ell} d\tau^2 - \frac{\ell^2}{\rho^2} d\rho^2 - r_0^2 d\phi^2. \tag{4.24}$$

This metric belongs to the allowed metrics analyzed in Ref. [16], and near-horizon values of the charges for this metric are

$$Q[\xi^\tau] = Q[\xi^\rho] = Q[\xi^\phi] = 0. \tag{4.25}$$

This motivates us to treat a massless OTT as the black hole vacuum.

V. CONSTRUCTION OF MICROSTATES

This section is devoted to further development of the idea that state space of the OTT black hole can be constructed from near-horizon and asymptotic symmetry algebra. From now on, we pass from the Poisson bracket to the commutator. Commutation relations are the Poisson bracket multiplied by imaginary unit.

A. Unitary irreducible representations of the in algebra

We will search for the representations of the following algebra:

$$[L_n, L_m] = (n - m)L_{n+m}, \qquad [J_n, J_m] = n\delta_{n+m,0},$$
$$[L_n, J_m] = -mJ_{n+m}. \tag{5.1}$$

Note that we redefined J_n so that the ones we use in the rest of the paper are divided with $\sqrt{\kappa}$; this is the reason for the absence of central charge κ in previous commutation relations.

The reality of charges on the quantum level becomes the condition

$$J_n^\dagger = J_{-n}, L_n^\dagger = L_{-n}. \tag{5.2}$$

We construct irreducible representation starting from the highest state vector $|j,l\rangle$, which satisfies

$$J_0|j,l\rangle = j|j,h\rangle, \qquad L_0|j,h\rangle = h|j,h\rangle. \tag{5.3}$$

The operators with positive n we interpret as annihilation operators

$$J_n|j,h\rangle = L_n|j,h\rangle = 0, \qquad n > 0, \tag{5.4}$$

and operators with negative n we interpret as creation operators. Then, we construct the whole representation by acting with creation operators

$$J_{-n}, L_{-n}, n > 0; \tag{5.5}$$

the arbitrary state is of the form

$$L_{-k_1}\ldots L_{-k_i} J_{-n_1}\ldots J_{-n_m}|j,l\rangle. \tag{5.6}$$

Because Virasoro algebra has zero central charge norm of the state $L_{-n}|j,l\rangle$ is zero,

$$\| L_{-n}|j,h\rangle\|^2 = \langle j,h|[L_n, L_{-n}]|j,l\rangle = 0. \tag{5.7}$$

Consequently, unitarity implies that the Virasoro part, except L_0, of the near-horizon algebra is trivially represented. The expectation value in the vacuum state of L_0 gives the classical value of $L_0^{\text{classical}}$ in vacuum, which is equal to

$$\langle j,h|L_0|j,h\rangle = L_0^{\text{classical}} = 0; \tag{5.8}$$

this implies

$$h = 0. \tag{5.9}$$

Consequently, the generic vector in irreducible representation is a linear combination of vectors of the form

$$|\{k_i\}\rangle = J_{k_1}\ldots J_{-k_i}|j,0\rangle. \tag{5.10}$$

We assume that in state space is a unitary irreducible representation of in algebra, which we constructed previously. The structure of the in Hilbert space is of the form

$$\mathcal{H}^{\text{in}} = \oplus\, \mathcal{H}^{(n)}; \tag{5.11}$$

this is a structure of the Fock space.

B. Unitary irreducible representations of the out algebra

The same as we do in the case of in state space, we assume that out space is a unitary irreducible representation of out algebra. First, we need to specify what is our out algebra. The $u(1)$ Kac-Moody algebra is certainly present; the problematic part is Virasoro algebra. Comparing charges of asymptotic and near-horizon Virasoro algebra. we see that they share the same structure. This motivates us to interpret them as charges of the same transformation.

Further argument, supporting this claim, is that near-horizon and asymptotic charges have the same value on OTT background. So, we come to the conclusion that Virasoro algebra acting on out space is the sum of asymptotic and near-horizon algebras

$$L_n = L_n^{as} - L_n^{nh}, \tag{5.12}$$

with commutation relations

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n^3\delta_{n+m,0}, \tag{5.13}$$

where the central charge is the same as the one of asymptotic algebra

$$c = \frac{3\ell}{G}. \tag{5.14}$$

Because near-horizon and asymptotic charges of the Virasoro algebra have the same values on the OTT black hole solution [15,16], recalling the discussion of Sec. II, we derive that values of Virasoro charges on OTT background are zero,

$$L_n|_{\mathrm{OTT}} = 0. \tag{5.15}$$

Classical values of the charges are expectation values of the corresponding operators in state describing the desired geometry, in our case the OTT black hole

$$\langle L_n \rangle_{\mathrm{OTT}} = L_n|_{\mathrm{OTT}}. \tag{5.16}$$

To enforce the previous relations, we are forced to give a further restriction on the relevant representations of out algebra. We demand that Virasoro algebra is trivially represented. Representation of $u(1)$ Kac-Moody algebra is constructed from vacuum $|j\rangle$,

$$J_0|j\rangle = j|j\rangle, \tag{5.17}$$

$$J_k|j\rangle = 0, \qquad k > 0, \tag{5.18}$$

and the rest of states are constructed by acting with creation operators J_{-k}, $k > 0$,

$$|\{n_m\}\rangle = J_{-n_1} \ldots J_{-n_m}|j, 0\rangle. \tag{5.19}$$

Now, after we determined relevant irreducible representations of the out algebra, we proceed with construction of the complete state space.

C. Hilbert state space

We construct the state space of the OTT black hole using insight from Sec. II, in which we came to some general conclusions. Hilbert state space is of the form

$$\mathcal{H}^{\mathrm{full}} = \mathcal{H}^{\mathrm{in}} \otimes \mathcal{H}^{\mathrm{out}}/\mathrm{Constraint\,(Continuity\,of\,charges)}, \tag{5.20}$$

states in $\mathcal{H}^{\mathrm{in}}$ we denote with $|\{k_i\}\rangle$, states in $\mathcal{H}^{\mathrm{out}}$ are labeled by $|\{n_i\}\rangle$, and the tensor product state we label with

$$|\{k_i\}\rangle \otimes |\{n_i\}\rangle = |\{k_i\},\{n_i\}\rangle. \tag{5.21}$$

On the classical level, the constraint is that $Q_{as}[\xi^r] = Q_{\mathrm{full}}[\xi^r]$ is small gauge from the infinity perspective, i.e., zero, and that $Q_{\mathrm{in}}[\xi^r] = -Q_{\mathrm{out}}[\xi^r]$. On the quantum level, we enforce this by requiring that J_n^{full} annihilates all states in full state space for every non-negative n. Demanding that J_n^{full} annihilates physical states for every n is too strong a demand that trivializes state space. Our approach is similar to Gupta-Bleuler quantization of electromagnetic field.

Acting on states which are the tensor product of in and out states, this translates into

$$J_n^{\mathrm{full}}|\{k_i\},\{n_i\}\rangle = (J_n^{\mathrm{in}} \otimes I - I \otimes J_n^{\mathrm{out}})|\{k_i\},\{n_i\}\rangle = 0, \quad n \geq 0. \tag{5.22}$$

If we take J_0 in the previous constraint, we derive that both $\mathcal{H}^{\mathrm{in}}$ and $\mathcal{H}^{\mathrm{out}}$ have the same value of j; they are representations with the same highest weight.

Further constraints, for $n > 0$, acting on states of this form gives the following restriction:

$$\{k_i\} = \{n_i\}. \tag{5.23}$$

This has the important consequence that the generic state in $\mathcal{H}^{\mathrm{full}}$ is of the form

$$|\{n_i\},\{n_i\}\rangle. \tag{5.24}$$

We also have

$$\langle \{n_i\}|J_0|\{n_i\}\rangle = J_0^{\mathrm{classical}} = 0, \tag{5.25}$$

from which we conclude that

$$j = 0. \tag{5.26}$$

We introduce creation a_{-n} and annihilation a_n operators acting on $\mathcal{H}^{\mathrm{full}}$, which create and annihilate modes in state space of the black hole

$$a_{-n}|0\rangle = |n, n\rangle. \tag{5.27}$$

These operators satisfy the same commutation relations as J_n,

$$[a_n, a_m] = n\delta_{n+m,0}. \tag{5.28}$$

In the end, we note that we have isomorphism

$$\mathcal{H}^{\text{full}} \cong \mathcal{H}^{\text{in}} \cong \mathcal{H}^{\text{out}}; \tag{5.29}$$

the isomorphism is realized by

$$|0\rangle \cong |0\rangle_{\text{in}} \cong |0\rangle_{\text{out}} \tag{5.30}$$

$$a_n \cong J_n^{\text{in}} \cong J_n^{\text{out}}. \tag{5.31}$$

Interpretation of this result is that the black hole can effectively be described by the scalar near-horizon degree of freedom propagating in two-dimensional $t-\varphi$ spacetime. Carlip [17] arrived at a similar conclusion in his analysis of asymptotic dynamics of radial diffeomorphisms. The presence of a boundary breaks radial diffeomorphisms and leads to the appearance of dynamical degrees of freedom on a boundary.

Recently, Hamiltonian reduction was applied to general relativity in three dimensions [18] in which horizon is treated as a boundary with specific boundary conditions. The authors obtained that, in the set up of their paper, the dynamics of a black hole is effectively described by Floreanini-Jackiw scalar theory on the horizon.

D. Action of asymptotic algebra on state space

Now, we have to specify how the asymptotic algebra, which is Virasoro in our case, acts in state space $\mathcal{H}^{\text{full}}$. Because state space $\mathcal{H}^{\text{full}}$ is constructed solely from the action of $u(1)$ Kac-Moody algebra, this implies that Virasoro algebra can be constructed from Kac-Moody algebra. We will now do this using the well-known Sugawara-Sommerfeld construction [19]. This is the same as the approach taken in Ref. [3].

Operators $L_n^{(1)}$ of the Virasoro algebra with central charge $c=1$ are given as a bilinear combination of Kac-Moody operators

$$L_n^{(1)} = \frac{1}{2}\sum_{p=-\infty}^{\infty} :a_{n-p}a_p:, \tag{5.32}$$

where $::$ stands for normal ordering.

Virasoro algebra with arbitrary integer central charge c can be obtained in the manner [20]

$$L_n^{(c)} = \frac{1}{c}L_{cn}^{(1)} \tag{5.33}$$

or explicitly

$$L_n^{(c)} = \frac{1}{2c}\sum_{p=-\infty}^{\infty} :a_{cn-p}a_p:. \tag{5.34}$$

The representation of the asymptotic Virasoro algebra is identified as

$$L_n^+ = L_n^{(c)}, \tag{5.35}$$

with the assumption that central charge is an integer, which is supported by the results in Ref. [21]. Because we do not have a full microscopical description, we are not able to deduce the origin of central charge. We, nonetheless, have its value from the asymptotic analysis.

For us, the most important operator is Virasoro zero mode

$$L_0^{(c)} = \frac{1}{c}\sum_{p=0}^{\infty} a_{-p}a_p = \frac{1}{c}N, \tag{5.36}$$

where we introduced the number operator

$$N = \sum_{p=0}^{\infty} a_{-p}a_p. \tag{5.37}$$

Generic state is linear combination of states of the form $\sum_i a_{n_i}^\dagger|0\rangle$, for which we define the level as the $\sum_i n_i$. The number operator, as the name suggests, counts to which level state belongs which is obvious from the commutation relation

$$[N, a_n] = -na_n \tag{5.38}$$

and the construction of the unitary irreducible representations of $u(1)$ Kac-Moody algebra.

E. Microstate counting

We start from the well-known observation that the classical value of charge is given by the expectation value of the corresponding quantum generator in the correct microstate. Because states in $\mathcal{H}^{\text{full}}$ are a linear combination of $|\{k_i\}\rangle$, it is natural to interpret them as the underlying states of an OTT black hole. Quantitatively, this discussion is expressed as

$$\langle\{k_i\}|L_n^+|\{k_i\}\rangle = \delta_{n,0}L_0^+ = \delta_{n,0}\frac{r_0^2}{2\ell G}. \tag{5.39}$$

Alternatively, from Sugawara-Sommerfeld construction of Virasoro algebra, we obtain

$$\langle\{k_i\}|L_n^+|\{k_i\}\rangle = \delta_{n,0}\frac{1}{c}\langle\{k_i\}|N|\{k_i\}\rangle = \delta_{n,0}\frac{1}{c}\sum k_i. \tag{5.40}$$

From the previous relations, we conclude that

$$k = \sum k_i = cL_0^+. \tag{5.41}$$

Assuming $k \gg 1$, we can use the Hardy-Ramanujan formula for the number of partitions $[k]$ of natural number k, which states that the $[k]$ is asymptotically

$$[k] \propto \frac{1}{4\sqrt{3}k} e^{2\pi\sqrt{\frac{k}{6}}}. \tag{5.42}$$

In fact, it is expected for both c and L_0 to be separately much larger than 1, so our assumption is a very reasonable one.

The Boltzmann formula for entropy from the number of microstates W,

$$S = \ln W, \tag{5.43}$$

after identification $W = [k]$ gives

$$S = 2\pi\sqrt{\frac{cL_0^+}{6}}, \tag{5.44}$$

which is the same as the entropy obtained in Refs. [15,16] by different methods.

VI. CONCLUSION

We constructed the state space of an extremal OTT black hole using its asymptotic and near-horizon symmetry algebras. The crucial difference between asymptotic and near-horizon algebra is the presence of $u(1)$ Kac-Moody algebra in the latter, which can be identified as an algebra of creation and annihilation operators. The Virasoro part of asymptotic and near-horizon algebra is recognized as the charges of the same transformation calculated at infinity and at the horizon. We further assumed that state space is constructed from unitary irreducible representations of in and out algebras of symmetry.

The classically observable quantities of the black hole are conserved charges far away from the horizon, meaning that we will differentiate only black holes with different values of asymptotic charges. This way, we identified the microstates which correspond to the same macrostate, and using the Boltzmann formula, we reproduced the black hole entropy.

This construction is essentially quantum, although we did not include any quantum corrections, because we worked with Hilbert spaces, which is in line with our understanding that black hole entropy is quantum in nature.

We also obtained that there is isomorphism of full state space with state spaces *inside* and *outside* of a black hole. This is in agreement with discussion of Sec. II, in which we concluded that knowledge of the exterior of a black hole should be sufficient for a derivation of a black hole entropy in semiclassical approximation.

This approach essentially relies on the existence of $u(1)$ Kac-Moody near-horizon algebra of radial diffeomorphisms and anti-de Sitter asymptotic; consequently, it is expected that this analysis can be applied, possibly with some modifications, in any case which fulfilling previously mentioned requirements. For example, this approach should be applicable to extremal BTZ with(out) torsion [22].

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Velocity memory effect without soft particles

B. Cvetković[*] and D. Simić[†]
Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

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We study the behavior of geodesics in the plane-fronted wave background of the three-dimensional gravity with propagating torsion, which possesses only *massive* degrees of freedom. We discover the *velocity memory effect*, in contrast to the current belief that its existence is due to the presence of soft particles.

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I. INTRODUCTION

Memory effect for gravitational waves was first discovered by Zeldovich and Polnarev [1] and got its name from Braginsky and Grishchuk [2]. The conclusion of [1,2] is that massive test particles, initially at rest, will suffer permanent displacement after the passage of gravitational wave. For this reason, this displacement is called memory effect.

The memory effect [1,2] is described in linear approximation. A nonlinear contribution to the memory effect is discovered in Ref. [3]; for less technical derivation, see [4].

In the recent years, we have witnessed great new discoveries connecting asymptotic symmetries, soft theorems, and displacement memory effect [5]. This line of reasoning applied to the black holes [6] offers new insights into black hole physics.

Permanent displacement implies that relative velocity of massive test particles is zero. This conclusion is questioned in Refs. [7,8], where velocity memory effect is derived on the contrary to displacement memory effect. The main result of [7,8] is that passage of gravitational wave will be encoded not in the permanent displacement but in the nonzero relative velocity of test masses. For recent development on velocity memory effect, see [9], where, among other things, authors concluded that velocity memory effect is connected with soft gravitons.

The goal of this paper is to investigate if there is a memory effect for gravitational waves within the framework of Poincaré gauge theory [10–12]. We shall consider the solutions of the three-dimensional (3D) gravity with propagating torsion [13,14], a theory in which all modes are massive. If the soft modes are the ones that cause memory effect (according to the already mentioned conclusions in [9]) there should be no memory effect in this theory.

Let us note that 3D general relativity (GR) is a topological theory, and there are consequently no gravitational wave solutions in vacuum. The gravitational waves with torsion in 3D are solutions in which the metric function *crucially* depends on torsion [14], in the sense that in the absence of torsion, the metric function becomes trivial and the wave solution "disappears." This offers us an interesting opportunity to study the effects of torsion already at the level of geodesic motion of spinless particles.

The paper is organized as follows. First, we review the theory of gravity under consideration and its gravitational pp wave solutions. Next, we derive the geodesic equations in this pp wave space-time. Thereafter, we investigate the solutions of these equations. Unfortunately, the geodesic equations are not analytically solvable except in a very special case, which we have also analyzed; thus, we solved the geodesic equations numerically for some characteristic choices of the coefficients which appear in the gravitational wave solutions.

Our conventions are as follows. The Latin indices $(i, j, \dots)$ refer to the local Lorentz (co)frame and run over 0,1,2)), b^i is the tetrad (one form), h_i is the dual basis (frame), such that $h_i \lrcorner b^k = \delta^i_k$; the volume three form is $\hat{\epsilon} = b^0 \wedge b^1 \wedge b^2$, the Hodge dual of a form α is $^\star\alpha$, with $^\star 1 = \hat{\epsilon}$, totally antisymmetric tensor is defined by $^\star(b_i \wedge b_j \wedge b_k) = \varepsilon_{ijk}$ and normalized to $\varepsilon_{012} = +1$; the exterior product of forms is implicit.

II. RIEMANNIAN PP WAVES

In this section, we give an overview of Riemannian 3D pp waves. For details, see [13].

A. Geometry

The metric of pp waves can be written as

$$ds^2 = du(Sdu + dv) - dy^2, \tag{2.1a}$$

where

[*]cbranislav@ipb.ac.rs
[†]dsimic@ipb.ac.rs

$$S = \frac{1}{2} H(u, y). \tag{2.1b}$$

Next, we choose the tetrad field (coframe) in the form

$$b^0 := du, \qquad b^1 := S du + dv, \qquad b^2 := dy, \tag{2.2a}$$

so that $ds^2 = \eta_{ij} b^i \otimes b^j$, where η_{ij} is the half-null Minkowski metric

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The corresponding dual frame h_i is given by

$$h_0 = \partial_u - S\partial_v, \qquad h_1 = \partial_v, h_2 = \partial_y. \tag{2.2b}$$

For the coordinate $x^\alpha = y$ on the wave surface, we have

$$x^c = b^c{}_\alpha x^\alpha = y, \qquad \partial_c = h_c{}^\alpha \partial_\alpha = \partial_y,$$

where $c = 2$.

Starting from the general formula for the Riemannian connection one form,

$$\omega^{ij} := -\frac{1}{2}[h^i \lrcorner\, db^j - h^j \lrcorner\, db^i - (h^i \lrcorner\, h^j \lrcorner\, db^k) b_k],$$

one can find its explicit form; for $i < j$, its nonvanishing component reads as

$$\omega^{12} = -\partial_y S b^0. \tag{2.3a}$$

Introducing the notation $i = (A, a)$, where $A = 0$, 1 and $a = 2$, one can rewrite ω^{ij} in a more compact form as follows:

$$\omega^{Ac} = k^A b^0 \partial^c S, \tag{2.3b}$$

where $k^i = (0, 1, 0)$ is a null propagation vector, $k^2 = 0$.

The above connection defines the Riemannian curvature

$$R^{ij} = 2 b^0 k^{[i} Q^{j]}, \tag{2.4a}$$

where

$$Q^2 = \partial_{yy} S b^2. \tag{2.4b}$$

The Ricci one-form $Ric^i := h_m \lrcorner\, Ric^{mi}$ is given by

$$Ric^i = b^0 k^i Q, \qquad Q = h_c \lrcorner\, Q^c = \frac{1}{2}\partial_{yy} H, \tag{2.5a}$$

and the scalar curvature vanishes

$$R = 0. \tag{2.5b}$$

B. Dynamics

1. pp waves in GR

Starting with the action $I_0 = -\int d^4x (a_0 R + 2\Lambda_0)$, one can derive the GR field equations in vacuum,

$$2a_0 G^n{}_i = 0, \tag{2.6}$$

where $G^n{}_i$ is the Einstein tensor. As a consequence, the metric function H must obey

$$\partial_{yy} H = 0. \tag{2.7}$$

However, the solution of this equation is trivial,

$$H = C(u) + y D(u),$$

since the corresponding radiation piece of curvature vanishes.

III. PP WAVES WITH TORSION

A. Geometry of the ansatz

We assume that the form of the triad field (2.2) remains unchanged, whereas the connection is

$$\omega^{ij} = \tilde{\omega}^{ij} + \frac{1}{2}\varepsilon^{ij}{}_m k^m k_n b^n G, \tag{3.1a}$$

$$G := S' + K. \tag{3.1b}$$

Here, the new term $K = K(u, y)$ describes the effect of torsion as follows:

$$T^i := \nabla b^i = \frac{1}{2} K k^i k_m {}^\star b^m. \tag{3.2}$$

The only nonvanishing irreducible piece of T^i is its tensorial piece

$${}^{(1)}T^i = T^i,$$

while the curvature is

$$R^{ij} = \varepsilon^{ijm} k_m k^{n\star} b_n G',$$
$$Ric^i = \frac{1}{2} k^i k_m b^m G',$$
$$R = 0. \tag{3.3}$$

The nonvanishing irreducible components of the curvature R^{ij} are

$$ {}^{(4)}R^{ij} = \frac{1}{2}\varepsilon^{ijm}k_m k^{n\star} b_n p G' $$

and the quadratic curvature invariant vanishes $R^{ij} * R_{ij} = 0$. For details on irreducible decomposition of torsion and curvature, see [15].

The geometric configuration defined by the triad field (2.2) and the connection (3.1) represents a generalized gravitational plane-fronted wave of GR_Λ, or the *torsion wave* for short.

B. Massive torsion waves

The field equations take the following form [14]:

$$ a_0 G' - a_1 K' = 0, \qquad \Lambda = 0, $$
$$ K'' + m^2 K = 0, \qquad m^2 = \frac{a_0(a_1 - a_0)}{b_4 a_1}, \quad (3.4) $$

with $G = S' + K$ and $S = H/2$. The solution has a simple form,

$$ K = A(u)\cos my + B(u)\sin my, $$
$$ \frac{1}{2}H = \frac{a_1 - a_0}{a_0 m}(A(u)\sin my - B(u)\cos my) + h_1(u) + h_2(u)y. \quad (3.5) $$

Disregarding the integration "constants" h_1 and h_2, the metric and the torsion functions, H and K, become both periodic in y. In the absence of torsion, the metric function becomes trivial. This is an expected result since 3D general relativity is a theory which possesses no propagating degrees of freedom.

The vector field $k = \partial_v$ is the Killing vector for both the metric and the torsion; moreover, it is a null and covariantly constant vector field. This allows us to consider the solution (3.5) as a generalized pp wave.

IV. GEODESIC MOTION

In this section, we shall examine the *geodesic motion* of particles in the field of the massive wave with torsion.

At first sight that might be puzzling, since the motion of spinless particles is not affected by torsion [12,16], and in the gravitational field, they follow geodesic lines, which are influenced by the Riemannian connection depending on the metric. However, gravitational waves with torsion in 3D are interesting solutions, which are intrinsically different from the well-known spherically symmetric (static or stationary) solutions of Poincaré gauge theory [17] (for review, see [18]). The metric of these solutions is "independent" of torsion in the sense that it represents Schwarzschild (or Schwarzschild anti–de Sitter, Kerr, etc.) metric and the motion of spinless particles is not affected by the presence of torsion. However, for the gravitational wave solution (3.5), the metric crucially depends on torsion as we noted in the previous section. This offers us an interesting opportunity to study the effects of torsion already at the level of geodesic motion.

Christoffel connection. The nonvanishing components of Christoffel (torsion free) connection are given by

$$ \tilde{\Gamma}^v{}_{uu} = \frac{1}{2}\partial_u H, $$
$$ \tilde{\Gamma}^v{}_{uy} = \frac{1}{2}H', \qquad \tilde{\Gamma}^v{}_{yu} = \frac{1}{2}H', $$
$$ \tilde{\Gamma}^y{}_{uu} = \frac{1}{2}H'. \quad (4.1) $$

Let us mention that nontrivial contribution to metric function and consequently Christoffel connection stems from the presence of torsion.

Geodesic equations. The geodesic equation for u takes the expected form

$$ \frac{d^2u}{d\lambda^2} = 0. \quad (4.2) $$

Therefore, without the loss of generality, we can assume $u \equiv \lambda$.

The equation for y is given by

$$ \ddot{y} + \frac{1}{2}H' = 0 \quad (4.3a) $$

or more explicitly

$$ \ddot{y} + \frac{a_1 - a_0}{a_0}(A\cos my + B\sin my) = 0. \quad (4.3b) $$

Finally, the equation for v reads as

$$ \ddot{v} + \frac{1}{2}\partial_u H + H'\dot{y} = 0. \quad (4.4) $$

In the special case, when H does not explicitly depend on u, the equation (4.4) can be integrated as

$$ \dot{v} + Hy = C, $$

where C is a integration constant. Hence, consequently, we get

$$ v = \int (C - Hy)du. \quad (4.5) $$

A. Exact solutions

Interestingly, the geodesic equations admit the existence of exact solutions in particular cases. The simplest case is when $A(u)$ and $B(u)$ are constants. In that case, Eq. (4.3b) can be rewritten in the form

$$ \frac{1}{2}\frac{d\dot{y}^2}{dy} + \frac{a_1 - a_0}{a_0}(A\cos my + B\sin my) = 0. $$

If we impose *initial conditions*

$$y(0)=0, \qquad \dot y(0)=0, \tag{4.6}$$

by integrating the previous equation, we obtain

$$\frac{1}{2}\dot y^2+\frac{a_1-a_0}{a_0 m}(A\sin my-B(\cos my-1))=0,$$

or equivalently.

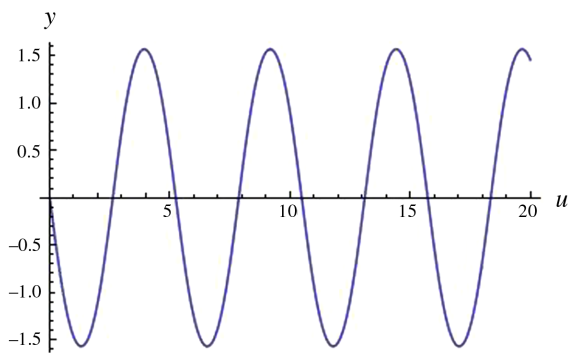

FIG. 1. The plot for the particle position in units $m=2$, for $\bar A=1$.

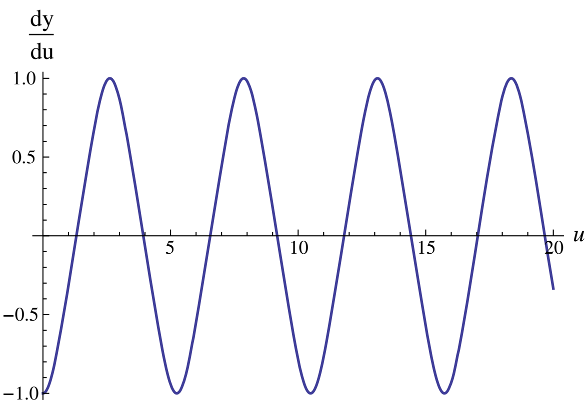

FIG. 2. The plot for the particle velocity in units $m=2$, for $\bar A=1$

$$\frac{dy}{\sqrt{\bar A\sin my-\bar B(\cos my-1)}}=du,$$

$$\bar A \coloneqq \frac{2(a_0-a_1)}{a_0 m}A, \qquad \bar B \coloneqq \frac{2(a_0-a_1)}{a_0 m}B,$$

which after integration yields the following equation for y:

$$\frac{4i\sqrt{2\bar A\,\bar B-(\bar A^2+\bar B^2)\frac{\cos\frac{my}{2}}{\sin^2\frac{my}{4}}}}{m\sqrt{\bar B+\sqrt{\bar A^2+\bar B^2}}\sqrt{\bar A\sin my-\bar B(\cos my-1)}} \times \sin\frac{my}{2}\sqrt{\tan\frac{my}{4}}F\left(i\mathrm{Arcsinh}\left(\frac{\sqrt{\frac{\bar B+\sqrt{\bar A^2+\bar B^2}}{A}}}{\sqrt{\tan\frac{my}{4}}}\right)\middle| -\frac{\bar A^2+2\bar B(\bar B-\sqrt{\bar A^2+\bar B^2}}{\bar A^2}\right)=u, \tag{4.7}$$

where $F(\phi|k)$ represents the elliptic integral of the first kind [19].

The choice $\bar A\neq 0$, $B(u)=0$ yields the following exact solution for $y(u)$:

$$y(u)=-\frac{2\mathrm{am}(\frac{1}{2}\sqrt{\bar A}mu|2)}{m}, \tag{4.8}$$

where $\mathrm{am}(z|m)$ is a Jacobi amplitude function. H does not explicitly depend on u; we get that v is given by the expression (4.5).

The characteristic plots for particle position y and velocity $\dot y$ (for $m=2$, $\bar A=1$) are shown in Figs. 1 and 2, respectively.

B. Velocity memory effect

The velocity memory effect is present in the case when functions $A(u)$ and $B(u)$ vanish for large u.

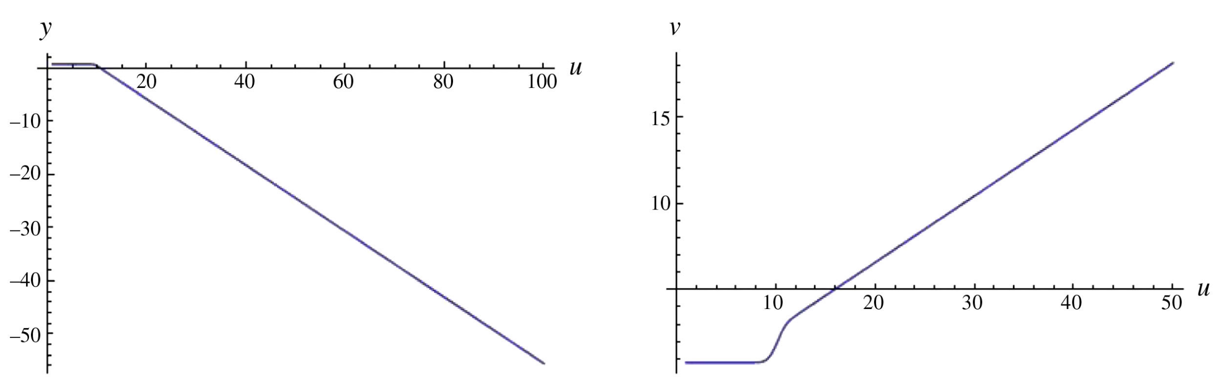

FIG. 3. The plot for the particle position y and v in units $m=1$, for $\bar B=-e^{-(u-10)^2}$.

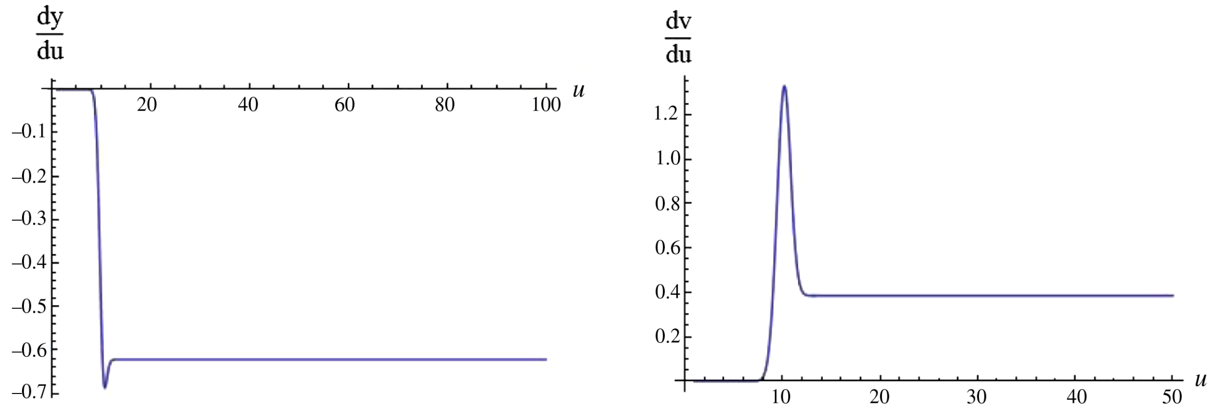

FIG. 4. The plot for the particle velocity $\dot{y}$ and $\dot{v}$ in units $m = 1$, for $\bar{B} = -e^{-(u-10)^2}$.

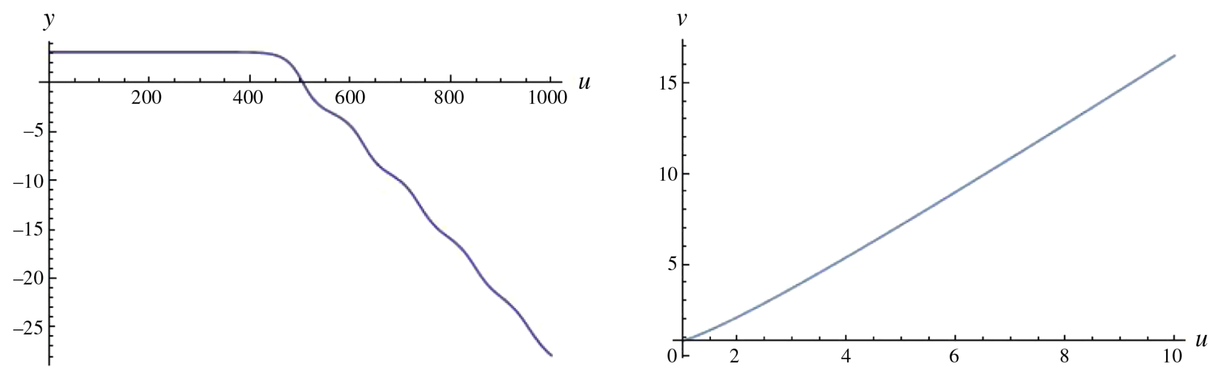

FIG. 5. The plot for the particle position y and v in units $m = 1$, for $\bar{B} = -1/u$.

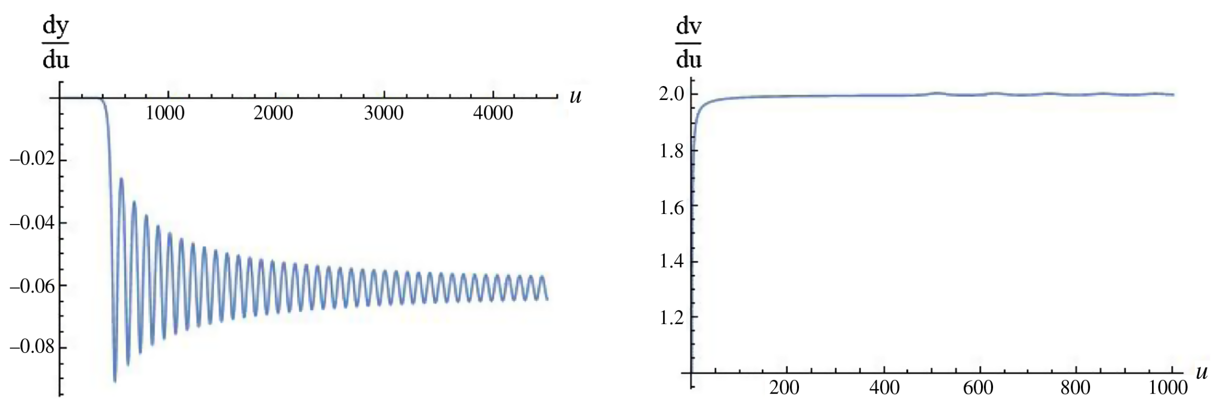

FIG. 6. The plot for the particle velocity $\dot{y}$ and $\dot{v}$ in units $m = 1$, for $\bar{B} = -1/u$.

1. Shockwave case

In the shock wave case, when functions $A(u) = 0$ and $B(u)$ vanish exponentially, for example, $B(u) \sim e^{-(u-10)^2}$, numerical solutions of the geodesic equations lead to the plots for the particle position y and v shown in Fig. 3 and velocity $\dot{y}$ and $\dot{v}$ shown in Fig. 4.

2. Slow fallof

In the case when $A(u) = 0$ and $B(u) \sim 1/u$, numerical solutions lead to the following plots for the particle position y and v shown in Fig. 5 and velocity $\dot{y}$ and $\dot{v}$ shown in Fig. 6.

V. DISCUSSION

We studied the geodesic motion in asymptotically flat pp wave space-time, and we discovered the presence of velocity memory effect. The effect is present for the very fast falloff of the gravitational wave, as well as for the slow one. Analysis of this paper provides the first example of memory effect for gravitational waves with torsion. We demonstrated that torsion waves lead to the memory effect same as the torsion-less waves do. This is also the first account of the memory effect in three dimensions to authors' knowledge. It would be interesting to see is there a connection to BMS_3 symmetry. Because theory has no massless modes, without a doubt, we can conclude that there can be no soft particles responsible for the memory effect. Consequently, the belief that soft particles are responsible for the velocity memory effect is demonstrated to be incorrect. For the related work on massive gravity, see [20].

Intuitively, we can say that memory effect is due to energy transfer. Passing gravitational wave transfers energy to the test particle which after the passage of the gravitational wave continues to move with constant velocity, in which intensity is dictated by the amount of energy transferred. Looking at the memory effect in this way we conclude that displacement memory effect is not possible, except, maybe, in some special cases where the total amount of transferred energy would be zero. To make this intuitive discussion precise, it is required to define energy in asymptotically flat space-times in a satisfying manner; this is left for further investigation.

The theory we considered is three-dimensional, while the four-dimensional case is realistic and relevant for applications. The next step in investigation is to study geodesic motion for massive gravitational waves with torsion in four dimensions. The metric of the gravitational waves with torsion in 4D has a nontrivial contribution stemming from the tensorial component of torsion [21] as in 3D, which affects geodesic motion. Consequently, it is expected that in 4D, we shall obtain the velocity memory effect similar to the one noticed in 3D case. Also, there is a possible difference compared to the memory effect in general relativity, which, in principle, may be observable. This will be the possible experimental setup for detection of torsion.

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Memory effect of the pp waves with torsion

B. Cvetković[a], **D. Simić**[b]

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

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Abstract We analyse the motion of test particles in the spacetime of the plane-fronted (pp) waves with torsion in four-dimensions. We conclude that there is a velocity memory effect in the direction of advanced time and along radial direction, while we have rotation of particles in angular direction. The velocity memory effect in the aforementioned directions is severely affected by the value of the tordion mass and probably it is not observable. A very interesting, probably observable effect, steams from the rotation, which is insensitive to the tordion mass.

1 Introduction

When a gravitational wave passes through a system of test particles it induces an observable disturbance of the system [1–4]. In other words a system remembers that a wave passed, and for this reason it is known as the memory effect.

There are two possible outcomes when a wave passes, disregarding the trivial possibility that everything is translated or boosted in the same way which is unobservable. The first scenario is that relative velocity of test particles is zero while they suffer a permanent displacement depending on their initial conditions. This is known as the displacement memory effect [1–4], for the results on nonlinear contribution to the memory effect see [5,6]. The appearance of displacement memory effect is questioned in [7–9], where authors concluded that test particles will have non-zero relative velocity. This variation of the memory effect is known as the velocity memory effect, for recent development see [10–12].

All of the previously mentioned results are obtained in the framework of general relativity and memory effect is not much investigated beyond it. Some of the results, known to the authors, are memory effect for massive graviton investigated in Ref. [13] and memory effect for pp waves in General relativity [14], while, the memory effect of the gravitational waves with torsion in the Poincaré gauge theory (PGT) has been investigated only in three-dimensions in Ref. [15].

The aim of this paper is to fill the gap in the literature, namely to extend results about the memory effect to the gravitational waves with torsion in four-dimensions.

Basic dynamical variables in PGT [16–19] are the tetrad field b^i and the Lorentz connection $\omega^{ij} = -\omega^{ji}$ (1-forms), and the associated field strengths are the torsion $T^i = db^i + \omega^i{}_k \wedge b^k$ and the curvature $R^{ij} = d\omega^{ij} + \omega^i{}_k \wedge \omega^{kj}$ (2-forms). By construction, PGT is characterized by a Riemann-Cartan geometry of spacetime, and its physical content is directly related to the existence of mass and spin as basic characteristics of matter at the microscopic level. General PGT Lagrangian L_G is at most quadratic in the field strengths. The number of independent (parity invariant) terms in L_G is nine, which makes the corresponding dynamical structure rather complicated.

The paper is organized as follows. First, we review the gravitational pp wave solutions with torsion in four-dimensions. After that, we derive the geodesic equations in this pp wave spacetime. We finally numerically solve the geodesic equations.

Our conventions are as follows. The Latin indices $(i, j, ...)$ refer to the local Lorentz (co)frame and run over (0, 1, 2, 3), b^i is the tetrad (1-form), h_i is the dual basis (frame), such that $h_i \rfloor b^k = \delta^i_k$; the volume 4-form is $\hat{\epsilon} = b^0 \wedge b^1 \wedge b^2 \wedge b^3$, the Hodge dual of a form α is $^\star\alpha$, with $^\star 1 = \hat{\epsilon}$, totally antisymmetric tensor is defined by $^\star(b^i b^j b^k b^l) = \varepsilon^{ijkl}$ and normalized to $\varepsilon_{0123} = +1$; the exterior product of forms is implicit in all expressions.

2 Review of the pp waves

In this section, we give an overview of 4D pp waves in PGT. For details see [20].

[a] e-mail: cbranislav@ipb.ac.rs

[b] e-mail: dsimic@ipb.ac.rs (corresponding author)

2.1 pp waves without torsion

2.1.1 Geometry

In local coordinates $x^\mu = (u, v, y, z)$, the metric of the pp waves is of the form

$$ds^2 = du(Hdu + 2dv) - (dy^2 + dz^2), \tag{2.1}$$

where the unknown metric function $H = H(u, y, z)$ is to be obtained from the field equations. The advanced time v is an affine parameter along the null geodesics $x^\mu = x^\mu(v)$, and u is retarded time such that $u =$ const. are the spacelike surfaces parameterized by $x^\alpha = (y, z)$. Since the null vector $\xi = \xi(u)\partial_v$ is orthogonal to these surfaces, they are regarded as wave surfaces, and ξ is the null direction (ray) of the wave propagation.

We choose the tetrad field (coframe) to be of the form

$$b^0 := du, \qquad b^1 := \frac{H}{2}du + dv,$$
$$b^2 := dy, \qquad b^3 := dz, \tag{2.2a}$$

so that $ds^2 = \eta_{ij} b^i \otimes b^j$, where η_{ij} is the half-null Minkowski metric:

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The corresponding dual frame h_i is given by

$$h_0 = \partial_u - \frac{H}{2}\partial_v, \qquad h_1 = \partial_v, \qquad h_2 = \partial_y, \qquad h_3 = \partial_z. \tag{2.2b}$$

For the coordinates $x^\alpha = (y, z)$ on the wave surface, we have:

$$x^c = b^c{}_\alpha x^\alpha = (y, z), \qquad \partial_c = h_c{}^\alpha \partial_\alpha = (\partial_y, \partial_z),$$

where $c = 2, 3$. After introducing the notation $i = (A, a)$, where $A = 0, 1$ and $a = (2, 3)$, one can find the compact form of the Riemannian connection ω^{ij}:

$$\omega^{Ac} = \frac{1}{2}k^A b^0 \partial^c H, \tag{2.3}$$

where $k^i = (0, 1, 0, 0)$ is a null propagation vector, $k^2 = 0$.

The above connection defines the Riemannian curvature $R^{ij} = d\omega^{ij} + \omega^i{}_m \omega^{mj}$; for $i < j$, it is given by

$$R^{ij} = 2b^0 k^{[i} Q^{j]} \tag{2.4a}$$

where Q^c is a 1-form introduced by Obukhov [21],

$$Q^2 = \frac{1}{2}\partial_{yy}Hb^2 + \frac{1}{2}\partial_{yz}Hb^3,$$
$$Q^3 = \frac{1}{2}\partial_{zz}Hb^3 + \frac{1}{2}\partial_{yz}Hb^2. \tag{2.4b}$$

The Ricci 1-form $Ric^i := h_m \rfloor Ric^{mi}$ is given by

$$Ric^i = b^0 k^i Q,$$
$$Q = h_c \rfloor Q^c = \frac{1}{2}\left[\partial_{yy}H + \partial_{zz}H\right], \tag{2.5}$$

and the scalar curvature $R := h_i \rfloor Ric^i$ vanishes.

2.1.2 pp waves in GR

Starting with the action $I_0 = -\int d^4x a_0 R$, one can derive the GR field equations in vacuum:

$$2a_0 G^n{}_i = 0, \tag{2.6}$$

where $G^n{}_i$ is the Einstein tensor. As a consequence, the metric function H must obey

$$\partial_{yy}H + \partial_{zz}H = 0. \tag{2.7}$$

There is a simple solution of these equations,

$$H_c = A(u) + B_\alpha(u)x^\alpha, \tag{2.8}$$

for which Q^a vanishes. This solution is trivial (or pure gauge), since the associated curvature takes the background form, $R^{ij} = 0$.

2.2 pp waves with torsion

2.2.1 Geometry of the ansatz

We assume that the form of the triad field (2.2) remains unchanged, while looking at the Riemannian connection (2.3), one can notice that its radiation piece appears only in the ω^{1c} components:

$$(\omega^{1c})^R = \frac{1}{2}(h^{c\alpha}\partial_\alpha H)b^0.$$

This motivates us to construct a new connection by applying the rule

$$\frac{1}{2}\partial_\alpha H \to \frac{1}{2}\partial_\alpha H + K_\alpha, \qquad K_\alpha = K_\alpha(u, y, z), \tag{2.9a}$$

where K_α is the component of the 1-form $K = K_\alpha dx^\alpha$ on the wave surface. Thus, the new form of $(\omega^{ij})^R$ reads

$$(\omega^{ic})^R := k^i h^{c\alpha}(\frac{1}{2}\partial_\alpha H + K_\alpha)\, b^0, . \tag{2.9b}$$

The geometric content of the new connection is found by calculating the torsion:

$$T^i = \nabla b^i + \omega^i{}_m b^m = k^i b^0 (b^2 K_y + b^3 K_z) = k^i b^0 b^c K_c. \tag{2.10}$$

The only nonvanishing irreducible piece of T^i is ${}^{(1)}T^i$.

The new connection modifies also the curvature, so that its radiation piece becomes

$$(R^{1c})^R = k^1 b^0 \Omega^c, \qquad \Omega^c := Q^c + \Theta^c, \tag{2.11a}$$

where the term Θ^c that represents the contribution of torsion is given by

$$\Theta^2 = \partial_y K_y b^2 - \partial_z K_y b^3, \qquad \Theta^3 = \partial_z K_z b^3 - \partial_y K_z b^2.$$

The covariant form of the curvature reads

$$R^{ij} = 2b^0 k^{[i}\Omega^{j]}, \tag{2.11b}$$

and the Ricci curvature takes the form

$$Ric^i = b^0 k^i \Omega, \qquad \Omega := h_c \rfloor \Omega^c = Q + \Theta. \tag{2.11c}$$

The torsion has no influence on the scalar curvature and it again vanishes. Thus, our ansatz defines a RC geometry of spacetime.

2.2.2 Massive torsion waves

The irreducible decomposition of the curvature implies (see [20])

$${}^{(3)}R_{ij} = 0, \qquad {}^{(5)}R_{ij} = 0, \qquad {}^{(6)}R_{ij} = 0 \tag{2.1}$$

whereas the remaining pieces ${}^{(n)}R^{ij}$ are defined by their non-vanishing components as

$$\begin{aligned}
&{}^{(2)}R^{1c} = \frac{1}{2}{}^\star(\Psi^1 b^c), \qquad {}^{(4)}R^{1c} = \frac{1}{2}(\Phi^1 b^c),\\
&{}^{(1)}R^{1c} = b^0\left(\Omega^{(ce)} - \frac{1}{2}\eta^{ce}\Omega\right) b_e,
\end{aligned} \tag{2.2a}$$

where the 1-forms Φ^i and Ψ^i are given by

$$\begin{aligned}
&\Phi^i = k^i b^0 (Q+\Theta), \qquad \Theta = \partial_y K_y + \partial_z K_z,\\
&\Psi^i = X^i = -k^i b^0 \Sigma, \qquad \Sigma = \partial_z K_y - \partial_y K_z.
\end{aligned} \tag{2.2b}$$

Having found ${}^{(1)}T_i$ and ${}^{(n)}R_{ij}$, we obtain the following form of the two PGT field equations [20]:

$$(1\mathrm{ST})\Lambda = 0, \qquad a_1\Theta - a_0(Q+\Theta) = 0, \tag{2.3a}$$

$$\begin{aligned}
&(2\mathrm{ND}) - (b_2+b_1)(\nabla\Psi^1)b^2 - (b_4+b_1)(\nabla\Phi^1)b^3\\
&\quad - 2\big(a_0 - A_1\big)T^1 b^3 = 0,\\
&\quad - (b_2+b_1)(\nabla\Psi^1)b^3 + (b_4+b_1)(\nabla\Phi^1)b^2\\
&\quad +2\big(a_0 - A_1\big)T^1 b^2 = 0.
\end{aligned} \tag{2.3b}$$

Leaving (1ST) as it is, (2ND) can be given a more clear structure as follows:

$$(\partial_{yy}+\partial_{zz})\Theta - m^2_{2+}\Theta = 0, \quad m^2_{2+} := \frac{2a_0(a_0-a_1)}{a_1(b_1+b_4)}, \tag{2.4a}$$

$$(\partial_{yy}+\partial_{zz})\Sigma - m^2_{2-}\Sigma = 0, \quad m^2_{2-} := \frac{2(a_0-a_1)}{b_1+b_2}. \tag{2.4b}$$

The parameters $m^2_{2\pm}$ have a simple physical interpretation. They represent masses of the spin-$2^\pm$ torsion modes with respect to the M_4 background [22,23],

$$\bar m^2_{2+} = \frac{2a_0(a_0-a_1)}{a_1(b_1+b_4)}, \qquad \bar m^2_{2-} = \frac{2(a_0-a_1)}{b_1+b_2}.$$

In M_4, the physical torsion modes are required to satisfy the conditions of no ghosts (positive energy) and no tachyons (positive m^2) [22–24]. However, for spin-2^+ and spin-2^- modes, the requirements for the absence of ghosts, given by the conditions $b_1+b_2 < 0$ and $b_1+b_4 > 0$, do not allow for both m^2 to be positive. Hence, only one of the two modes can exist as a propagating mode (with finite mass), whereas the other one must be "frozen" (infinite mass).

Important point to be noted is that the two spin-2 sectors have very different dynamical structures.

- In the spin-2^- sector, the infinite mass of the spin-2^+ mode implies $\Theta = 0$, while (1ST) gives $Q = 0$, which is nothing other than the GR field equation for metric. Consequently, the presence of torsion has no influence on the metric.
- In the spin-2^+ sector, the infinite mass of the spin-2^- mode leads to $\Sigma = 0$, whereas (1ST) gives that Q is proportional to Θ, with $\Theta \neq 0$. Leading to the conclusion that the torsion function Θ has a decisive dynamical influence on the metric.

We shall focus our attention on the spin-2^+ sector, where the metric appears to be a genuine dynamical effect of PGT.

2.2.3 Solutions in the spin-2^+ sector

After introducing polar coordinates $y = \rho\cos\varphi, z = \rho\sin\varphi$, Eq. (2.4a) takes the form

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)\Theta - m^2\Theta = 0. \tag{2.5a}$$

Looking for a solution of Θ in the form of a Fourier expansion,

$$\Theta = \sum_{n=0}^{\infty}\Theta_n(\rho)(c_n e^{in\varphi} + \bar c_n e^{-in\varphi}),$$

we obtain:

$$\Theta''_n + \frac{1}{\rho}\Theta'_n - \left(\frac{n^2}{\rho^2} + m^2\right)\Theta_n = 0, \tag{2.5b}$$

where prime denotes $d/d\rho$.

The general solution of Eq. (2.5b) has the form

$$\Theta_n = c_{1n}J_n(-im\rho) + c_{2n}Y_n(-im\rho), \qquad n = 0,1,2,\ldots \tag{2.6}$$

where J_n and Y_n are Bessel functions of the 1st and 2nd kind, respectively.

2.2.4 *Solutions for the metric function* $\boldsymbol{H}$

For a given Θ, the first PGT field equation $a_0 Q = (a_1 - a_0)\Theta$, with Q defined in (2.5), represents a differential equation for the metric function H:

$$(\partial_{yy} + \partial_{zz})H = \frac{2(a_1 - a_0)}{a_0}\Theta. \tag{2.7}$$

This is a second order, linear nonhomogeneous differential equation, and its general solution can be written as

$$H = H^h + H^p,$$

where H^h is the general solution of the homogeneous equation, and H^p a particular solution of (2.7). One finds that there is a simple particular solution for H:

$$H^p = \sigma V, \qquad \sigma = \frac{2(a_1 - a_0)}{m^2 a_0}. \tag{2.8a}$$

On the other hand, H^h coincides with the general vacuum solution o GR_Λ, see (2.7). Since our idea is to focus on the genuine torsion effect on the metric, we choose $H^h = 0$ and adopt H^p as the most interesting PGT solution for the metric function H. Thus, we have

$$H_n = \sigma \Theta_n. \tag{2.8b}$$

The solutions for torsion functions are given in Appendix A.

3 Geodesic motion

In this section we shall examine the *geodesic motion* of particles in the field of the massive gravitational wave with torsion. We shall consider the motion of spinless particles in a gravitational field, which follow geodesic lines. It is known that torsion affects the motion of the particles with spin by causing its precession [19,25]. However, the gravitational waves with torsion, which we are considering are intrinsically different from the well-known spherically symmetric (static or stationary) solutions of PGT [26,27] (for review see [28]). The metric of these spherically symmetric solutions is "independent" of torsion in the sense that it represents Schwarzschild (or Schwarzschild AdS, Kerr etc.) metric and the motion of spinless particles is not affected by the presence torsion. For the gravitational wave solution (2.5), metric crucially depends on torsion, as we noted in the previous section.

Christoffel connection The non-vanishing components of Christoffel connection in polar coordinates are given by

$$\begin{aligned}
&\tilde{\Gamma}^v{}_{uu} = \frac{1}{2}\partial_u H, \qquad \tilde{\Gamma}^v{}_{u\rho} = \frac{1}{2}H', \qquad \tilde{\Gamma}^v{}_{u\varphi} = \frac{1}{2}\partial_\varphi H,\\
&\tilde{\Gamma}^\rho{}_{uu} = \frac{1}{2}H', \qquad \tilde{\Gamma}^\rho_{\varphi\varphi} = -\rho,\\
&\tilde{\Gamma}^\varphi{}_{uu} = \frac{1}{2\rho^2}\partial_\varphi H, \qquad \tilde{\Gamma}^\varphi{}_{\rho\varphi} = \frac{1}{\rho},
\end{aligned} \tag{3.1}$$

where $H' := \partial_\rho H$.

Let us mention that we shall consider the solution with non-trivial contribution to metric function (and consequently Christoffel connection) stemming from the presence of torsion.

Geodesic equations. The geodesic equation for u takes the expected form

$$\frac{d^2u}{d\lambda^2} = \ddot{u} = 0. \tag{3.2}$$

Therefore without the loss of generality we can assume $u \equiv \lambda$.

The equation for v, ρ and φ are given by:

$$\ddot{v} + \frac{1}{2}\partial_u H + H'\dot{\rho} + \partial_\varphi H\dot{\varphi} = 0. \tag{3.3}$$

$$\ddot{\rho} + \frac{1}{2}H' - \rho\dot{\varphi}^2 = 0, \tag{3.4}$$

$$\ddot{\varphi} + \frac{1}{2\rho^2}\partial_\varphi H + \frac{2}{\rho}\dot{\rho}\dot{\varphi} = 0. \tag{3.5}$$

We shall solve the geodesic equations numerically, but let us first make some reasonable simplifications.

First, v appears only as a second derivative because H is independent of it. Consequently, we have a shift symmetry

$$v \to v + c_0 + c_1 u, \tag{3.6}$$

which means that initial conditions at time u_i can be chosen as

$$v[u_i] = v'[u_i] = 0. \tag{3.7}$$

Second, in the metric function H there is a factor

$$\sigma = \frac{2(a_1 - a_0)}{m^2 a_0}, \tag{3.8}$$

where $a_0 = \frac{1}{16\pi G}$ is coupling constant of general relativity and a_1 corresponds to correction in the action stemming from torsion. Experimental results suggest that a_1 is much smaller than a_0 so we can approximate

$$\sigma \approx -\frac{2}{m^2}. \tag{3.9}$$

Also, we can introduce reduced variables

$$\mathrm{v} = m^2 v,\ r = m\rho, \tag{3.10}$$

while φ remains the same. In these variables geodesic equations do not have explicit dependence on m and have more suitable form for numerical calculations.

3.1 Memory effect

We have one more unknown in geodesic equations and that is the form of functions c_{1n} and c_{2n}. We expect that their exact form is not specially important, as long as they sufficiently fast tend to zero at infinity. But, we encountered numerical

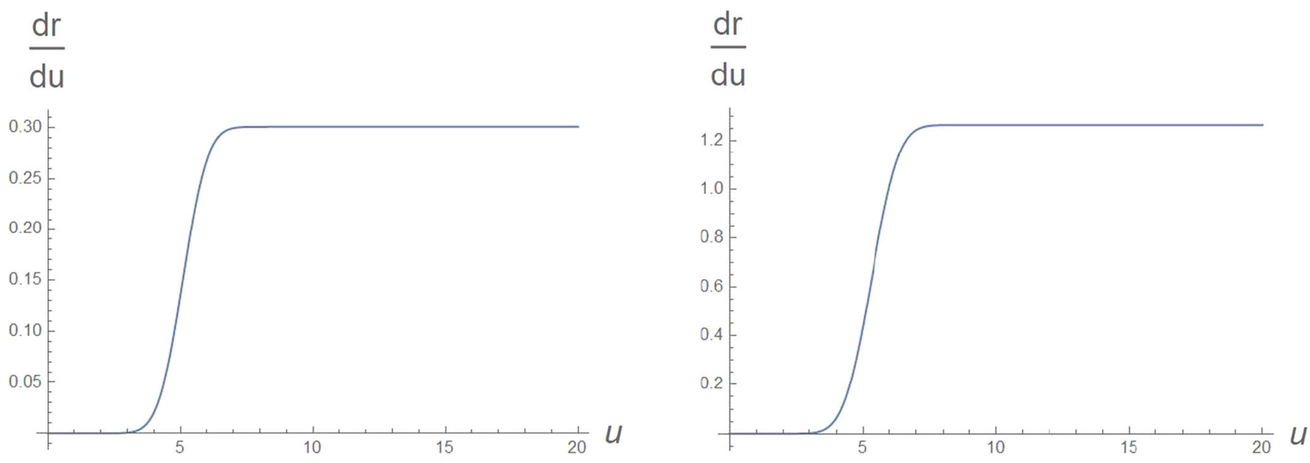

Fig. 1 The plot for the particle velocity $\dot{r}$ for $r[0] = 1$ and $r[0] = 2$

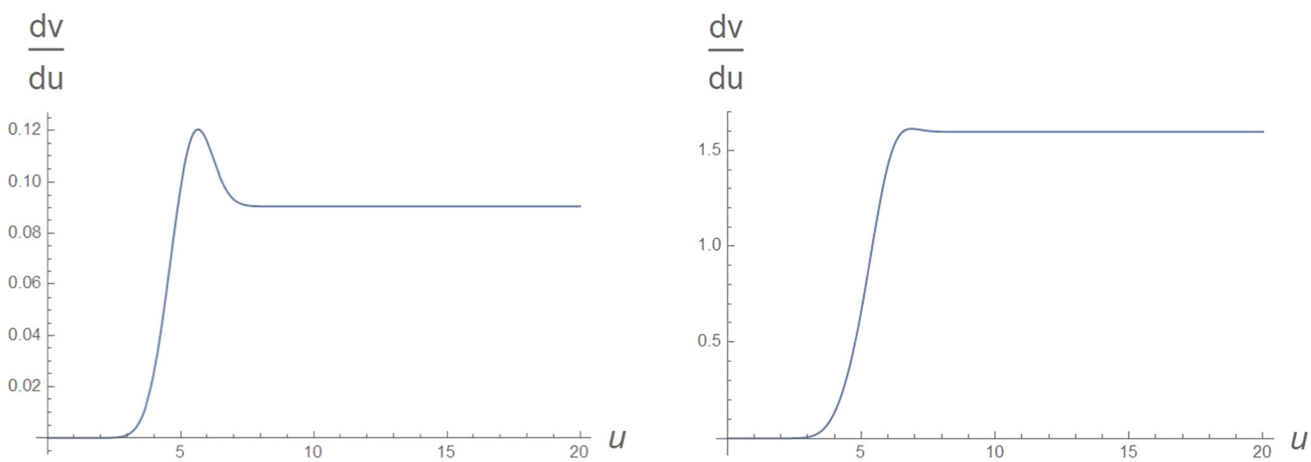

Fig. 2 The plot for the particle velocity $\dot{\text{v}}$ for $r[0] = 1$ and $r[0] = 2$

problems because for polynomial fall-off the software cannot handle the computational complexity. Because, of this problem we decided to focus to Gaussian form of functions, more precisely to the form $e^{-(u-5)^2}$. For the initial time we chose $u = 0$. As we already noted the initial conditions for $\tilde{v}$ are

$$v[0] = \dot{v}[0] = 0, \tag{3.11}$$

and we assume that particle is initially at rest

$$\dot{\rho}[0] = \dot{\varphi}[0] = 0. \tag{3.12}$$

So, the only variable inputs are $\rho[0]$ and $\varphi[0]$ as well as the modes c_{1n} and c_{2n} we are including.

Mode J_0. In this case we set $H = J_0(-ir)e^{-(u-5)^2}$. Because nothing explicitly depends of φ it remains the same as at initial time. In Fig. 1, we plot radial velocity $\dot{r}$ in function of u. While the Fig. 2 shows the value of velocity $\dot{v}$.

Mode J_2. In this case we set $H = J_2(-ir)e^{-(u-5)^2}\sin(2\varphi)$. In Fig. 3, we plot the radial velocity $\dot{r}$. In Fig. 4, we show the value of angle φ. We see that in angular direction we have displacement memory effect in contrary to the others where we have velocity memory effect.

Mode J_4. In this case we set $H = J_4(-ir)e^{-(u-5)^2}\sin(4\varphi)$. In Fig. 5, we plot the radial velocity $\dot{r}$. In Fig. 6, we show the value of angle φ.

4 Discussion

We studied the geodesic motion of test particle in the presence of the pp wave with torsion. Our analysis discovered that particles, after the passage of the wave, show a combination of displacement and velocity memory effect. In the angular direction we discovered that pp waves induce displacement memory effect, for comparison velocity memory effect takes place for axial gravitational waves [29]. After the passage of axial wave burst particles rotate with constant angular velocity around the symmetry axis.

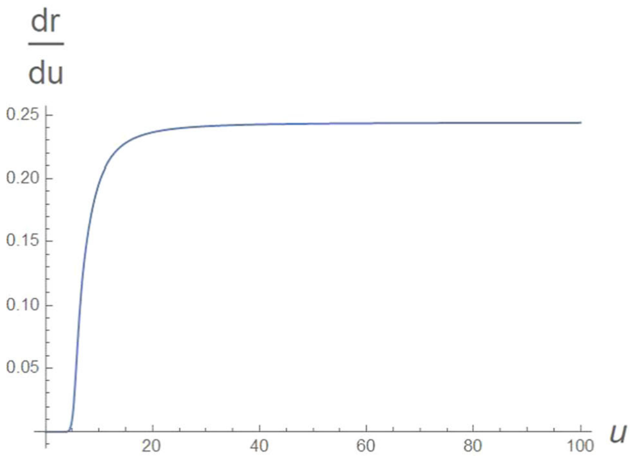

Fig. 3 The plot for the particle velocity $\dot{r}$ for $r[0] = 1$ and $r[0] = 2$ in both plots $\varphi[0] = 0$

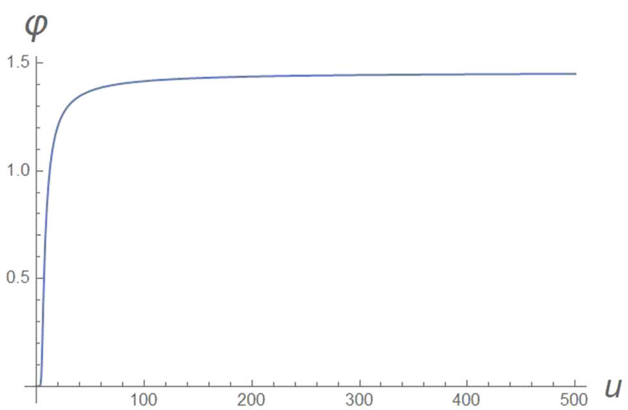

Fig. 4 The plot for the particles angular position φ for $\varphi[0] = 0$ and $\varphi[0] = \frac{\pi}{2}$ in both plots $r[0] = 1$

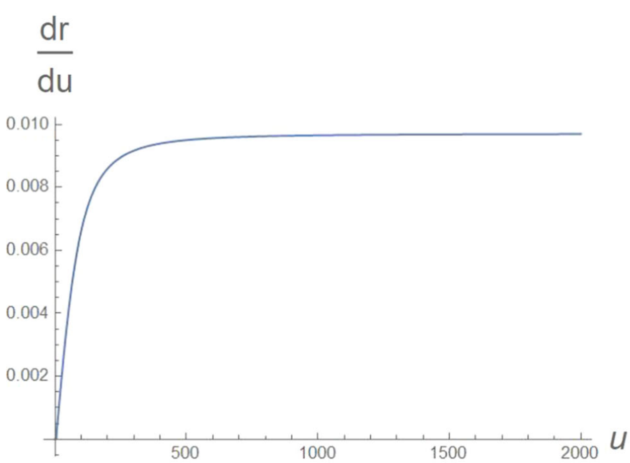

Fig. 5 The plot for the particle velocity $\dot{r}$ for $r[0] = 1$ and $r[0] = 2$ in both plots $\varphi[0] = 0$

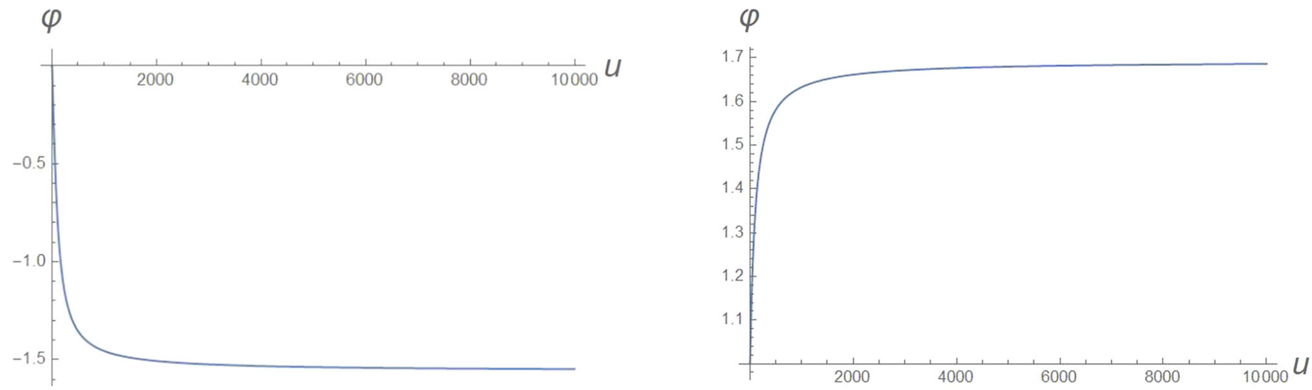

Fig. 6 The plot for the particles angular position φ for $\varphi[0] = 0$ and $\varphi[0] = 1$ in both plots $r[0] = 1$

Because we defined new variables $\text{v} = m^2 v$ and $r = m\rho$ our results have to be interpreted with taking into account the reasonable value of m. According to the CERN results we expect that possible mass of tordion is no less that 10 TeV

$$m \geq 10\,\text{TeV}. \tag{4.1}$$

This order of magnitude of the mass is equivalent to the length scale ℓ_m

$$\ell_m \approx 10^{-20}\,\text{m}. \tag{4.2}$$

Due to the very large mass of tordion or, equivalently, very small length scale the physical values of v and ρ are very small and probably not observable. Fortunately φ is insensitive to the value of the mass and offers a possible observable effect. We see from Figs. 4 and 6 that depending on the initial angular position the particle will be rotated by a different angle. Consequently, the particles initially set at some positions on a circle will be rotated relatively to each other. This is the possible experimental setup for the detection of torsion waves.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the data that we obtained are included in the article.]

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A Solutions for the torsion functions K_α

In the spin-2^+ sector, the torsion functions K_α can be determined (by using the condition $\Sigma = 0$) from the equations:

$$\partial_y \Theta + m^2 K_y = 0, \qquad \partial_z \Theta + m^2 K_z = 0. \tag{A.1}$$

Going over to polar coordinates,

$$K_y = K_\rho \cos\varphi - \frac{K_\varphi}{\rho} \sin\varphi, \qquad K_z = K_\rho \sin\varphi + \frac{K_\varphi}{\rho} \cos\varphi,$$

the previous equations are transformed into

$$K_\rho = -\frac{1}{m^2} \partial_\rho V, \qquad K_\varphi = -\frac{1}{m^2} \partial_\varphi V, \tag{A.2a}$$

or equivalently, in terms of the Fourier modes,

$$K_{\rho n} = -\frac{1}{m^2} \frac{p}{q} \partial_\rho \Theta_n, \qquad K_{\varphi n} = -\frac{1}{m^2} n \Theta_n, \tag{A.2b}$$

where $K_\varphi = \sum_{n=1}^{\infty} (d_n e^{in\varphi} + \bar{d}_n e^{-in\varphi})$ with $d_n = -ic_n$, and similarly for K_ρ.

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Note on asymptotic symmetry of massless scalar field at null infinity

Dejan Simić *

Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade-Zemun, Serbia

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In this article we address the question of asymptotic symmetry of massless scalar field at null infinity. We slightly generalize notion of asymptotic symmetry in order to make sense for the theory without gauge symmetry. Derivations of the results are done in two different ways, using Hamiltonian analysis and using covariant phase space. The results are in agreement with the ones previously obtained by various authors for dual 2-form field and with the results obtained starting from scalar soft theorem.

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I. INTRODUCTION

Asymptotic symmetries at null infinity [1] have attracted substantial amount of attention in last decade (see Ref. [2] for overview) because of their connection with soft theorems and memory effect.

A massless scalar field has a soft theorem associated to it [3] and we expect that there is the corresponding asymptotic symmetry. A problem arises when we try to understand what this asymptotic symmetry is, because there is no gauge symmetry in the theory.

There are many approaches to understand asymptotic symmetry of a massless scalar and its connection to the soft scalar theorem. Starting from the soft scalar theorem the authors in [4] derived what the asymptotic symmetry should be and generalized the result to all even dimensions [5]; a scalar field is dual to 2-form field which is the theory with gauge symmetry. By passing on the dual 2-form formulation of scalar field the standard approach is used to derive the asymptotic symmetry [6,7]. This lack of symmetry in one and presence in other formulation of the theory led some to the conclusion that the symmetry theory is a union of symmetries in all formulations of the theory [8]. This is an unsatisfactory solution because dual theories should have same number of symmetries. To better understand this problem an approach that relies on compact extra dimensions [9] is proposed. There is also related work in spacelike infinity based on the invariance of the symplectic form under Poincaré transformations [10] which, also, did not yield symmetry for the scalar field but recovered results obtained at null infinity for 2-form field. This work suggests that a scalar field search for missing symmetry is not a problem of finding boundary degrees of freedom as it was in electromagnetism [11]. This approach is generalized to any massless boson in [12,13] with the same conclusion that there is no apparent symmetry for a massless scalar.

Differences between our approach and previous approaches is that we work with scalar field theory and not with dual 2-form or some extended formulation and propose a generalization of the notion of asymptotic symmetry at null infinity. The standard understanding of asymptotic symmetry starts with gauge symmetry, with some asymptotic conditions imposed; the part of gauge symmetry that respects the asymptotic conditions is allowed. The next step is the derivation of the associated conserved charges, that are given as an integral over the corner at infinity—generically some allowed gauge transformations will have identically zero charges and are called trivial gauge transformations. Allowed gauge transformations with nonzero charges are an asymptotic symmetry of the theory, or in other words, allowed modulo the trivial gauge transformations. To extend the standard notion of asymptotic symmetry we start from the observation that all the calculations are done asymptotically and that everything is ultimately about charges and their conservation. Then, it is natural to propose what the asymptotic symmetry for any theory, with or without gauge symmetry, should be. The proposed generalization is as follows. If an asymptotic transformation, not *a priori* defined in whole spacetime, can be represented with a nonzero and conserved charge then it is an asymptotic symmetry of the theory. Conservation of charges in this setup can have a more subtle meaning; for example, conservation for symmetries at null infinity means that charges at past and future null infinity are equal, and this is established by more detailed inspection of the properties of the solutions. Starting from this definition of asymptotic symmetry we can, at least in principle, obtain a globally defined transformation if it is

*dsimic@ipb.ac.rs

consistent with equations of motions. The globally defined transformation is the one which maps solutions onto solutions and in the asymptotic region coincides with the asymptotic symmetry; this would explicitly be done by the boundary-bulk propagator. Notice, that when dealing with gauge theory we can think of asymptotic symmetry in this way, with the additional need to fix trivial gauge transformations in order to obtain a unique boundary-bulk propagator. This generalization of asymptotic symmetry seems justified for null infinity, as will be demonstrated in the main part of the paper, but to see if it is possible to apply it to more general case requires more investigation.

We demonstrate in the rest of the article how this extended asymptotic symmetry can be obtained using the example of massless scalar field. We do it both in the Hamiltonian and covariant phase-space formalism as these are two dominant approaches both with their own pros and cons.

II. HAMILTONIAN CHARGES

A. Coordinates

We will work in four spacetime dimensions although it is trivial to see that results hold in other dimensions. We use mostly minus convention for the Minkowski metric. In the Hamiltonian approach we use light cone coordinates; $u = t - r$, $v = t + r$, and x^a are the coordinates on the sphere. The metric is given by

$$ds^2 = dudv - g_{ab}dx^a dx^b, \tag{2.1}$$

with

$$g_{ab} = \left(\frac{u - v}{2}\right)^2 \gamma_{ab}, \tag{2.2}$$

where γ_{ab} is the metric on the unit sphere. An explicit form of the metric on the sphere γ_{ab} is not important for the details of our analysis and can be taken as the standard form or in complex coordinates as in [1,2]. Future null infinity J^+ is reached in the limit $v \to \infty$ with the other coordinates fixed. Additionally, with past null infinity J^- we obtain in the limit, $u \to -\infty$, with other coordinates fixed.

In the covariant phase-space formalism we can use light cone coordinates or outgoing coordinates for future null infinity with the metric

$$ds^2 = du^2 + 2dudr - r^2\gamma_{ab}dx^a dx^b, \tag{2.3}$$

where in limit, $r \to \infty$, we reach future null infinity J^+. The metric in ingoing coordinates, that are suitable for past null infinity, is given by

$$ds^2 = dv^2 - 2dvdr - r^2\gamma_{ab}dx^a dx^b, \tag{2.4}$$

and past null infinity J^- we get in $r \to \infty$ limit.

B. Canonical analysis

Action of self-interacting massless scalar in light cone coordinates is given by

$$S = \int d^4x\sqrt{-g}\left(\partial_u\phi\partial_v\phi - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right), \tag{2.5}$$

where $V(\phi)$ is any polynomial with degree higher than 3; the reason why ϕ^3 interaction must be excluded will be described later in text. We will focus on future null infinity J^+ as it is trivial to see that the same calculation is valid for past null infinity J^-, just by changing u and v. Massless particles evolve along the v direction and for $v \to \infty$ they go to J^+. For this reason it is valid to take v as an evolution parameter in Hamiltonian dynamics. For more details on Hamiltonian dynamics see Refs. [14,15].

The impulse is obtained by the standard definition

$$\pi = \frac{\partial L}{\partial\partial_v\phi} = \sqrt{-g}\partial_u\phi. \tag{2.6}$$

Because the right-hand side does not contain a derivative over v we get the constraint (at every point)

$$\psi = \pi - \sqrt{-g}\partial_u\phi \approx 0. \tag{2.7}$$

The Poisson bracket of constraints at different points is

$$\Omega(v, x, x') = \{\psi(v, x), \psi(v, x')\} = -2\partial_u\delta(x - x'), \tag{2.8}$$

where we introduced abbreviation $x = (u, x^a)$.

The total Hamiltonian determines the evolution of the system [14] and is given by

$$H_T = H + \int dud^2x^a\Lambda\psi, \tag{2.9}$$

where H is Hamiltonian that we calculate in usual manner

$$\begin{aligned} H &= \int dud^2x^a(\pi\partial_v\phi - L) \\ &= \int dud^2x^a\sqrt{-g}\left(\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi + V(\phi)\right). \end{aligned} \tag{2.10}$$

Since the Hamiltonian is obtained when integrating over a Cauchy surface, and $v = \text{const}$ is a Cauchy surface only when v goes to infinity, the rest of analysis is valid only in immediate vicinity of J^+. The consistency condition of the constraint

$$\{\psi, H_T\} = 0, \tag{2.11}$$

gives the solution for a multiplier Λ up to arbitrary u-independent function. The presence of an arbitrary function means that there is a first-class constraint hidden in the second-class constraint ψ. An alternative way to see that there is first-class constraint hidden among the second class is by looking for the presence of a zero mode in $\Omega(v, x, x')$, see Ref. [15]. The eigenvalue equation takes form

$$\int d^3x' \Omega(v, x, x') k(x') = 2\partial_u k(x), \tag{2.12}$$

and $k(x)$ is a zero mode if and only if it does not depend on u.

C. Charges

In order to construct the charge we will adopt the approach used in [16] where the same situation appeared. We construct the generator using the whole constraint ψ and not only the first-class part but we multiply it with the u-independent function Λ and not completely arbitrary one

$$\Psi = \int dud^2x^a \Lambda \psi. \tag{2.13}$$

This is enough to select the first class from the full constraint, see Ref. [16] for more details.

We demand that the variation of the generator is well-defined, meaning that there is no surface term

$$\delta\Psi = \int A\delta\phi + B\delta\pi, \tag{2.14}$$

which leads to the need to add a surface term to the generator [17]

$$\tilde{\Psi} = \Psi + Q = \int \Lambda\pi, \tag{2.15}$$

where the surface term is given by

$$Q = \int dud^2x^a \sqrt{g} \Lambda \partial_u \phi, \tag{2.16}$$

and represents the charge associated to the transformation. The asymptotic behavior of scalar field near null infinity is

$$\phi \propto \frac{2\varphi(u, x^a)}{v} + \mathcal{O}\left(\frac{1}{v^2}\right), \tag{2.17}$$

where this is the usual $\frac{\varphi}{r}$ asymptotic used in [4,10] (just in light cone coordinates). We immediately derive the behavior of Λ from

$$\delta_\Lambda \phi = \{\phi, \tilde{\Psi}\} = \Lambda, \tag{2.18}$$

and get

$$\Lambda \propto \frac{2\lambda(x^a)}{v} + \mathcal{O}\left(\frac{1}{v^2}\right). \tag{2.19}$$

Substituting the asymptotic behaviors of the field and parameter Λ into the expression for charge, we obtain after simple algebra,

$$\begin{aligned} Q &= \int dud^2x^a \sqrt{\gamma} \lambda(x^a) \partial_u \varphi(u, x^a) \\ &= -\int d^2x^a \sqrt{\gamma} \lambda(x^a) \varphi(-\infty, x^a). \end{aligned} \tag{2.20}$$

The last equality follows from the fact that there are no massive particles because then the field φ goes to zero at $u \to \infty$ (see Refs. [2,4]). The result is in agreement with the expressions for charge obtained in [4–7].

We can repeat the same calculation at past null infinity J^- and obtain the charge

$$\begin{aligned} Q_{J^-} &= \int dvd^2x^a \sqrt{\gamma} \lambda(x^a) \partial_v \varphi(v, x^a) \\ &= \int d^2x^a \sqrt{\gamma} \lambda(x^a) \varphi(\infty, x^a). \end{aligned} \tag{2.21}$$

At first glance these two charges are unrelated as the parameter λ at past and future null infinity are not connected, and it is not obvious how to establish conservation in any sense. Careful inspection of the equations of motion [4] reveals that the asymptotic values of the field at past and future null infinity are connected. Namely, they are equal for antipodal points that approach spatial infinity

$$\varphi_{J^+}(u = -\infty, x^a) = \varphi_{J^-}(v = +\infty, -x^a). \tag{2.22}$$

The reason for this can be traced back to the discontinuity of boosts at spatial infinity [2]. If we impose the antipodal matching condition $\lambda_{J^+}(x^a) = -\lambda_{J^-}(-x^a)$ on parameter λ, then we obtain equality of charges at past and future null infinity and retrieve the conservation of charges.

III. COVARIANT PHASE SPACE

Now we analyze the symmetry at null infinity of massless scalar field in the covariant phase-space approach (for an introduction see Ref. [18]). How to systematically construct charges is well elaborated on in [19].

A. Symplectic form

The starting point is the variation of the action

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\right), \tag{3.1}$$

which is easy to calculate

$$\delta S = \int d^4x\Big(-\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)\delta\phi - \sqrt{-g}\frac{\partial V}{\partial\phi}\delta\phi + \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\delta\phi)\Big), \tag{3.2}$$

from which we obtain the equation of motion (EOM)

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + \sqrt{-g}\frac{\partial V}{\partial\phi} = 0, \tag{3.3}$$

as well as the presymplectic potential

$$\theta^\mu = \sqrt{-g}g^{\mu\nu}\partial_\nu\phi\delta\phi. \tag{3.4}$$

The symplectic form on spacelike surfaces is given by the standard formula

$$\Omega = \delta\int_\Sigma n_\mu\theta^\mu, \tag{3.5}$$

where Σ is Cauchy surface with the orientation given by unit normal n_μ that points toward the future, see Ref. [19]. Generally, the symplectic form is a surface term obtained by applying Stokes theorem on $\int d^4x\partial_\mu\delta\theta^\mu$ and keeping only the relevant surface term. For future null infinity only terms at J^+ and at past null infinity only terms at J^- with the additional change of sign because normal points to the "past".

At future null infinity, assuming the same asymptotic behavior of the scalar field as before, the symplectic form is given by

$$\Omega = \int_{J^+} dud^2x^a\sqrt{\gamma}\delta\partial_u\varphi\delta\varphi. \tag{3.6}$$

The same result, only with u replaced by v, holds at past null infinity.

B. Symmetry at null infinity

The symplectic form is always invariant under the transformation

$$\delta_\lambda\varphi = \lambda, \tag{3.7}$$

where λ is field independent $\delta\lambda = 0$. This is the trivial invariant of the symplectic form that is always present and does not automatically imply symmetry. An additional condition that the transformation must fulfill to be a symmetry is that it maps solution onto a solution.

The equation of motion in (u, r, x^a) coordinates is

$$\sqrt{\gamma}r^2\partial_u\partial_r\phi + \sqrt{\gamma}\partial_r(r^2\partial_u\phi) + \partial_a(\sqrt{\gamma}\gamma^{ab}\partial_b\phi) + \sqrt{\gamma}r^2\frac{\partial V}{\partial\phi} = 0. \tag{3.8}$$

Assuming the same asymptotic behavior of the scalar field $\frac{\varphi}{r}$ as before, we see that if there was $\frac{g}{3}\phi^3$ term in the potential V, the leading term is of order $\mathcal{O}(1)$ and reads

$$g\varphi^2 = 0, \tag{3.9}$$

namely ϕ^3 interaction makes asymptotic theory trivial. Next orders of expansion of the EOM give the solution for subleading terms as functions of free data φ. Completely analogous equations hold in ingoing coordinates (v, r, x^a) at past null infinity.

The invariance of symplectic form under transformation is equivalent to the claim that we can obtain the charge associated to that transformation via the following equation [19]:

$$\delta Q = -I_{X_\lambda}\Omega, \tag{3.10}$$

where I_{X_λ} is contraction that acts as $I_{X_\lambda}\delta\varphi = \lambda$. Charge Q is easily calculated using previous equation

$$Q = \int dud^2x^a\sqrt{\gamma}(\lambda\partial_u\varphi - \partial_u\lambda\varphi), \tag{3.11}$$

where there is an additional term proportional to $\partial_u\lambda$ in comparison to the result for charge in the Hamiltonian approach. We point out again that charge is not useful if there is not some kind of conservation associated to it. We see that if parameter λ is u independent we get the same result for charge as in the Hamiltonian approach and it is conserved with the addition of the antipodal matching condition. We argue that for any λ that does depend on u this is not possible. By doing partial integration, the charge can be transformed into

$$Q_{J^+} = -\int d^2x^a\sqrt{\gamma}\lambda_{J^+}\varphi_{J^+}(u=-\infty) - 2\int dud^2x^a\sqrt{\gamma}\partial_u\lambda_{J^+}\varphi_{J^+}, \tag{3.12}$$

and analogous for past null infinity J^-. The first term is the same as Hamiltonian charge that is conserved due to the antipodal matching condition. This implies that second term must be conserved separately as

$$\int dud^2x^a\sqrt{\gamma}\partial_u\lambda_{J^+}\varphi_{J^+} = \int dvd^2x^a\sqrt{\gamma}\partial_v\lambda_{J^-}\varphi_{J^-}. \tag{3.13}$$

We expect that fields at past and future null infinity can be related via a transformation that should be nonlinear due to the interaction

$$\varphi_{J^+} = \int dvd^2x'^a S(u, x, v, x'; \varphi_{J^-})\varphi_{J^-}, \tag{3.14}$$

which after substitution into the above equation leads to

$$\int dud^2x^a S(u,x,v,x';\varphi_{J^-})\partial_u\lambda_{J^+} = \partial_v\lambda_{J^-}(\varphi_{J^-}). \tag{3.15}$$

This means that starting from the field-independent parameter λ_{J^+} we get field-dependent parameter λ_{J^-} at past null infinity. This contradicts the starting and crucial assumption; for construction of the charge the parameter λ must be field independent. Consequently, we are forced into taking u independent λ_{J^+} and v independent λ_{J^-}, in agreement with the Hamiltonian approach.

IV. DISCUSSION

We derived asymptotic symmetry of massless scalar field at null infinity directly and not by passing to the dual 2-form field formulation.

The first derivation is the Hamiltonian and relies crucially on the presence of constraints in the theory. In the scalar field case, constraints appear only is special coordinates, we worked in light cone coordinates, and in most of other coordinates the symmetry is completely hidden.

Covariant phase space offers a more direct and simple way of deriving symmetry. We search for transformations that leave the symplectic form at null infinity invariant and we construct the charge via the variational equation if and only if it is conserved, meaning that if it is the same at past and null infinity the transformation is really asymptotic symmetry. This gives a computational approach that can be applied to any theory and unravel the hidden asymptotic symmetries. This topic is left for future research.

Besides the direct derivation of asymptotic symmetry another unanswered question is how do these charges act. We offer our view on this in the context of the extended notion of asymptotic symmetry.

Globally defined symmetry maps solutions onto solutions. Starting from this obvious claim we can demand that action of asymptotic symmetry is extended on to the whole spacetime in a way that satisfies this requirement. This can be done when asymptotic symmetry shifts initial and final conditions at null infinity in a way that is consistent with the EOM. For an explicit expression of the action of symmetry transformation we would need boundary-bulk propagator i.e., an explicit solution with given initial or final conditions at null infinity. Because in the case of the scalar field antipodal matching conditions for field φ and parameter λ have opposite signs asymptotic symmetry cannot be extended into the bulk and is defined only asymptotically.

An open problem that remains is the derivation of the symmetry at spatial infinity. The approach of [10] shows that there is no justification for adding boundary degrees of freedom at spatial infinity for the scalar field; that is a necessary ingredient in their approach and it seems like the same holds for the approach of this article. We must conclude that there is still a long way to go if we want to fully understand symmetries in field theory.

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PECULIAR FIVE-DIMENSIONAL BLACK HOLES†

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Dejan Simić

Institute of Physics, University of Belgrade, Pregrevića 118, 11080 Zemun, Belgrade, Serbia

Abstract. *In this article, we review two black hole solutions to the five-dimensional Lovelock gravity. These solutions are characterized by the non-vanishing torsion and the peculiar property that all their conserved charges vanish. The first solution is a spherically symmetric black hole with torsion, which also has zero entropy in the semi-classical approximation. The second solution is a black ring, which is the five-dimensional uplift of the BTZ black hole with torsion in three dimensions.*

Key words: *black hole, torsion, alternative theories of gravity*

1. INTRODUCTION

Gravity is the interaction which we are aware of for the longest period of time of all the known interactions but, paradoxically, it is also the one we know the least about. Quantum gravity is the goal which drives the most of modern research in high-energy physics. Unfortunately, the realm of quantum gravity is beyond our current experimental abilities and researchers have to come up with ingenious ideas how to go around this. Fortunately, the effects of quantum gravity are visible in black hole physics already in the semi-classical level. This makes black holes the most important objects in gravity and this is the very reason why they were extensively studied in the past century. Now, it is well known that general relativity cannot be the whole story and for this reason, for different purposes, research went in the direction of alternative theories of gravity. Some hope to construct a good theory of quantum gravity in this way, others, less ambitious, hope to gain a small insight into the quantum effect of gravity.

Lovelock gravity is an interesting generalization of general relativity, which is a unique ghost-free higher derivative theory of gravity with second-order field equations. In three and four dimensions, Lovelock gravity reduces to general relativity. Originally,

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Corresponding author: Dejan Simić
Institute of Physics, University of Belgrade, Pregrevića 118, 11080 Zemun, Belgrade, Serbia
E-mail: dsimic@ipb.ac.rs
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Lovelock gravity is formulated in the second order, i.e. the metric tensor is a dynamical field. More interesting is the first order formulation in which we treat the vielbein and spin connection as independent dynamical variables of our theory. A theory formulated in this way is no longer torsion-less but may and does contain solutions with non-trivial torsion. The first order formulation is convenient because it contains torsion-less theory as a limit and is conceptually necessary for coupling with fermionic matter fields and supersymmetric extensions of the theory. The Einstein-Cartan theory, the first-order formulation of general relativity, has the property that all vacuum solutions are without torsion, the vacuum solutions of the Lovelock gravity are more complicated since there exist solutions with non-zero torsion. Lovelock gravity possesses a large number of black hole solutions (see Aros et al., 2001; Boulware and Deser, 1985; Camanho and Edelstein, 2013; Canfora et al., 2007; Canfora et al., 2008; Cai Rong-Gen and Ohta, 2006; Cai Rong-Gen et al., 2010; Cvetković and Simić, 2016; Cvetković and Simić, 2018; Dotti et al., 2007; Garraffo and Giribet, 2008; Kastor and Mann, 2006; Maeda et al., 2011, and references therein). Many of them possess exotic properties, for example, zero mass (Cai Rong-Gen et al., 2010; Cvetković and Simić, 2016; Cvetković and Simić, 2018), peculiar topology of the event horizon (Cvetković and Simić, 2016; Kastor and Mann, 2006; Maeda et al., 2011; Ray, 2015), etc. This brings us to the problem of black hole uniqueness. Solutions of general relativity are highly constrained, but things change when we go into higher dimensions. In higher dimensions, black hole solutions appear which have the non-spherical topology of the event horizon, more precisely, black string, black ring and black brane (Emparan and Reall, 2002; Emparan and Reall, 2008; Horowitz and Strominger, 1991; Kastor and Mann, 2006). It is not uncommon that these black holes suffer from various instabilities, for example, black strings and branes suffer from Gregory-Laflamme instability (Gregory and Lafflamme, 1993), and will decay into a black hole with a spherical horizon. We see that gravity in higher dimensions is a very interesting area of research, full of surprising discoveries, whose importance is in its applications in many different areas.

In the end, a few words on notation. We use the following conventions: Lorentz signature is mostly minus, local Lorentz indices are denoted by the middle letters of the Latin alphabet, while space-time indices are denoted by the letters of the Greek alphabet. For notational simplicity, we mostly use differential forms instead of coordinate notation, in all formulas the wedge product is not written explicitly.

2. Lovelock Gravity

Lovelock gravity (Lovelock, 1971; Lovelock, 1972) is a minimalistic generalization of general relativity and is one of many alternative theories of gravity which is under constant investigation.

The first-order formulation of gravity as dynamical variables has the vielbein e^i 1-form and the spin connection $\omega^{ij} = -\omega^{ji}$ 1-form. In local coordinates x^μ, we can expand the vielbein and the connection 1-forms as $e^i = e^i_\mu dx^\mu$, $\omega^{ij} = \omega^{ij}_\mu dx^\mu$. Gauge symmetries of the theory are local translations (diffeomorphisms) and local Lorentz rotations, parameterized by ξ^μ and ε^{ij}, respectively.

From the dynamical variables, we can construct field strengths torsion T^i and curvature R^{ij} (2-forms), which are given as

$$T^i = \nabla e^i \equiv de^i + \omega^{ik} e_k ,$$

$$R^{ij} = d\omega^{ij} + \omega^i_k \omega^{kj} ,$$

where $\nabla = dx^\mu \nabla_\mu$ is the exterior covariant derivative.

Metric tensor $g_{\mu\nu}$ can be constructed from the vielbein e^i_μ and flat metric η_{ij}

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu , \quad \eta_{ij} = (+,-,-,\ldots) .$$

The anti-symmetry of ω^{ij} is equivalent to the metric condition $\nabla g = 0$. The geometry whose connection is restricted by the metric condition (metric-compatible connection) is known as Riemann-Cartan geometry.

The connection ω^{ij} determines the parallel transport in the local Lorentz basis. Because parallel transport is a geometric operation it is independent of the basis. This property is encoded into PGT via the so-called *vielbein postulate*, which implies

$$\omega_{ijk} = \Delta_{ijk} + K_{ijk} ,$$

where Δ is the Levi-Civita connection, and $K_{ijk} = -\frac{1}{2}(T_{ijk} - T_{kij} + T_{jki})$ is the contortion.

The Lagrangian of Lovelock gravity in D dimensions is given by

$$L = \sum_{p=0}^{[D/2]} \frac{\alpha_p}{D-2p} L_p ,$$

where α_p are real parameters and L_p is dimensionally continued Euler density defined in the following manner

$$\varepsilon_{i_1 i_2 \ldots i_D} R^{i_1 i_2} \ldots R^{i_{2p-1} j_{2p}} e^{D-2p} \ldots e^D .$$

Because we will be concerned with Lovelock gravity in five dimensions, we will only give equations of motion for this case. A variation of the action with respect to the vielbein e^i and spin-connection ω^{ij} leads to the following field equation

$$\varepsilon_{ijkln} (\alpha_0 e^j e^k e^l e^n + \alpha_1 R^{jk} e^l e^n + \alpha_2 R^{jk} R^{ln}) = 0 \qquad (1)$$

and

$$\varepsilon_{ijkln} (e^k e^l + 2\alpha_2 R^{kl}) T^n = 0 . \qquad (2)$$

Note that from the equation (2) it does not follow that torsion is zero in the vacuum, the explicit examples of this are given in the following sections where vacuum solutions with non-vanishing torsion are constructed.

Finding the solution to the equations (1) and (2) is greatly simplified in the torsion-less sector because the equations (2) are automatically solved in this case. For this very reason finding solutions with non-trivial torsion that exist for arbitrary values of parameters is extremely hard and still out of reach.

3. SPHERICALLY SYMMETRIC BLACK HOLE

This section is based on the results in the reference (Cvetković and Simić, 2018).

3.1. Killing vectors

We search for the static solution of equations that possesses SO(4) symmetry. Killing vectors that correspond to this symmetry are.

$$\begin{aligned}
&\xi_0 = \partial_t, \\
&\xi_1 = cos\theta\,\partial_\psi - cot\psi\, sin\theta\,\partial_\theta, \\
&\xi_2 = cos\varphi\,\partial_\theta - cot\theta\, sin\varphi\,\partial_\varphi, \\
&\xi_3 = \partial_\varphi, \\
&\xi_4 = sin\theta\cos\varphi\partial_\psi + cot\psi\, cos\theta\, cos\varphi\,\partial\theta - \frac{cot\psi}{sin\theta} sin\varphi\partial_\varphi, \\
&\xi_5 = sin\theta\, sin\varphi\,\partial_\psi + cot\psi\, cos\theta\, sin\varphi\partial_\theta + \frac{cot\psi}{sin\theta} cos\varphi\,\partial_\varphi, \\
&\xi_6 = sin\varphi\,\partial_\theta + cot\theta\, cos\varphi\,\partial_\varphi.
\end{aligned} \tag{3}$$

Independent Killing vectors are ξ_0, ξ_1, ξ_2 and ξ_3. Others are obtained as the commutator of the later. Besides the invariance under Killing vectors, we have invariance under local Lorentz transformations which form is fixed and the only non-zero are given by

$$\varepsilon^{23} = -\frac{\sin\theta}{\sin\psi}, \quad \varepsilon^{34} = -\frac{\sin\varphi}{\sin\theta}.$$

3.2. Form of the vielbein and spin connection

Invariance under Killing vectors greatly restricts the most general form of the vielbein and spin connection.

The most general metric which is invariant under Killing vectors in coordinates $x^\mu = (t, r, \psi, \theta, \phi)$ is of the form

$$ds^2 = N^2 dt^2 - B^{-2} dr^2 - r^2 (d\psi^2 + \sin\psi^2 d\theta^2 + \sin\psi^2 \sin\theta^2 d\varphi^2), \tag{4}$$

where functions N and B depend only on r.

The vielbeins e^i are chosen to be of the form

$$\begin{aligned}
&e^0 = Ndt, \quad e^1 = B^{-1}dr, \quad e^2 = r\,d\psi, \\
&e^3 = r\sin\psi\, d\theta, \quad e^4 = r\sin\psi \sin\theta\, d\varphi.
\end{aligned} \tag{5}$$

The most general form of the spin connection is

$$\omega^{01} = A_0 dt + A_1 dr, \quad \omega^{02} = A\_2 d\psi,$$
$$\omega^{03} = A_2 \sin\psi d\theta, \quad \omega^{04} = A_2 \sin\psi \sin\theta,$$
$$\omega^{12} = A_3 d\psi, \quad \omega^{13} = A_3 \sin\psi d\theta \tag{6}$$
$$\omega^{14} = A_3 \sin\psi \sin\theta d\varphi, \quad \omega^{23} = \cos\psi d\theta + A_4 \sin\psi \sin\theta d\varphi,$$
$$\omega^{24} = -A_4 \sin\psi d\theta + \cos\psi \sin\theta d\varphi, \quad \omega^{34} = A_4 d\psi + \cos\theta d\varphi,$$

where A_i are arbitrary functions of the radial coordinate.

3.3. Solution

Solution to the equations of motion (1) and (2) with the most general form of the vielbein (5) and spin connection (6) is found by straightforward computation with the use of computer assistance.

A solution exists only if functions N and B are equal and there are two solutions one of which is a well-know Boulware-Deser black hole (Boulware and Deser, 1985) and the other is

$$N^2 = B^2 = -\frac{\alpha_1}{8\alpha_2}\left(r^2 - \frac{16\alpha_2 C}{7\alpha_1} r - \frac{r_+^2}{r^2}\right). \tag{7}$$

The solution for the functions that determine the spin connection is as follows

$$A_1 = A_2 = A_3 = 0,$$
$$A_0 = \frac{\alpha_1}{\alpha_2} r + C, \tag{8}$$
$$A_4 = \sqrt{1 - 2\frac{\alpha_0}{\alpha_1} r^2}.$$

Constants $C$and r_+ characterize the solution. For simplicity, we take $C = 0$ in the following analysis of the properties. This solution exists only if the constraint between parameters holds

$$\alpha_1^2 - 12\alpha_0\alpha_2 = 0, \tag{9}$$

and if the ratio α_1/α_2 is negative. From the formula (7) we see that the metric of the black hole is asymptotically Anti de Sitter.

3.4. Properties of the solution

For the definitions and properties of curvature and torsion invariants see reference (Obukhov, 2006). Now we will give the results for the most important invariants of the black hole.

The scalar Cartan curvature is of the form

$$R = -\frac{2\alpha_1}{\alpha_2}. \tag{10}$$

The scalar Riemann curvature is given by

$$\breve{R} = \frac{\alpha_1}{2\alpha_2}(-5 - \frac{12\alpha_2}{\alpha_1 r^2} + \frac{3r_+^8}{r^8}) \ . \tag{11}$$

The quadratic torsion invariant is

$$*(T^i * T_i) = \frac{3\alpha_1}{2\alpha_2}(1 - \frac{r_+^8}{r^8}) \ . \tag{12}$$

From these invariants, we conclude that there is a singularity in the center of the black hole. An interesting point is that singularity is not visible in the Cartan curvature invariant but in the torsion invariant, which is an unusual property.

Next, we turn to the thermodynamics of the black hole and give the results for its temperature and entropy.

The temperature of the black hole is given by

$$T = \frac{(N^2)'}{4\pi} = -\frac{\alpha_1}{4\alpha_2} r_+ \ . \tag{13}$$

The proportionality of the temperature to the radius of the event horizon is not common for the black holes with a spherical topology of the event horizon, except in the case of three dimensions, and it is a nice illustration that interesting things can happen in alternative theories of gravity.

The entropy is calculated in the semi-classical level, by calculating the Euclidean partition function which has an interpretation of free energy, and it is concluded that it vanishes

$$S = 0 \, . \tag{14}$$

This result is very interesting because it is drastically different from the one in general relativity. As such it is a good check for any entropy formula. Also, because it is expected that the explanation of black hole microstates is universal, it is puzzling why this solution has such a low number of states compared to black holes in general relativity.

To every Killing vector ξ_i, we can associate conserved charge $Q(\xi_i)$, the charges are calculated in the original reference using the Nester formula (Nester, 1991) and it is obtained that all charges are zero

$$Q(\xi_i) = 0 \, . \tag{15}$$

4. Black Ring

This section is based on the results obtained in the reference (Cvetković and Simić, 2016).

4.1. Ansatz

The search for a new class of solutions is inspired by Canfora et al. (Canfora et al., 2007), who found a solution of the type $AdS_2 \times S^3$ when the coefficients in the Lagrangian satisfy the relation

$$\alpha_1^2 - 12\alpha_0\alpha_2 = 0\,. \tag{16}$$

We shall now construct another class of solutions of the "complementary" type $\Sigma^3 \times S^2$, where Σ^3 is a three-dimensional space-time and S^2 is a two-dimensional sphere. We start from the following ansatz for curvature

$$\begin{aligned} R^{ab} &= qe^a e^b, \\ R^{3a} &= R^{4a} = 0, \\ R^{34} &= -\frac{1}{r_0^2} e^3 e^4, \end{aligned} \tag{17}$$

and torsion

$$\begin{aligned} T^a &= p\varepsilon^{abc} e_b e_c, \\ T^3 &= T^4 = 0. \end{aligned} \tag{18}$$

In the ansatz we have three real number q, r_0 and p which are a priory arbitrary before substituting the ansatz in the equations of motion (1) and (2), which will lead to a relation among them. We decomposed the indices $a,b,c... = 0,1,2$ and 3 and 4 which are written explicitly, and we also defined $\varepsilon^{abc} = \varepsilon^{abc34}$.

4.2. Solution

The three-dimensional space-time remains arbitrary after substituting the ansatz in the equations of motion, but there is only one reasonable black hole solution in this number of dimensions which is a BTZ black hole with(-out) torsion (Garcia et al., 2003; Obukhov, 2003). Because of this, the vielbein takes the following form.

$$\begin{aligned} &e^0 = Ndt, \quad e^1 = N^{-1}dr, \quad e^2 = r(d\varphi + N_\varphi dt), \\ &e^3 = r_0 d\theta, \quad e^4 = r_0 sin\theta\, d\chi, \end{aligned} \tag{19}$$

where the functions N and N_φ are given by

$$N^2 = -2m + \frac{r^2}{l^2} + \frac{j^2}{r^2}, \quad N_\varphi = \frac{j}{r^2}\,. \tag{20}$$

We introduced the AdS radius in the following manner

$$\frac{1}{l^2} = \mathrm{q} + \frac{p^2}{4}, \tag{21}$$

and m and j are parameters of the solution which are related to mass and angular momentum, respectively.

The spin connection is of the form

$$\begin{aligned} \omega^{ab} &= \breve{\omega}^{ab} - \frac{p}{2}\varepsilon^{abc} e_c, \\ \omega^{34} &= -\cos\theta\, d\chi, \end{aligned} \tag{22}$$

where ω^{ab} is the Riemann spin connection given by the following expressions

$$\breve{\omega}^{01} = -\frac{r}{l^2}dt - \frac{j}{r}d\varphi,$$
$$\breve{\omega}^{02} = -\frac{j}{Nr^2}dr, \tag{23}$$
$$\breve{\omega}^{12} = Nd\varphi.$$

As previously stated, the equations of motion introduce a relation among the parameters in the ansatz, which reads

$$q = -\frac{1}{2r_0^2}\,. \tag{24}$$

Also, as usual for the solutions with torsion of Lovelock gravity, a solution does not exist generally but in the sector of the theory in which a constraint between the coefficients in the Lagrangian holds

$$\alpha_1^2 - 8\alpha_0\alpha_2 = 0\,. \tag{25}$$

4.3. Properties of the black ring

The black ring as the product manifold of a BTZ black hole and a two-dimensional sphere inherits their Killing vectors. The complete set of Killing vectors consists of those originating from the BTZ black hole

$$\xi_0 = \partial_t, \quad \xi_1 = \partial_\varphi\,, \tag{26}$$

and those inherited from the sphere

$$\xi_2 = \partial_\chi, \quad \xi_3 = \sin\chi\partial_\theta + \cot\theta\cos\chi\partial_\chi, \quad \xi_4 = \cos\chi\partial_\theta - \cot\theta\sin\chi\partial_\chi\,. \tag{27}$$

For every Killing vector, we have conserved charge $Q(\xi_i)$, the charges are calculated in reference (Cvetković and Simić, 2016) using the Nester formula and, as in the previous solution, it is concluded that all the charges are zero

$$Q(\xi_i) = 0\,. \tag{28}$$

This is even more striking than in the case of a spherically symmetric black hole for which, because it does not rotate, only zero mass was an unexpected result. Namely, the black ring is a five-dimensional generalization of a rotating BTZ black hole which has non-zero angular momentum in three dimensions but, as we see, the black ring has a vanishing angular momentum nonetheless.

5. CONCLUSION

In this paper, we gave a short review of two black hole solutions that exist in five-dimensional Lovelock gravity.

First, we reviewed a spherically symmetric black hole. We explained what its Killing vectors are and what is the most general form of the metric and spin connection compatible with them. Afterward, we presented the solution itself and gave its properties. An interesting property is that all conserved charges vanish, which means that the mass of this solution is zero, too. This is a peculiar property which is in conflict with our intuition that black holes are made by the collapse of ordinary matter. Another peculiar property of this black hole is zero entropy. The vanishing entropy in the semi-classical approximation does not imply that the entropy calculated in full quantum theory is zero. It tells us that the entropy is much smaller than expected by the factor $1/G$, which immediately leads to the conclusion that this black hole has far fewer microstates than the usual black hole with non-vanishing entropy in the semi-classical approximation. For this reason, the solution is very interesting as a consistency check of every proposal for the black hole micro-states.

Second, we constructed a black ring with torsion which is a black hole which horizon of events does not have a spherical topology but the topology $S^1 \times S^2$. This is the reason for its name. The black ring also has all charges equal to zero, including its mass and angular momentum. This is, again, counter-intuitive, even more, if we take into account that this solution is nothing more than a rotating BTZ black hole times a two-dimensional sphere. Because a rotating BTZ black hole possesses mass and angular momentum, it is not clear what makes black rings so different from it.

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ZANIMLJIVE PETODIMENZIONALNE CRNE RUPE

U ovom radu ćemo proučiti dve crne rupe koje su rešenja petodimenzionalne Lavlokove gravitacije. Ova rešenja su karakterisana nenultom torzijom i interesantnom osobinom da su svi njihovi očuvani naboji nula. Prvo rešenje je sfernosimetrična crna rupa sa torzijom koja takođe ima nultu entropiju u semiklasičnoj aproksimaciji. Drugo rešenje je crni prsten, koji je petodimenzionalna generalizacija BTZ crne rupe sa torzijom u tri dimenzije.

Ključne reči: *crna rupa, torzija, alternativne teorije gravitacije*

Velocity memory effect for gravitational waves with torsion*

Dejan Simić†
Institute of Physics, University of Belgrade,
Pregrevica 118, 11080 Belgrade, Serbia

Abstract

We study the geodesic motion of massive test particle in the presense of the torsional plane-fronted (pp) wave in the three-dimensional (3D) gravity. The idea of this investigation is to test the appearance of the memory effect for the torsional waves. Our analysis discovers that the *velocity memory effect* happens for all waves that go to zero as, retarded time, u goes to infinity at sufficient fast rate.

1. Introduction

Does some observable change occur when a gravitational wave passes through a system of test particles in Minkowski spacetime? The answer is affirmative and is know as the gravitational memory [1, 2]. This is the effect that happens when a gravitational wave passes through a system of test particles, in asymptotically flat spacetime, which are initially at rest. If, after the passage of a gravitational wave, permanent displacement of particles occur we call this displacement memory effect [1, 2, 3, 4] and if particles have non-zero relative velocity we call it velocity memory effect [5, 6, 7]. This is important effect because it represents a possible experimental set up for the detection of gravitational waves and investigation of their properties.

In this article we will analyze the geodesic motion of massive test particles in the background of the pp wave with torsion. The reason why we undertook this investigation is to see is there a memory effect for gravitational waves with torsion in the Poincaré gauge theory [8, 9, 10]. To investigate this we will use the solutions with propagating torsion [11]. Important thing to note is that gravitational pp waves in 3D are solutions which do not exist without torsion [11], meaning that in the absence of torsion metric becomes trivial. This offers us an interesting opportunity to study the effects of torsion at the level of geodesic motion.

* This work was partially supported by the Serbian Science Foundation under Grant No. 171031.
† e-mail address: dsimic@ipb.ac.rs

The paper is organized as follows. First, we review pp waves without torsion in three dimensions and show that this solutions do not exist in 3D general relativity. After that, we analyze the pp waves with torsion in 3D Poincaré gauge theory of gravity. Next, we derive the geodesic equations for the metric of the pp wave with torsion. Unfortunately, the geodesic equations cannot be solved analytically except in a very special case, which is not interesting from the aspect of memory effect because it is not asymptotically flat. Due to this technical problem we had to solve geodesic equations numerically and results are given as plots of velocity in a function of retarded time u.

Conventions we are using are the following. The Latin indices $(i, j, ...)$ refer to the local Lorentz coordinates and run over $(0, 1, 2)$. The spacetime indices are denoted with letters of Greek alphabet. The contraction of vector with a form we label with $\rfloor$. The e^i is triad 1-form and the dual basis h_i is defined by the following equation $h_i \rfloor e^j = \delta_i^j$. For the Hodge dual we use the standard symbol *, and the Hodge dual of triad is $^\star e^i = \frac{1}{2}\varepsilon^{ijk} e_j e_k$. The exterior product of forms is implicit in all formulas.

2. The pp waves without torsion in three-dimensions

To better understand the nature of the pp waves with torsion we start with the Riemannian pp waves. For more details see Ref. [11].

The metric of the pp waves in Brinkmann coordinates is

$$ds^2 = 2du(Sdu + dv) - dy^2 \,, \tag{1a}$$

we, also, introduce an auxiliary function H

$$S = \frac{1}{2} H(u, y) \,. \tag{1b}$$

From the metric it is easy to derive the form of the triad e^i so that $ds^2 = \eta_{ij} e^i \otimes e^j$ holds, where η_{ij} is the half-null flat metric

$$\eta_{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} .$$

The triad is given by

$$e^0 := du \,, \qquad e^1 := Sdu + dv \,, \qquad e^2 := dy \,, \tag{2a}$$

The dual frame h_i define by the requirement $h_i \rfloor e^j = \delta_i^j$, where $\rfloor$ is a label for contraction, reads

$$h_0 = \partial_u - S\partial_v \,, \qquad h_1 = \partial_v \,, \qquad h_2 = \partial_y \,. \tag{2b}$$

Starting from the general formula for the Riemann connection

$$\omega^{ij} := -\frac{1}{2}\Big[h^i \rfloor de^j - h^j \rfloor de^i - (h^i \rfloor h^j \rfloor de^k) e_k\Big]\,,$$

we derive that the only non-zero component is

$$\omega^{12} = -\partial_y S\, e^0\,. \tag{3a}$$

From the above connection the Riemannian curvature is easily obtained

$$R^{ij} = 2e^0 k^{[i} Q^{j]}\,, \tag{4a}$$

where $k^i = (0,1,0)$ is a null vector and

$$Q^i = \partial_y^2 S e^2 \delta_2^i\,. \tag{4b}$$

The Ricci 1-form $Ric^i := h_m \rfloor Ric^{mi}$ is given by

$$Ric^i = e^0 k^i Q\,, \qquad Q = h_i \rfloor Q^i = \frac{1}{2}\partial_y^2 H, \tag{5a}$$

while the scalar curvature is zero

$$R = 0\,. \tag{5b}$$

Up to this point discussion was valid for any theory of gravity, now we want to look at what happens in general relativity in 3D. Action of general relativity $I = -a_0 \int d^3x R$ leads to the vacuum field equations

$$2a_0 G^n{}_i = 0\,, \tag{6}$$

where $G^n{}_i$ is the Einstein tensor. Einstein equations, after substitution of the metric, give

$$\partial_y^2 H = 0\,, \tag{7}$$

which only has a trivial solution

$$H = C(u) + yD(u)\,,$$

due to the fact that the curvature identically vanishes and this solution is diffeomorphic to Minkowski spacetime. This is to be expected because general relativity in three-dimensions does not have propagating local degrees of freedom and consequently should not have a wave solution.

3. The pp waves with torsion

The theory we will consider is a quadratic Poincaré theory of gravity which generalizes general relativity with 6 additional quadratic terms and as many free parameters $a_0, \ldots, a_2$ and $b_0, \ldots, b_2$. We will not write the action interested reader can find more details in Ref. [11].

The pp wave with torsion is a generalization of Riemannian pp wave which is obtained under assumption that the triad field (2) remains unchanged, while the connection takes the form

$$\omega^{ij} = \tilde{\omega}^{ij} + \frac{1}{2}\varepsilon^{ij}{}_m k^m k_n e^n G\,, \tag{8a}$$

$$G := S' + K\,. \tag{8b}$$

The function $K = K(u, y)$ is added to account for the effect of torsion, which is seen from the following expression for torsion

$$T^i := \nabla e^i = \frac{1}{2} K k^i k_m {}^\star e^m\,. \tag{9}$$

The curvature 2-form, Ricci 1- form and Ricci scalar are given by

$$\begin{aligned} R^{ij} &= \varepsilon^{ijm} k_m k^{n\star} e_n\, G'\,, \\ Ric^i &= \frac{1}{2} k^i k_m e^m G'\,, \\ R &= 0\,. \end{aligned} \tag{10}$$

The geometric configuration defined by the triad field (2) and the connection (8) represents a generalized gravitational plane-fronted wave of GR_Λ, or the *torsion wave* for short. The vector field $k = \partial_v$ is the Killing vector for both the metric and the torsion; moreover, it is a null and covariantly constant vector field. This allows us to consider the solution (12) as a generalized pp-wave.

The field equations [11] are given by

$$\begin{aligned} &a_0 G' - a_1 K' = 0\,, \qquad \Lambda = 0\,, \\ &K'' + m^2 K = 0\,, \qquad m^2 = \frac{a_0(a_1 - a_0)}{b_4 a_1}\,, \end{aligned} \tag{11}$$

with $G = S' + K$ and $S = H/2$. The solution of this equations is

$$\begin{aligned} K &= A(u)\cos my + B(u)\sin my\,, \\ \frac{1}{2}H &= \frac{a_1 - a_0}{a_0 m}\left(A(u)\sin my - B(u)\cos my\right) + h_1(u) + h_2(u)y\,. \end{aligned} \tag{12}$$

As we already said the h_1 and h_2 do not contribute to the radiation part of the curvature and can be discarded as trivial solution. Consequently, when the torsion is not present the metric becomes trivial. This is to be expected since general relativity in three-dimensions is a theory without propagating local degrees of freedom. Because the metric is crucially related to the torsion we can extract information about the torsion already on the level of the metric and geodesic motion.

4. Motion of massive test particle

In this section we investigate the geodesic motion of massive test particle in the presence of the massive pp wave with torsion described in the previous section. The geodesic motion of the test particle is obtained by solving a geodesic equation in which appear Christoffel (Riemannian) connection. So first we have to find the Christoffel connection for the metric of the masive pp wave with torsion.

4.1. Christoffel connection

The Christoffel connection is easily derived from the metric using the well known formula

$$\tilde{\Gamma}^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\alpha} \left(\partial_\nu g_{\alpha\rho} + \partial_\rho g_{\alpha\nu} - \partial_\alpha g_{\nu\rho} \right) , \tag{13}$$

and its non-zero components are given by

$$\begin{aligned} \tilde{\Gamma}^{v}{}_{uu} &= \frac{1}{2}\partial_u H \,, \\ \tilde{\Gamma}^{v}{}_{uy} &= \frac{1}{2}H' \,, \qquad \tilde{\Gamma}^{v}{}_{yu} = \frac{1}{2}\partial_y H \,, \\ \tilde{\Gamma}^{y}{}_{uu} &= \frac{1}{2}\partial_y H \,. \end{aligned} \tag{14}$$

Let us note that existence of a non-trivial metric of the pp wave is due to the presence of torsion. This allows us to see effects of torsion on the level of metric and, consequently, in geodesic motion of test particles [12].

4.2. Geodesic equations

The geodesic equation for u is

$$\frac{d^2u}{d\lambda^2} = 0 \,. \tag{15}$$

Consequently, we take $u \equiv \lambda$.

The equation for y reads

$$\ddot{y} + \frac{1}{2}\partial_y H = 0 \,, \tag{16a}$$

after substitution of explicit form of the function H it becomes

$$\ddot{y} + \frac{a_1 - a_0}{a_0}(A(u)\cos my + B(u)\sin my) = 0\,. \tag{16b}$$

The equation for v is

$$\ddot{v} + \frac{1}{2}\partial_u H + \partial_y H \dot{y} = 0\,, \tag{17}$$

or explicitly

$$\begin{aligned} &\ddot{v} + \frac{a_1 - a_0}{m a_0}(A'(u)\sin my - B'(u)\cos my) \\ &+ \frac{a_1 - a_0}{a_0}(A(u)\cos my + B(u)\sin my)\dot{y} = 0\,, \end{aligned} \tag{18}$$

4.3. Velocity memory effect

The velocity memory effect is present in the case when functions $A(u)$ and $B(u)$ vanish for large u. For numerical calculations it is better to introduce functions $\bar{A}(u) = \frac{a_1 - a_0}{a_0}A$ and $\bar{B}(u) = \frac{a_1 - a_0}{a_0}B(u)$ which we will use later in the text instead of $A(u)$ and $B(u)$. The velocity changes as one changes initial conditions, so this is a true observable effect. We do not show plots for different initial conditions because we wanted the presentation to be as short as possible.

4.4. Shockwave case

In the shock wave case when functions $\bar{A}(u) = 0$ and $\bar{B}(u)$ vanishes exponentially $\bar{B}(u) = e^{-(u-10)^2}$ numerical solutions of the geodesic equations gives the plots [12]for the particle velocities $\dot{y}$ and $\dot{v}$ shown in the Figure 1.

4.5. Slow fall off

In the case when $\bar{A}(u) = 0$ and $\bar{B}(u) = 1/u$ numerical solutions lead to the following plots [12] for the particle velocities $\dot{y}$ and $\dot{v}$ shown in the Figure 2.

5. Conclusion

We investigated a motion of massive test particles in asymptotically flat pp wave spacetime with torsion. The meaning of this is that test particle is initially well described by a particle in Minkowski spacetime and at some point a pp wave passes by and at time-like infinity a particle is again well described by its motion in Minkowski spacetime. Consequently, properties of particles motion at initial time and at infinity can be consistently compared.

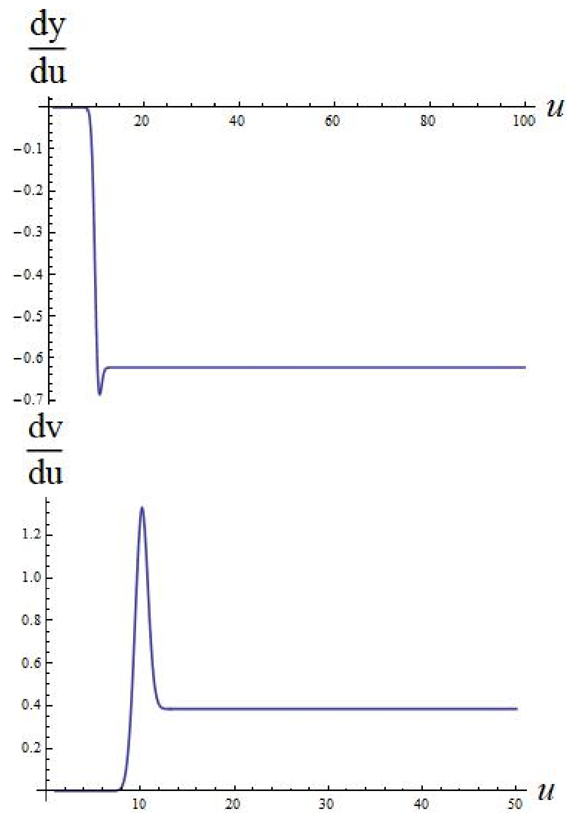

Figure 1: The plot for the particle velocity $\dot{y}$ and $\dot{v}$ in units $m = 1$, for $\bar{B} = -e^{-(u-10)^2}$

The conclusion is that velocity memory effect happens both for the exponentially fast fall-off of the gravitational wave as well as for the arbitrary polynomial fall-off. For the related work on memory effect for massive gravitons see Ref. [13]. This is the first time the memory effect for gravitational waves with torsion is analyzed. To authors knowledge, this is also the first example of the memory effect in three-dimensional gravity.

In the last few years there was a lot of effort on connecting asymptotic symmetries, soft theorems and memory effect [14]. This approach based on BMS symmetry offers a new perspective on the black hole microstates and information loss [15]. It is an open problem to connect the memory effect described in this article with asymptotic symmetry of the theory.

It is very interesting to generalize the analysis of this paper to the pp waves in four dimensions [16]. Preliminary results [17] show that most of the conclusions of this paper transfer to the four-dimensional case. This is important effect because it offers a possible experimental set up for the

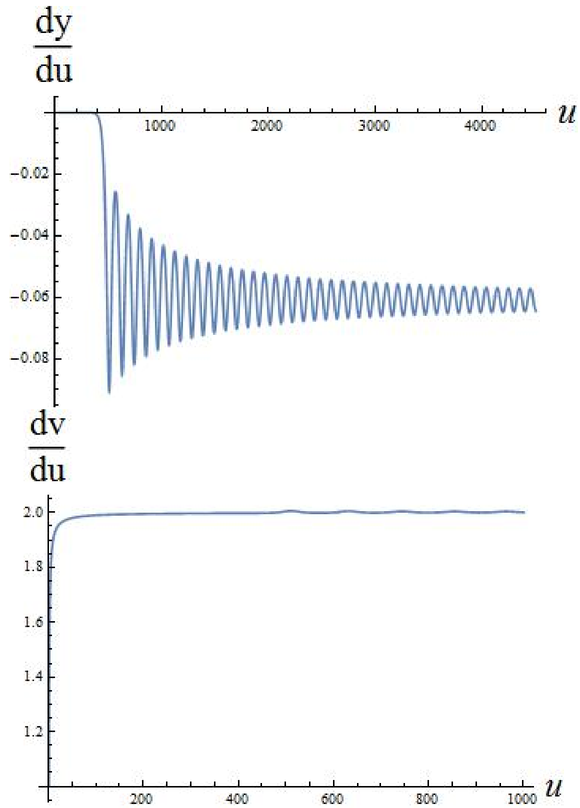

Figure 2: The plot for the particle velocity $\dot{y}$ and $\dot{v}$ in units $m = 1$, for $\bar{B} = -1/u$

detection of torsion.

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Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА
Комисија за стицање научних звања

Број: 660-01-00001/1089
24.02.2020. године
Б е о г р а д

На основу члана 22. став 2. члана 70. став 4. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка, 18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) и захтева који је поднео

Институт за физику у Београду

Комисија за стицање научних звања на седници одржаној 24.02.2020. године, донела је

ОДЛУКУ
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Дејан Симић

стиче научно звање

Научни сарадник

у области природно-математичких наука - физика

О Б Р А З Л О Ж Е Њ Е

Институт за физику у Београду

утврдио је предлог број 562/1 од 17.04.2019. године на седници Научног већа Института и поднео захтев Комисији за стицање научних звања број за доношење одлуке о испуњености услова за стицање научног звања ***Научни сарадник***.

Комисија за стицање научних звања је по претходно прибављеном позитивном мишљењу Матичног научног одбора за физику на седници одржаној 24.02.2020. године разматрала захтев и утврдила да именовани испуњава услове из члана 70. став 4. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка,18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) за стицање научног звања ***Научни сарадник***, па је одлучила као у изреци ове одлуке.

Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ

Др Ђурђица Јововић,
научни саветник

МИНИСТАР

Младен Шарчевић

2018 Workshop on Gravity, Holography, Strings and Noncommutative Geometry

1. February 2018, Belgrade, Serbia

Organization

The Workshop is organized by Group for Gravitation, Particles and Fields (Institute of Physics, University of Belgrade), within the framework of the **national project "Physical implications of modified spacetime"**, number ON 171031, of the Ministry of Education, Science and Technological Development, Serbia.

Scientific committee

Branislav Cvetkovic and Marko Vojinovic

Registration

It is important for all interested participants to register as soon as possible, so that we can reserve an appropriate number of places for lunch in the restaurant. Registration is now closed.

Programme

Lectures were held on Thursday, 1. February 2018, at the Institute of Physics, seminar room 360 (ex-room 300), third floor.

09:50 -- 10:00 --- Opening	**Introduction and opening of the workshop**	
10:00 -- 10:45 --- Lecture 10:45 -- 10:55 --- Discussion	**Speaker:** Rodrigo Olea (lecture slides)	**Title:** Brussels sprouts, black hole mass and pre-holography **Abstract:** We present the first evidence on the fact that topological invariants should renormalize anti-de Sitter gravity with quadratic-curvature corrections. This argument is based on the computation of energy for Einstein black holes in the theory, which appears as an alternative to linearized methods (e.g., Deser-Tekin formula).
10:55 -- 11:40 --- Lecture 11:40 -- 11:50 --- Discussion	**Speaker:** Olivera Miskovic (lecture slides)	**Title:** Thermodynamic instabilities of extremal black holes **Abstract:** We study static, charged extremal black holes in 4D gravity non-linearly coupled to a scalar field. We show that the system can exhibit a phase transition due to electric charge variations only in presence of a cosmological constant and if the scalar is massive. A near-critical analysis reveals that, on one side of the critical point, the hairy black hole has larger entropy than the non-hairy one, thus giving rise to a zero temperature phase transition. Our results are analytical and based on the second law of thermodynamics.
11:50 -- 12:10 --- Break	**Coffee break**	
12:10 -- 12:55 --- Lecture 12:55 -- 13:05 --- Discussion	**Speaker:** Maria Pilar Garcia del Moral (lecture slides)	**Title:** On global aspects of duality invariant theories: M2-brane versus double field theory **Abstract:** In this talk I will discuss the global description of a supermembrane compactified on a $T^2\times M_9$, where T^2 is the 2-dimensional torus and M_9 is a 9-dimensional noncompact spacetime. I will discuss the T-duality transformation of this model and compare global aspects of this construction with that of the double field theory.
13:05 -- 13:50 --- Lecture 13:50 -- 14:00 --- Discussion	**Speaker:** Mihailo Cubrovic (lecture slides)	**Title:** Three tales on boundary action in AdS/CFT: Berry phases, gauge fields and non-canonical Hamiltonians **Abstract:** We argue that AdS/CFT dictionary can be extended by adding new subleading terms to the boundary (surface) part of the AdS action, under the constraint that the bulk equations of motion at semiclassical level remain unchanged. This corresponds to modifying the state space and/or the Poisson structure in dual field theory without additional sources/operator insertions. We give three applications of the general idea. The Berry phase is obtained from a subleading boundary term for the bulk fermion which transforms as a spin on the sphere, hence it is crucial to start from global AdS and subsequently take the planar limit. A phenomenological (bottom-up) description of dynamical gauge fields is encoded in a singleton excitation on the boundary, decoupled from the bulk. The Poisson structure, defined by the time evolution operator in CFT, is modified by sourcing multi-particle states in the bulk from appropriate boundary sources. This opens a way toward constructing the gravity dual of non-canonical Hamiltonians as encountered, e.g., in fluid advection.

14:00 -- 16:00 --- Lunch	**Official workshop lunch at the IPB restaurant**	
16:00 -- 16:20 --- Lecture 16:20 -- 16:25 --- Discussion	**Speaker:** Dejan Simic (lecture slides)	**Title:** Near horizon of the OTT black hole, asymptotic symmetries and soft hair **Abstract:** We study near horizon geometry of extremal (non-)rotating Oliva-Tempo-Troncoso black hole. First we derive the corresponding geometries. Next, we analyze asymptotic structure and determine asymptotic symmetry, which consists of time reparametrization, chiral Virasoro and u(1) Kac-Moody algebra. In the end, interpretation in term of soft hair on the black hole is given.
16:25 -- 16:45 --- Lecture 16:45 -- 16:50 --- Discussion	**Speaker:** Biljana Nikolic (lecture slides)	**Title:** Some geometrical aspects of NC SO(2,3)* gravity **Abstract:** We construct gravity action on the Moyal-Weyl spacetime as a noncommutative SO(2,3)* gauge theory and expand it, using the Seiberg-Witten map, up to the second order in the deformation parameter. After the braking of symmetry of obtained action down to SO(1,3) gauge symmetry, we analyze the low energy sector of the model. We calculate the equations of motion, and discuss the noncommutative corrections as the source of the curvature and torsion. We find one solution: the NC correction to Minkowski spacetime. Using this solution, we explain breaking of the diffeomorphism symmetry as a consequence of working in a particular coordinate system given by the Fermi normal coordinates.
16:50 -- 17:10 --- Break	**Coffee break**	
17:10 -- 17:30 --- Lecture 17:30 -- 17:35 --- Discussion	**Speaker:** Dragoljub Gocanin (lecture slides)	**Title:** Birefringence property of the Moyal-Weyl noncommutative spacetime in SO(2,3)_* model **Abstract:** We demonstrate that flat noncommutative (NC) Moyal-Weyl spacetime acts as a birefringent medium for electrons propagating in it, and we present an action that predicts this "optical" effect. The action is obtained by NC Moyal-Weyl *-product deformation of a certain classical action invariant under local SO(2,3) transformations. After perturbative expansion via Seiberg-Witten map in powers of the deformation parameter θ and a suitable symmetry breaking down to the local Lorentz SO(1,3) symmetry, we get NC deformation of the Dirac action in curved spacetime with various new couplings. One of its significant features is the nonvanishing linear θ-correction which pertains even in flat spacetime. We analyse NC deformation of the Dirac equation and dispersion relation for electrons. The theory predicts Zeeman-like splitting of electron's undeformed (commutative) energy levels due to noncommutativity of the background spacetime. This splitting is helicity-dependent --- electrons with different helicity are affected differently by NC background. NC correction to the electron's energy levels is linear in θ, which brings us closer to the potential observation.
17:35 -- 17:55 --- Lecture 17:55 -- 18:00 --- Discussion	**Speaker:** Dragan Prekrat (lecture slides)	**Title:** Phase transitions on the truncated Heisenberg space **Abstract:** We discuss the phase structure of matrix models on non-commutative spaces. We examine the connection between the geometry of the truncated Heisenberg space, the renormalizability and the striped phase and present the first numerical evidence of the modification of the phase diagram due to the coupling between the matrix field and the curvature.
18:00 -- 18:30 --- Closing	**Final discussion and closing**	

List of participants

- Milutin Blagojevic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Bojana Brkic (Faculty of Physics Belgrade, Serbia)
- Mihailo Cubrovic (Scientific Computing Laboratory, Institute of Physics Belgrade, Serbia)
- Branislav Cvetkovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Ljubica Davidovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Aleksandra Dimic (Group for Quantum and Mathematical Physics, Faculty of Physics Belgrade, Serbia)
- Ivan Dimitrijevic (Faculty of Mathematics Belgrade, Serbia)
- Marija Dimitrijevic Ciric (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Maria Pilar Garcia del Moral (Departamento de Fisica, Universidad de Antofagasta, Chile)
- Dragoljub Gocanin (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Ilija Ivanisevic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Filip Jurukovic (Faculty of Physics Belgrade, Serbia)
- Dusko Latas (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Olivera Miskovic (Instituto de Fisica, Pontificia Universidad Catolica de Valparaiso, Chile)
- Luka Nenadovic (Group for Gravitation, Particles and Fields, College for Medicine, Business and Technology, Sabac, Serbia)
- Biljana Nikolic (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Bojan Nikolic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Rodrigo Olea (Departamento de Ciencias Fisicas, Universidad Andres Bello, Santiago, Chile)
- Dragan Prekrat (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Tijana Radenkovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Voja Radovanovic (Group for Gravitation, Particles and Fields, Faculty of Physics Belgrade, Serbia)
- Igor Salom (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Branislav Sazdovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Dejan Simic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)
- Jelena Stankovic (Department of Mathematics, Teacher Education Faculty, Belgrade, Serbia)
- Marko Vojinovic (Group for Gravitation, Particles and Fields, Institute of Physics Belgrade, Serbia)

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