ИНСТИТУТ ЗА ФИЗИКУ

ПРИМЉЕНО: 03-02-2025			
Рад.јед.	број	Арх.шифра	Прилог
0801-	161/1		

Научном већу Института за Физику, Београд

Београд 31.01.2025

Предмет: Молба за покретање реизбора у звање виши научни сарадник за Николу З. Петровића

Молим научно веће Инстиута за физику да у складу са Правилником о поступку и начину вредновања и квантитивном исказивању научно-истраживачких резултата истраживача покрене поступак за мој реизбор у звање виши научни сарадник.

У прилогу достављам:

1. Мишљење руководиоца пројекта са предлогом комисије за реизбор у звање и потврдом о руковођењу задатком
2. Стручну биографију
3. Преглед научне активности
4. Елементе за квалитативну оцену научног доприноса
5. Елементе за квантитативну оцену научног доприноса
6. Списак објављених радова и њихове копије
7. Податке о цитираности радова
8. Фотокопију документа о избору у садашње звање

Са поштовањем,

Никола Петровић

др Никола З Петровић,

виши научни сарадник

Институт за физику, Београд

Научном већу Института за физику у Београду

Предмет: Мишљење руководиоца лабораторије за реизбор др Николе Петровића у звање виши научни сарадник

Др Никола Петровић је запослен у Институту за физику у Београду у Лабораторији за нелинеарну физику. Он се бави налажењем егзактних аналитичких решења за разне облике нелинеарне Шредингерове једначине и једначине Грос-Питаевског. Он је развио изузетно висок степен самосталног рада, што је показао објављивањем неколико радова где је он једини аутор и још великог броја радова где је он први аутор. Поред тога, др Петровић има развијену сарадњу са Фалуктетом за науку и нњжењеринг Хамад Бин Калифа универзитета у Ќатару, некадашњег Тексас A&M универзитета у Катару, преко ког је сарађивао са неколико великих имена у тој области широм света.

Др Никола Петровић испуњава све критеријуме прописане Правилником за избор у научна звања Министарства науке, технолошког развоја и иновација, те сам сагласан за реизбор др Николе Петровића у звање Виши научни сарадник.

Предлажем да комисију за реизбор др Николе Петровића у звање виши научни сарадник чине:

1. др Милан Петровић, научни саветник Института за физику у Београду
2. др Александра Стринић, научни саветник Института за физику у Београду
3. др Жељко Шљиванчанин, дописни члан САНУ и научни саветник Нуклеарног института Винча у Београду

Београд,

31.01.2025

Руководилац пројекта

Милан Петровић

др Милан Петровић

научни саветник

Институт за физику

3. Стручна биографија кандидата Никола З Петровић

Никола Петровић је рођен 12. 03. 1980. године у Београду.

Завршио је Математичку Гимназију 1999. године као ученик генерације са просеком 5.00. У гимназији се такмичио на такмичењима из физике и математике на свим нивоима националних такмичења. На међународним такмичењима је освојио више медаља од којих треба издвојити две сребрне и једну бронзану медаљу на Међународним олимпијадама из математике. Захваљујући тим успесима постао је стипендиста Министарства за науку и технолошки развој.

Дипломирао је физику и математику у јуну 2003. године на Масачусетс институту за технологију (Massachusetts Institute of Technology) са просеком 4.5 (на скали од 0 до 5). Дипломски рад је био на тему кодова за исправљање грешака у квантним компјутерима: "**Constructing an Infinite Class of Perfect Codes**", са оценом Б (9). Ментор је био проф. Исак Чуанг (Isaac Chuang).

Објавио је са још три коаутора књигу **"The IMO Compendium"** са свим задацима предложеним на Међународним математичким олимпијадама (Спрингер-Верлаг, Берлин, 2006, друго издање 2011). Дугогодишњи је сарадник Истраживачког центра Петница и члан Државне комисије за такмичења из математике. Као члан комисије за такмичења из физике учествовао је у припреми и оцени задатака на националним такмичењима.

Од 2004. године Никола Петровић је у радном односу са Институтом за Физику у Београду. Његов статус је замрзнут од августа 2005. године када одлази на Тексашки A&M универзитет у Катару (Texas A&M University at Qatar), од 2024. године преиначен у Факултет за науку и ињжењеринг Хамад Бин Калифа универзитета, где је запослен као лабораторијски координатор и ради такође као асистент све до јула 2012. године, када се враћа у Институт за Физику. У септембру 2012. године је изабран у звање истраживача сарадника. Докторску дисертацију под насловом: **"Тачна таласна и солитонска решења генералисане нелинеарне Шредингерове једначине"** је одбранио 16. октобра 2013. године на Физичком факултету Универзитета у Београду.

У звање научни сарадник изабран је 3. Маја 2014. У Београду. У звање вишег научног сарадника изабран је 10. 06. 2020 године. Ангажован је на пројекту 171006 под руководством др Душана Јовановића ''Нелинеарна динамика локализованих самоорганизованих структура у плазми, нано-композитним материјалима, течним и фотоничним кристалима и ултрахладним кондензатима'' и члан је Лабораторије за нелинеарну физику Института за Физику.

Ожењен је са супругом Ташаном и има двоје деце: Бориса и Емилију.

4. Никола З Петровић: Преглед научне активности

4.1 Општи подаци о активностима Никола З Петровића

Под руководством ментора проф. др Миливоја Белића, Никола Петровић је почео 2005. године да се бави и истраживањима у нелинеарној оптици.

Никола Петровић се бави проналажењем егзактних решења за широке класе нелинеарних еволутивних парцијалних диференцијалних једначина, углавном користећи се методом експанзије по Јакобијевим елиптичким функцијама. Овај метод примењен је досад на неколико облика нелинеарне Шредингерове једначине, на једначину Грос-Питаевског као и на многе друге системе. Такође, досад је користио и Хирота методу и уз то радио линеарну анализу стабилности добијених решења. Тренутно је ангажован на пројекту Министарства просвете и науке ОИ 171006 под руководством др Милана Петровића.

Никола Петровић је досад објавио 23 рада у угледним међународним часописима (М21а, М21, М22, М23), од којих је један објављен у престижном часопису Physics Review Letters. Укупан број цитата његових радова до сада је укупно 604, према ISI Web of Knowledge.

Важно је поред научних резултата напоменути и вишегодишњи педагошки рад др Николе Петровића као асистента на Тексас А&М универзитету, где је за 7 генерација студената држао вежбе и лабораторију из механике, електромагнетике и модерне физике. У току свог истраживачког рада, др Никола Петровић је такође био и ментор најталентованијим студентима, који су под окриљем проф. др Миливоја Белића добили прилику да учествују у научном истраживању и буду ко-аутори на Николиним радовима.

Никола Петровић је тренутно запослен као виши научни сарадник у Институту за Физику у Лабораторији за нелинеарну физику.

4.2 Активности пре избора у звање виши научни сарадник и опис његових доприноса

У овом делу ћемо представити научну активност др Николе Петровића, уз напомену да су бројеви референце конзистентни са коначном листом радова из секције 7.

Никола Петровић се у свом раду бавио применом и модификацијом такозване методе развоја по Јакобијевим елиптичним функцијама, у циљу проналажења нових класа егзактних и аналитичких решења мултидимензионих генералисаних нелинеарних Шредингерових једначина, као и других једначина. Кључни допринос кандидата је била генерализација методе Јакобијевих елиптичних функција на нелинеарну Шредингерову једначину са кубичном нелинеарношћу у 3 димензије [3], која је дотад претходно примењена на 2 димензије, у раду у којем је и Никола Петровић био укључен [2]. Рад у коме су ови резултати презентовани је објављен у Physical Review Letters и постао је високо цитирани рад који је отворио целу једну подобласт математичке физике. Добијени су и тамни и светли солитони, у оба случаја са и без просторног чирпа. Контролишући параметар Јакобијевих функција добија се солитонски талас као гранични случај решења која описују бесконачан низ путујућих таласа. Добијена решења имају велику флексибилност у зависности од параметара једначине: коефицијената дифракције, нелинеарности, и губитака; једино један од три параметра мора бити дефинисан у функцији осталих. За разлику од претходних радова са једначином у 2 димензије, у овом новом раду је улога чирп функције коначно разјашњена.

У наредним радовима је метода Јакобијевих елиптичних функција модификована да би се пронашла решења за случај нормалне дисперзије, који има много ширу физичку примену од случаја аномалне дисперзије и који дотад није био урађен [4]. Др Петровић је открио на који начин да се промени облик решења како би се узела у обзир анти-симетрија времена у односу на остале трансверзне варијабле. Иако се физички систем нормалне дисперзије квалитативно знатно разликује од случаја аномалне дисперзије, показало се да се модификацијом само неколико параметара могу добити решења и за овај случај.

Метода Јакобијевих елиптичних функција је затим генералисана на системе са нелинеарношћу вишег степена, пригодном модификацијом степена решења. Уз одређене специфичне услове пронађена су солитонска решења и за кубично-квинтични (qubic-quintic) и за кубично-квинтично-септични (qubic-quintic-septic) модел [6]. Ово истраживање је отворило могућност евентуалног налажења решења са сатурабилном нелинеарношћу.

Потом је Никола Петровић применио методу на једначину Грос-Питаевског (Gross-Pitaevski), која има облик нелинеарне Шредингерове једначине са укљученим параболичним потенцијалом [5]. Никола Петровић је установио да је најпре потребно решити такозвану Рикатијеву (Riccati) диференцијалну једначину да би се добило решење једначине Грос-Питаевског. С обзиром на то да је Рикатијеву једначину немогуће решити у општем случају, кандидат је истражио случајеве који имају позната решења а од физичког су значаја. За константне вредности параметра дифракције и јачине потенцијала добио је решења која опадају или имају сингуларитет и утврдио да је решења која опадају могуће стабилизовати додатним напајањем енергије (gain) у тачно одређеној мери. Са друге стране, за синусоидни облик параметра дифракције и јачине потенцијала добио је стабилна таласна и солитонска решења [7].

Др Петровић и студент Анас Ал Бастами, коме је Никола био ментор, су утврдили да је могуће за компликованије облике параметара свести Рикатијеву једначину на решиву линеарну једначину другог степена, у ком случају се добија широка класа нових решења једначине Грос-Питаевски, укључивши и решења за случај Фешбахове (Feschbach) резонанце. Решења добијена би могла имати широку примену са обзиром на то да се једначина Грос-Питаевски користи у проучавању Боз-Ајнштајнових (Bose-Einstein) кондензата [8,9].

Кандидат је даље у сарадњи са студентом Хусеин Захредином модификовао методу за случај да потенцијал није параболичан него линеаран и у том случају су пронађена решења за константну вредност параметра дифракције и јачине потенцијала, за синусоидалан облик ова два параметара, као и за оба мешана случаја, тј. кад је један од параметара константан а други синусоидалан [10].

Метода је од стране др Петровића такође по први пут примењена и на двокомпонентне, тзв. Манаковљеве системе, тачније на пар ко- и контра-пропагирајућих таласа [13]. Пронађена су решења за случај кад је однос укрштено-фазне (cross-phase) и само-фазне (self-phase) модулације једнак 3. Упркос томе што није било могуће овом методом добити општа решења Манаковљевог система, системе са овим односом двеју модулација је могуће направити.

Др Никола Петровић у овом периоду довршио рад [19] у којем се анализира стабилност многобројних решења које је добио методом развоја по Јакобијевим елиптичним функцијама објавио у већ споменутим радовима публикованим у периоду од 2008. до

2011. године. У сарадњи са Др. Најданом Алексићем и Проф. Др. Миливојем Белићем, урађена је анализа стабилности решења нелинеарне Шредингерове једначине са нормалном и аномалном дисперзијом и једначине Грос-Питајевског. Најпре је урађена трансформација која нелинеарну Шредингерову једначину са дистрибуираним коефицијентима своди на једначину са константним коефицијентима. Затим је конструисан одговарајући Лагранжијан и под претпоставком постојања модулационе нестабилности су добијене једначине за њихову целокупну амплитуду, тј. њен реалан и имагинаран део, у функцији од таласног броја пертурбација. Затим је систем једначина решен да би се добило да ли параметри дивергирају или не и тиме одредило да ли решења имају стабилност. Утврђено је да у свим случајевима решења поседују или апсолутну стабилност или стабилност уз присуство такозваног менажирања дисперзије, тј. алтернирања знака коефицијента дисперзије уз помоћ метаматеријала. Апсолутна стабилност је утврђена у три димензије за тамне солитоне у аномалној дисперзији, и за светле временске солитоне у нормалној дисперзији, док је у две димензије апсолутна стабилност утврђена за све тамне солитоне. Ови резултати су проверени компјутерским симулацијама и добијено је скоро потпуно слагање у решењима без чирпа и изузетно добро квалитативно слагање које у сваком случају потврђује критеријуме апсолутне стабилности у решењима са чирпом. Др. Никола Петровић је као први аутор учествовао у свим аспектама овог рада осим компјутерских симулација.

Др Никола Петровић је у овом периоду написао и рад [18] у коме је једини аутор. Он је генералисао своје методе зна системе нелинеарних Шредингерових једначина где степен нелинеарности није цео број, као и где постоје два члана, један са дупло већим степеном од другог. Ово је урађено помоћу трансформације која је сводила систем на систем са коефицијентима целобројног степена. Посебна пажња је посвећена такозваним кубично-квинтичним системима код којих су нађене велике класе нових решења јер се случај са тим вредностима испоставља као специјалан случај. Добијена су не само решења заснована на Јакобијевој елиптичној функцији, него и решења која садрже такозвани чирп. Наравно, све прорачуни и резултати у раду су изведени од стране Др. Николе Петровића.

Након почетних резултата укључених у његову докторску дисертацију, Никола Петровић је проширио свој домен рада и продубио сарадњу са колегама из Кине које се баве сличном облашћу.

У сарадњи са професором Веипинг Жонгом (Wei-Ping Zhong), Никола је учествовао у раду на утврђивању постојања контролисаних параболично-цилиндричних дивљих таласа (rogue waves) [14]. Дивљи таласи су тренутно врло актуелна тема у свету нелинеарне оптике (а и шире) јер настају изненада и имају велики интензитет, те њихово проучавање је јако битно у циљу успешне примене нелинеарних оптичких система. У раду су добијени дивљи таласи чија амплитуда је пропорцционална параболично-цилиндричној функцији. Др. Никола Петровић је учествовао у налажењу и провери исправности датих решења.

Затим је Др. Никола Петровић учествовао у дугогодишњој и плодоносној сарадњи са физичарем из Кине Силију Суом (Si-Liu Xu). У серији од неколико радова др Никола Петровић је дао велики допринос у реализацији идеја, провери тачности, прављењу илустрација и писању радова које је заједно са др Суом објавио.

У раду [15] је коришћена такозвана самослична трансформација да би се добила решења нелинеарне тродимензионе Шредингерове једначине са четвртим степеном нелинеарности. Добијени су и тамни и светли солитони као решења за неколико различитих математичких облика коефицијента дифракције и проучавано је динамичко понашање светлости у датим срединама.

У раду [16] су нађена решења за (3+1)-димензиону нелинеарну Шредингерову једначину са нецелобројним степеном и такозваним ПТ (parity-time) симетричним потенцијалом. Урађена је трансформација сличности и добијене једначине такве да за свааки облик решења постоји одговарајући потенцијал такав да је оригиналнаа нелинеарна Шредингерова једначина испуњена. Ово отвара могућност налажења решења локализованих у свим трансверзалним координатама, такозваних светлосних метака.

У раду [17], нађена су решења нелинеарне Шредингерове једначине четвртог степена у цилиндричним координатама. За параметар везан за амплитуду је добијена конфлуентна хипергеометријска диференцијална једначина чија су решења такозване Сонине функције. Утврђено је да су решења стабилна кад је тополошко наелектрисање мање од 1, а нестабилна кад је веће од 2.

У раду [20], нађена су решења у Нелинеарној Шредингеровој једначини са ПТ-симетричним потенцијалном и супротстављеним нелинеарностима степена 3 и 2k+1. Добијена су локализована решења у свим координатама на бази хиперболичког секанса.

У раду [21] из категорије (М22) су нађена решења за нелокални и нелинеарни систем, дефинисан двема једначинама, једном за решење и другом која одређује јачину индекса преламања у датој тачки. Добијена решења се заснивају на Јакобијевим елиптичним функцијама. Најдзад, урађена је основна анализа стабилности и утврђено да су за велике апсолутне вредности коефицијента дифракције решења стабилна, док у малим вредностима настају нестабилности.

У раду [22] из категорије (М22) су нађена решења за двокомпоненту нелинеарну Шредингерову једначину која су заснована на Перегриновим, Акмедијевим и Маовим решењима.

Коначно, у раду [23] из категорије (М23) је др Никола Петровић у сарадњи са својим студентом Моизом Бохром нашао решења заснована на општем облику елиптичне диференцијалне једначине где је квадрат извода једнак општем полиному четвртог степена оригиналне функције, дакле где се за разлику од једначине за Јакобијеву елиптичну функцију укључују чланови првог и трећег степена. Нађена су решења на основу Вајерштрасове елиптичне функције и на основу општих елиптичних функција које нису симетричне у односу на средњу вредност максимума и минимума функције.

4.3 Активности после избора у звање виши научни сарадник и опис пет истакнутих радова из тог периода

4.3.1. Радови са доминантном улогом кандидата

1. У раду [24] из категорије М21а, кандидат је применио методу Јакобијеве елиптичне функције на систем једначина који описује кретање светла кроз Нематичке течне кристале са нелинеарношћу трећег степена, такозваном Керовом нелинеарношћу. Испоставља се да треба изразити и таласну функцију и угаону функцију кристала као полином другог степена Јакобијеве елиптичне функције. Уз то коефицијент уз други степен и слободни коефицијент морају да буду у специфичним односима. Као последица добијају се карактеристична решења, као што су двоструки тамни солитон који је већ био описан у другим радовима, али захваљујућу методи Јакобијеве елиптичне функције такође може садржати чирп који утиче и на позадину. Добијају се и светли солитони, као и путујући таласи. Такође се добија за случај две трансверзне димензије да се могу наћи решења кад су сви коефицијенти константни осим једног чак и у случају чирпа. Генерално систем пружа велики степен флексибилности уз само 2 ограничења за параметре система диференцијалних једначина. Угаоне функције генерално прате облик таласних функција што одсликава интеракцију светлости са нематским течним кристалима. У изјави доприноса сваког аутора при крају рада види се да је др Никола Петровић био одговоран за идеју, реализацију, цртање графова, проверу решења и писање рада, тојест да је дао доминантни допринос у овом раду.

2. У раду [26] из категорије М22 су нађена решења за Кунду-Наскар-Мукрџи једначину. У овој једначини је разбијена симетрија између два трансверзална правца, те се добијају значајно другачији облици функција, а у правцу чији се извод не појављује у самој

једначини се добијају широке периодичне екстензије функције, такозване 'кресте'. Чирп је могућ само у овом и параметар који га одређује је ограничен да буде константан, те се не добија модулација амплитуде, али се добија изузетно комплексна модулација самих решења, поготово у режиму путујућих таласа. Др Никола Петровић као једини аутор је наравно извео целокупне прорачуне и анализу у самом раду, као и цртање графова и писање рада.

4.3.2. Опис осталих репрезентативних радова

3. У раду [25] из категорије М22 су нађена решења за Дејви-Стјуартсонов систем једначина. У овом случају је таласна функција полином првог степена Јакобијеве елиптичне функције, док је такозвана функција средњег проточног поља полином другог степена. Иако се добију решења са значајним ограничењима на већину параметара испоставља се да слободан члан средњег проточног поља не подлеже никаквим ограничењима, те је могућа изузетно велика флексибилност у конструкцији решења. Др Никола Петровић је био једини аутор овог рада.

4. У раду [26] из категорије М24 су примењени резултати из [24] на нематичне течне кристале са такозваном кубичном-квинтичном нелинеарношћу. Овде добијамо решења код којих је таласна функција полином првог степена Јакобијеве елиптичне функције док је угаона функција кристала полином другог степена. Пошто систем има један степен слободе више од система описаног у [24], а и даље је број ограничења параметара два, то даје још већу флексибилност у конструкцији нових решења од система. Др Никола Петровић је био једини аутор овог рада.

У случају да је неопходно да репрезентативни рад буде од претходног избора у звање:

5. У конференцијском раду [29] из категорије М33, генералисано је решење једначине Грос-Питаевски са сферно-симетричног потенцијала на цилиндрично- и планарно-симетричне потенцијале, разбивши тиме симетрију између три трансверзне димензије. Др Никола Петровић је био једини аутор овог рада.

Поред ових пет радова наведених за период од последњег избора пошто кандидат ради већину својих радова потпуно самостално или у колаборацијама доминантно води налажење егзактних математичких решења то се као илустрација његове самосталности могу навести и радови споменути у секцији 4.2. У случају да репрезентативни рад не мора да буде од претходног избора у звање конференцијски рад из тачке 5. можемо заменити са радом [19] чији ћемо резиме овде поновити:

5. У раду [19] се анализира стабилност решења које је др Никола Петровић добио методом развоја по Јакобијевим елиптичним функцијама и објавио у периоду од 2008. до 2011. године. У сарадњи са др Најданом Алексићем и проф. др Миливојем Белићем, урађена је анализа стабилности решења нелинеарне Шредингерове једначине са нормалном и аномалном дисперзијом и једначине Грос-Питајевског. Најпре је урађена трансформација која нелинеарну Шредингерову једначину са дистрибуираним коефицијентима своди на једначину са константним коефицијентима. Затим је конструисан одговарајући Лагранжијан и под претпоставком постојања модулационе нестабилности су добијене једначине за њихову целокупну амплитуду, тј. њен реалан и имагинаран део, у функцији од таласног броја пертурбација. Затим је систем једначина решен да би се добило да ли параметри дивергирају или не и тиме одредило да ли решења имају стабилност. Утврђено је да у свим случајевима решења поседују или апсолутну стабилност или стабилност уз присуство такозваног менажирања дисперзије, тј. алтернирања знака коефицијента дисперзије уз помоћ метаматеријала. Апсолутна стабилност је утврђена у три димензије за тамне солитоне у аномалној дисперзији, и за светле временске солитоне у нормалној дисперзији, док је у две димензије апсолутна стабилност утврђена за све тамне солитоне. Ови резултати су проверени компјутерским симулацијама и добијено је скоро потпуно слагање у решењима без чирпа и изузетно добро квалитативно слагање које у сваком случају потврђује критеријуме апсолутне стабилности у решењима са чирпом. Др Никола Петровић је као први аутор учествовао у свим аспектама овог рада осим компјутерских симулација.

5. ЕЛЕМЕНТИ ЗА КВАЛТИТАТИВНУ ОЦЕНУ НАУЧНОГ ДОПРИНОСА КАНДИДАТА: НИКОЛА З ПЕТРОВИЋ

5.1 Квалитет научних резултата

5.1.1 Научни ниво и значај резултата, утицај научних радова

Др Никола З Петровић је у досадашњој каријери био аутор или коаутор уз давање кључног доприноса у укупно 23 рада и два рада са конференција, објављених у међународним часописима са ISI листе (радови категорије M21a, M21, M22 и M23 и радови са конференције категорије M33). Од тога је 10 радова у категорији M21a (међународни часописи изузетних вредности), 5 у категорији M21 (врхунски међународни часописи), 6 у категорији M22 и 2 у категорији M23.

У периоду након одлуке Научног већа о предлогу за стицање претходног научног звања, др Никола З Петровић је објавио 3 радова у часописима са ISI листе. Од тога је 1 у часописима категорије M21a (међународни часописи изузетних вредности) и 2 у часописима категорије M22. Такође је објављен још један рад у домаћем часопису категорије M24.

Утицај научних радова се види и у секцији 5.1.2. кроз приказану цитираност.

Одржао је и предавање по позиву на научном скупу.

Као пет најзначајнијих радова у каријери др Николе Петровића могу се издвојити:

1. [3] M. Belić, N. Z. Petrović, W.-P. Zhong, R. H. Xie and G. Chen, “Analytical Light Bullet Solutions to the Generalized (3+1)-Dimensional Nonlinear Schrödinger Equation,’’ Phys. Rev. Lett. 101, 0123904 (2008). IF 7.180 (5/68) M21a, број цитата: 206

2. [2] W.-P. Zhong, R.-H. Xie, M. Belić, N. Z. Petrović, G. Chen and L. Yi, “Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients,’’ Phys. Rev. A 78,023821 (2008). IF 2.908 (6/64) M21a, број цитата: 153

3. [8] A. Al Bastami, N. Z. Petrović and M. R. Belić, “Special solutions of the Ricatti Equation with applications to the Gross-Pitaevskii nonlinear PDE,’’ Electron. J. Diff. Eqs., Vol. 2010, No. 66, 1 (2010). IF 0.427 (198/245 podaci za 2011) M23, број цитата: 50

4. [4] N. Z. Petrović, M. Belić, W.-P. Zhong, R.-H. Xie and G. Chen, “Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrödinger equation for both

normal and anomalous dispersion,’’ Opt. Lett. 34, 1609 (2009). IF 3.059 (6/71) M21a, број цитата: 44

5. [5] N. Z. Petrović, M. Belić and W.-P. Zhong, “Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation,’’ Phys. Rev. E 81, 016610 (2010). IF 2.352 (4/54) M21a, број цитата: 42

Као пет репрезентативних радова кандидата од претходног избора у звање могу се узети:

[24] N. Petrović, M. Belić, W. Krolikowski, ‘’Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions,’’ Chaos, Solitons & Fractals: X 13, 100121 (2024). IF=5.3 (2/57) SNIP=1.8, M21a

[25] N. Z. Petrović, ’’Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method,’’ Optical and Quantum Electronics 52 (6), 319 (2020). IF=2.084 (57/99) SNIP=0.78, M22

[26] N. Petrović, ‘’Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method,’’ Optical and Quantum Electronics 54 (10), 644 (2022). IF=3.0 (42/100) SNIP=0.87, M22

[27] N. Petrović, "Solitary and traveling wave solutions to nematic liquid crystal equations with cubic-quintic nonlinearity using the Jacobi elliptic function expansion method,’’ Facta Universatis: Electronics and Energetics (2025) (рад прихваћен у часопис), IF=0.6, M24.

[29] N. Petrović, ''Exact traveling and solitary wave solutions to the generalized Gross-Pitaevskii equation with cylindrical potential,'' Proceedings of the 2nd Conference on Nonlinearity, Serbian Academy of Nonlinear Science, p. 259 (2021), M33

Детаљан опис пет одабраних радова који је већ презентиран у одељку 4. укључује и два рада који се користе као доказ за самосталност кандидата, при чему је један категорије M21a, а други M22.

5.1.1.1 Радови са доминантном улогом кандидата

1. У раду [24] из категорије M21a, кандидат је применио методу Јакобијеве елиптичне функције на систем једначина који описује кретање светла кроз Нематичке течне кристале са нелинеарношћу трећег степена, такозваном Керовом нелинеарношћу. Испоставља се да треба изразити и таласну функцију и угаону функцију кристала као полином другог степена Јакобијеве елиптичне функције. Уз то коефицијент уз други степен и слободни коефицијент морају да буду у специфичним односима. Као последица добијају се

карактеристична решења, као што су двоструки тамни солитон који је већ био описан у другим радовима, али захваљујући методи Јаковијеве елиптичне функције такође може садржати чирп који утиче и на позадину. Добијају се и светли солитони, као и путујући таласи. Такође се добија за случај две трансверзне димензије да се могу наћи решења кад су сви коефицијенти константни осим једног чак и у случају чирпа. Генерално систем пружа велики степен флексибилности уз само 2 ограничења за параметре система диференцијалних једначина. Угаоне функције генерално прате облик таласних функција што одсликава интеракцију светлости са нематским течним кристалима. У изјави доприноса сваког аутора при крају рада види се да је др Никола Петровић био одговоран за идеју, реализацију, цртање графова, проверу решења и писање рада, тојест да је дао доминантни допринос у овом раду.

2. У раду [26] из категорије М22 су нађена решења за Кунду-Наскар-Мукрџи једначину. У овој једначини је разбијена симетрија између два трансверзална правца, те се добијају значајно другачији облици функција, а у правцу чији се извод не појављује у самој једначини се добијају широке периодичне екстензије функције, такозване 'кресте'. Чирп је могућ само у овом и параметар који га одређује је ограничен да буде константан, те се не добија модулација амплитуде, али се добија изузетно комплексна модулација самих решења, поготово у режиму путујућих таласа. Др Никола Петровић као соло аутор је наравно извео целокупне прорачуне и анализу у самом ради, као и цртање графова и писање рада.

5.1.1.2 Опис осталих репрезентативних радова

3. У раду [25] из категорије М22 су нађена решења за Дејви-Стјуартсонов систем једначина. У овом случају је таласна функција полином првог степена Јакобијеве елиптичне функције, док је такозвана функција средњег проточног поља полином другог степена. Иако се добију решења са значајним ограничењима на већину параметара испоставља се да слободан члан средњег проточног поља не подлеже никаквим ограничењима, те је могућа изузетно велика флексибилност у конструкцији решења. Др Никола Петровић је био једини аутор овог рада.

4. У раду [26] из категорије М24 су примењени резултати из [24] на нематичне течне кристале са такозваном кубичном-квинтичном нелинеарношћу. Овде добијамо решења код којих је таласна функција полином првог степена Јакобијеве елиптичне функције док је угаона функција кристала полином другог степена. Пошто систем има један степен слободе више од система описаног у [24], а и даље је број ограничења параметара два, то

даје још већу флексибилност у конструкцији нових решења од система. Др Никола Петровић је био једини аутор овог рада.

У случају да је неопходно да репрезентативни рад буде од претходног избора у звање:

5. У конференцијском раду [29] из категорије М33, генералисано је решење једначине Грос-Питаевски са сферно-симетричног потенцијала на цилиндрично- и планарно-симетричне потенцијале, разбивши тиме симетрију између три трансверзне димензије. Др Никола Петровић је био једини аутор овог рада.

У случају да је неопходно да репрезентативни рад не мора да буде од претходног избора у звање:

5. У раду [19] у којем се анализира стабилност многобројних решења које је добио методом развоја по Јакобијевим елиптичним функцијама објавио у неколико високо цитираних радова у престижним часописима у периоду од 2008. до 2011. године. У сарадњи са др Најданом Алексићем и проф. др Миливојем Белићем, урађена је анализа стабилности решења нелинеарне Шредингерове једначине са нормалном и аномалном дисперзијом и једначине Грос-Питајевског. Најпре је урађена трансформација која нелинеарну Шредингерову једначину са дистрибуираним коефицијентима своди на једначину са константним коефицијентима. Затим је конструисан одговарајући Лагранжијан и под претпоставком постојања модулационе нестабилности су добијене једначине за њихову целокупну амплитуду, тј. њен реалан и имагинаран део, у функцији од таласног броја пертурбација. Затим је систем једначина решен да би се добило да ли параметри дивергирају или не и тиме одредило да ли решења имају стабилност. Утврђено је да у свим случајевима решења поседују или апсолутну стабилност или стабилност уз присуство такозваног менажирања дисперзије, тј. алтернирања знака коефицијента дисперзије уз помоћ метаматеријала. Апсолутна стабилност је утврђена у три димензије за тамне солитоне у аномалној дисперзији, и за светле временске солитоне у нормалној дисперзији, док је у две димензије апсолутна стабилност утврђена за све тамне солитоне. Ови резултати су проверени компјутерским симулацијама и добијено је скоро потпуно слагање у решењима без чирпа и изузетно добро квалитативно слагање које у сваком случају потврђује критеријуме апсолутне стабилности у решењима са чирпом. Др Никола Петровић је као први аутор учествовао у свим аспектама овог рада осим компјутерских симулација.

• Остале показатеље, подељене у две групе (А и Б), процењује МОФ:

1	А	до 5 изабраних радова - **Приказано 5 радова**
2	А	утицајност (узимајући у обзир и 2.6) **Већина**

		радова из категорије М21а и М21. Један рад има преко 200 по систему Google Scholar и јоше један преко 100.
3	Б	додатни библиометријски показатељи* **Приказана табела**
4	Б	истакнутост, самосталност, дужина радова, **Радови у водећим часописима попут Physical Review Letter, рад са преко 200 цитата. Самосталност показана у часописима највишег ранга М21 и М21а, као и у осталим часописима**.
5	Б	применљивост, награде- **Обзиром да је рад претежно теоријског карактера не постоје техничке реализације. Ипак тематика се односи на простирање светлости поред осталог и кроз таласоводе као и реализацију квантно инжењерских система у оквиру оптике попут квантних рачунара па има индиректну примењивост и потенцијал за консултације.**

Додатни библиометријски показатељи (тачка 2 **П1П**) су:

	ИФ	М	СНИП
Укупно	10,984	26,5	3,45
Усредњено по чланку	2,746	2,65	1,15
Усредњено по аутору	7,45	19,83	2,25

5.1.2 Позитивна цитираност научних радова кандидата

Према бази WOS радови кандидата су цитирани укупно 604 пута, док је број цитата без аутоцитата 537. Према истој бази H–индекс кандидата је 11. Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 988 цитата (што укључује и 184 цитата књиге IMO Compendium) и H фактор 14.

Није примећена ниједна инстанца негативне цитираности, а у бројним случајевима методе из радова за које је основни доприност дао Никола З Петровић су примењиване у другим публикацијама.

5.1.3 Параметри квалитета часописа

И у периоду пре и у периоду после избора кандидат је већином објављивао радове у часописима категорије M21a и M21. Укупан фактор утицаја (збир импакт фактора) радова кандидата је **59,282**, а у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања тај фактор је **10,984**. Кандидат је објављивао радове у најугледнијим часописима из његове области. Посебно се међу њима истичу: *Phys. Rev. Lett.,* Nonlinear Dynamics, *Physical Review A u E*. и Optics Express.

У категорији M21a, M21, M22, M23 и M24 кандидат је објавио радове у следећим часописима, где су Посебно означени они часописи у којима је кандидат објављивао у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања:

Пре претходног избора:

• 1 рад у Asian J. Phys.: (2006) M24.

• 1 рад у Phys Rev A: (2008) IF=2.908 (6/64) M21a.

• 1 рад у Phys. Rev. Lett: (2008) IF=7.180 (5/68) M21a.

• 1 рад у Optics Letters: (2009) IF=3.059 (6/71) M21a.

• 1 рад у J Diff. Equations: (2010) IF=0.427 (198/245 подаци за 2011) M23.

• 5 радова у Phys. Rev. E:

(2010) IF=2.352 (4/54) M21a, (2011) IF=2.255 (6/55) M21, (2011) IF=2.255 (6/55) M21, (2011) IF=2.255 (6/55) M21, (2014) IF=2.288 (5/54) M21a.

• 2 рада у Phys. Scr.:

(2011) IF=1.204 (35/84) M22, (2013) IF=1.296 (40/78) M22.

• 4 рада у Nonlinear Dynamics:

(2015) IF=3.000 (8/135) M21a, (2016) IF=3.464 (8/133) M21a, (2016) IF=3.464 (8/133) M21a, (2018) IF=4.339 (8/134) M21a.

• 1 рад у Optics Express (2015) IF=3.148 (14/90) M21.

• 2 рада у Journal of Optics:

(2015) IF = 1.847 (36/90) M22, (2015) IF = 1.847 (36/90) M22.

• 1 рад у Europhysics Letters: (2016) IF= 1.957 (23/79) M21.

• 1 рад у Optical and Quantum Electronics: (2016) IF= 1.055 (70/92) M23.

После претходног избора:

• 2 рада у Optical and Quantum Electronics:

(2020) IF=2.084 (57/99) M22, (2022) IF=3.0 (42/100) M22.

• 1 рад у Chaos, Solitons & Fractals: X (2024) IF=5.3 (2/57) M21a

• 1 рад у Facta Universitatis Series: Electronics and Energetics: (2024) IF=0.6, M24.

5.1.4 Степен самосталности и степен учешћа у реализацији радова у научним центрима у земљи и иностранству

Кандидат је једини аутор у 4 рада (три од прошлог избора), укључујући и 1 рад у часопису M21a категорије, а водећи аутор у још 9 радова (један од прошлог избора), укључујући и 3 рада у часописима M21a категорије. Такође има рад са студентом додипломских студија у часопису M23 категорије Optical and Quantum Electronics. На свим тим радовима је дао основни допринос реализацији целога рада, а значајно је учествовао на свим осталим радовима. Значајним бројем радова као једини и као главни аутор др Никола Петровић је показао високи степен иницијативе и самосталности.

Као други аутор, дао је најважнији математички допринос раду публикованом у Phys. Rev. Lett који има преко 200 цитата на Google Scholar. У свим радовима дао је основни допринос техникама за решавање нелинеарних једначина. Тиме се може закључити да он има јасан домен главних доприноса који се састоји у налажењу и модификацији постојећих метода у циљу налажења егзактних решења различитих класа нелинеарних једначина и система.

При изради свих публикација кандидат је учествовао у развоју метода, провери ваљаности математичких резултата, као и у завршној анализи нелинеарних феномена и писању радова.

Два рада, према захтевима за реизбор у звање виши научни сарадник, одабрана да илуструју самосталност кандидата и његове основне доприносе су раније описани детаљније у секцији 4. и у секцији 5.1. То су:

1. [24] N. Petrović, M. Belić, W. Krolikowski, ‘’Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions,’’ Chaos, Solitons & Fractals: X 13, 100121 (2024). IF=5.3 (2/57) SNIP=1.8, M21a

Рад је реализовао потпуно самостално од идеје, реализације теорије и решавања свих једначина до писања рада и одговора рецензентима.

2. [26] N. Petrović, ‘’Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method,’’ Optical and Quantum Electronics 54 (10), 644 (2022). IF=3.0 (42/100) SNIP=0.87, M22

У приказу рада кандидата (секција 4) описани су доприноси кандидата и у другим радовима, а то је поновљено и у анализи пет радова.

5.1.5 Награде

Није обавезно за тражено звање.

Вредне помена су и бројне награде на такмичењима из математике и физике у средњој школи и на студијама укључујући две сребрне и једну бронзану медаљу на Међународним математичким олимпијадама, као и похвалу на елитном Вилијем Ловел Патнам такмичењу из математике за студенте у Америци, која се додељује само за 25-50 места у конкуренцији од неколико хиљада студената математике, углавном са најпрестижнијих универзитета.

5.2 Ангажованост у формирању научних кадрова

Током боравка на Универзитету у Катару кандидат је руководио дипломским радовима више студената. У том периоду није било могуће организовати докторске студије на том одсеку Универзитета али о комплексности пројеката којима је руководио у раду са студенатима говори и чињеница да је са три студента публиковао укупно шест радова, и то два рада у часопису Phys Rev E, два рада у часопису Physica Scripta, један рад у Optical and Quantum Electronics и један рад у Electron. J. Diff. Eqs.

Радио је у комисијама Друштва математичара и друштва физичара на припреми задатака за такмичења из физике и математика и на њиховом оцењивању. Учествовао је у припремама младих математичара. Књига решених проблема са Међународних математичких олимпијада је основни уџбеник за припреме за такмичења свуда у свету, била је цитирана (184 према сервису Google Scholar) у низу научних радова из области педагогије и наставе математике, као и рада са талентованим студентима. Треба напоменути да се кандидат бави Математичком физиком те да је математика основни алат у његовом раду.

Захваљујући томе да је кандидат био присутан у Дохи на његовом Универзитету је организован током неколико година турнир и припреме студената из десетак околних земаља за учешће на Међународним математичким олимпијадама. После одласка назад у Београд постојала је иницијатива да се он ангажује да настави рад на организацији припрема што је реализовано само током једне школске године. Учествовао је у бројним припремама у Србији за домаћа и међународна математичка такмичења.

5.3 Нормирање броја коауторских радова, патената и техничких решења

Пошто ниједан рад др Николе Петровића нема више од 3 аутора у овом изборном периоду, нема потребе за нормирањем.

5.4 Руковођење пројектима, потпројектима и пројектним задацима

Кандидат руководи пројектним задатком "*Аналитичко решавање нелинеарних једначина у оптици*" у оквиру пројекта ОН171006 "*Нелинеарна динамика локализованих самоорганизованих структура у плазми, нано-композитним материјалима, течним и фотоничним кристалима и ултрахладним кондензатима*" под руководством др Милана Петровића

5.5 Активност у научним и научно-стручним друштвима

Кандидат је био члан комисија за такмичење Друштва физичара Србије и Друштва математичара Србије.

Он је био члан комисије Друштва математичара Србије за такмичења из математике од 2014. године до 2022. године. Учествовао је у састављању задатака за све нивое такмичења и у припремама тима за Међународну математичку олимпијаду и друга међународна такмичења из математике. Такође је и аутор задатка број 6. на Међународној математичкој олимпијади 2022.

Др Никола Петровић је такође рецензент у неколико угледних часописа: Communications in Nonlinear Science and Numerical Simulation, Physica Scripta, Chinese Journal of Physics, Results in Physics и других.

5.6 Утицајност научних резултата

Овде понављамо одељак 4.1.3 уз допуну.

У категорији М21а, М21, М22, М23 и М24 кандидат је објавио радове у следећим часописима, где су посебно означени они часописи у којима је кандидат објављивао у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања:

Пре претходног избора:

• 1 рад у Asian J. Phys.: (2006) M24.

• 1 рад у Phys Rev A: (2008) IF=2.908 (6/64) M21a.

• 1 рад у Phys. Rev. Lett: (2008) IF=7.180 (5/68) M21a.

• 1 рад у Optics Letters: (2009) IF=3.059 (6/71) M21a.

• 1 рад у J Diff. Equations: (2010) IF=0.427 (198/245 подаци за 2011) M23.

• 5 радова у Phys. Rev. E:

(2010) IF=2.352 (4/54) M21a, (2011) IF=2.255 (6/55) M21, (2011) IF=2.255 (6/55) M21, (2011) IF=2.255 (6/55) M21, (2014) IF=2.288 (5/54) M21a.

• 2 рада у Phys. Scr.:

(2011) IF=1.204 (35/84) M22, (2013) IF=1.296 (40/78) M22.

• 4 рада у Nonlinear Dynamics:

(2015) IF=3.000 (8/135) M21a, (2016) IF=3.464 (8/133) M21a, (2016) IF=3.464 (8/133) M21a, (2018) IF=4.339 (8/134) M21a.

• 1 рад у Optics Express (2015) IF=3.148 (14/90) M21.

• 2 рада у Journal of Optics:

(2015) IF = 1.847 (36/90) M22, (2015) IF = 1.847 (36/90) M22.

• 1 рад у Europhysics Letters: (2016) IF= 1.957 (23/79) M21.

• 1 рад у Optical and Quantum Electronics: (2016) IF= 1.055 (70/92) M23.

После претходног избора:

• 2 рада у Optical and Quantum Electronics:

(2020) IF=2.084 (57/99) M22, (2022) IF=3.0 (42/100) M22.

• 1 рад у Chaos, Solitons & Fractals: X (2024) IF=5.3 (2/57) M21a

• 1 рад у Facta Universitatis Series: Electronics and Energetics: (2024) IF=0.6, M24.

Укупан фактор утицаја (збир импакт фактора) радова кандидата је **59,282**, а у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања тај фактор је **10,984**. Кандидат је објављивао радове у најугледнијим часописима из његове области. Посебно се међу њима истичу: *Phys. Rev. Lett., Nonlinear Dynamics*, *Chaos, Solitons & Fractals X*, *Physical Review E*. и *Optics Express*.

Према бази WOS радови кандидата су цитирани укупно 604 пута, док је број цитата без аутоцитата 537. Према истој бази H–индекс кандидата је 11. Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 984 цитата (што укључује и 184 цитата књиге IMO Compendium) и H фактор 14.

Најцитиранији радови у целој каријери су према Google Scholar:

1. Analytical light bullet solutions to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation, M Belić, N Petrović, WP Zhong, RH Xie, G Chen, Physical review letters 101 (12), 123904, 2008, цитата: 206

2. The IMO Compendium: A Collection of Problems Suggested for the International Mathematical Olympiads: 1959-2009 Second Edition, D Djukić, V Janković, I Matić, N Petrović, Springer Science & Business Media прво издање 2006 друго издање 2011, цитата: 184

3. Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients WP Zhong, RH Xie, M Belić, N Petrović, G Chen, L Yi, Physical Review A 78 (2), 023821, 2008, цитата: 153

4. Special solutions of the Riccati equation with applications to the Gross-Pitaevskii nonlinear PDE, A Al Bastami, MR Belić, NZ Petrović, Electronic Journal of Differential Equations (EJDE) 66, 1, 2010, цитата: 50

5. Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Schrödinger equation for both normal and anomalous dispersion, NZ Petrović, M Belić, WP Zhong, RH Xie, G Chen, Optics letters 34 (10), 1609-1611 2009, цитата: 44

6. Spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Gross-Pitaevskii equation, NZ Petrović, M Belić, WP Zhong, Physical Review E 81 (1), 016610, 2010, цитата: 42

7. Controllable parabolic-cylinder optical rogue wave, WP Zhong, L Chen, M Belić, N Petrović, Physical Review E 90 (4), 043201 2014, цитата: 41

5.7 Конкретан допринос кандидата у реализацији радова у научним центрима у земљи и иностранству

Сви радови кандидата спадају у домен нелинеарне оптике односно шире посматрано области Оптика из домена физичких наука. Један рад који се бави динамиком Bose Einsten кондензата спада више у домен атомске и молекуларне физике.

Кандидат има активну сарадњу и заједничке публикације са истраживачима у области нелинеаре оптике и нелинеарне динамике, као и математичке физике: проф. др Миливој Белић, Факултет за науку и инжењеринг Хамад бин Калифа универзитета, некадашњи Texas A&M универзитет, Доха Катар, Силију Су (Si-Liu Xu), Школа електронског и информационог инжењеринга, Ху-Беи универзитет науке и технологије, Сјенин, Кина , Веипинг Жонг (Wei-Ping Zhong), Шунде политехнички факултет, Шунде, Кина. Такође је у скорије време успостављена и сарадња са угледним стручњаком са Аустралијског Националног Универзитета (ANU) Веславом Кроликовским (Wieslaw Krolikowski)

5.8 Уводна предавања на конференцијама и друга предавања

Након претходног избора у звање, кандидат је одржао следећа предавања:

5.8.1. Предавање по позиву:

N. Petrović,’’Solitary and traveling wave solutions to equations governing nematic liquid crystals using the Jacobi elliptic function expansion method,’’ International Congress and Expo on Optics, Photonics and Lasers (EUROPL2023), p.19 (2023)

5.8.2. Остала предавања:

N. Petrović, ’’Spatio-temporal solitary and traveling wace solutions to the Kundu-Mukherjee-Naskar equation’’, VIII International School and Conference on Photonics (PHOTONICA2021), p.70 (2021)

N. Petrović, ’’Solitary and traveling two-component wave solutions in menatic liquid crystals using the Jacobi Elliptic function expansion method.’’ 3rd Conference on Nonlinearity, Serbian Academy of Nonlinear Science, (2023)

N. Petrović, ’’Solutions to nematic liquid crystal systems with cubic-quintic and septic nonlinearities using the Jacobi elliptic function expansion method,’’ IX International School and Conference on Photonics (PHOTONICA2023), p. 53 (2023)

N. Petrović, “Solitary and traveling wave solutions to the Nonlinear Schrödinger equation describing quantum droplets”, XI-th International Conference "Solitons, Collapses and Turbulence: Achievements, Developments and Perspectives," p. 60 (SCT2024) (2024)

6. Елементи за квантитативну анализу рада

Остварени резултати у периоду након одлуке Научног већа о предлогу за стицање претходног научног звања

Категорија	М бодова по раду	Број радова	Укупно М бодова
М21а	10	1	10
М22	5	2	10
М24	2	1	2
М32	1,5	1	1,5
М33	1	1	1
М34	0,5	4	2
Укупно			26,5

Поређење са минималним квантитативним условима за реизбор у звање виши научни сарадник:

М категорије	Услов	Остварено
Укупно	25	**26,5**
М10+М20+М31+М32+М33+М41+М42	20	**24,5**
М11+М12+М21+М22+М23	15	**20**

Додатни библиометријски показатељи (тачка 2 **П1П**) су:

	ИФ	М	СНИП
Укупно	10,984	26,5	3,45
Усредњено по чланку	2,746	2,65	1,15
Усредњено по аутору	7,45	19,83	2,25

Напомена, код усредњења су за ИФ су рачуната 4 рада са импакт фактором, за М катерогију су рачунате све ставке, а за СНИП су рачуната три рада која имају свој СНИП.

Према бази WOS радови кандидата су цитирани укупно 604 пута, док је број цитата без аутоцитата 537. Према истој бази H–индекс кандидата је 11. Прилог: подаци о цитираности са интернет странице WOS.

На бази Google Scholar има 988 цитата (што укључује и 184 цитата књиге IMO Compendium) и H фактор 14.

7. Списак објављених радова и других публикација
Никола З Петровић

СПИСАК РАДОВА ДО ПРЕТХОДНОГ ИЗБОРА У ЗВАЊЕ

[1] D. Jović, M. Petrović, D. Arsenović, S. Prvanović, M. Belić, N. Z. Petrović, “Counterpropagating beams in photorefractive media and optically induced photonic lattices’’, Asian J. Phys. 15, 283 (2006). M24

[2] W.-P. Zhong, R.-H. Xie, M. Belić, N. Z. Petrović, G. Chen and L. Yi, “Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients,’’ Phys. Rev. A 78,023821 (2008). IF 2.908 (6/64) M21a

[3] M. Belić, N. Z. Petrović, W.-P. Zhong, R. H. Xie and G. Chen, “Analytical Light Bullet Solutions to the Generalized (3+1)-Dimensional Nonlinear Schrödinger Equation,’’ Phys. Rev. Lett. 101, 0123904 (2008). IF 7.180 (5/68) M21a

[4] N. Z. Petrović, M. Belić, W.-P. Zhong, R.-H. Xie and G. Chen, “Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrödinger equation for both normal and anomalous dispersion,’’ Opt. Lett. 34, 1609 (2009). IF 3.059 (6/71) M21a

[5] N. Z. Petrović, M. Belić and W.-P. Zhong, “Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation,’’ Phys. Rev. E 81, 016610 (2010). IF 2.352 (4/54) M21a

[6] N. Z. Petrović, M. Belić and W.-P. Zhong, “Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrödinger equation with polynomial nonlinearity of arbitrary order,’’ Phys. Rev. E 83, 026604 (2011). IF 2.255 (6/55) M21

[7] N. Z. Petrović, N. Aleksić, A. Al Bastami and M. Belić, “Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential,’’ Phys. Rev. E 83, 036609 (2011). IF 2.255 (6/55) M21

[8] A. Al Bastami, N. Z. Petrović and M. R. Belić, “Special solutions of the Ricatti Equation with applications to the Gross-Pitaevskii nonlinear PDE,’’ Electron. J. Diff. Eqs., Vol. 2010, No. 66, 1 (2010). IF 0.427 (198/245 podaci za 2011) M23

[9] A. Al Bastami, M. R. Belić, D. Milović and N. Z. Petrović, “Analytical chirped solutions to the (3+1)-dimensional Gross-Pitaevskii equation for various diffraction and potential Functions,’’ Phys. Rev. E 84, 016606 (2011). IF 2.255 (6/55) M21

[10] N. Z. Petrović, H. Zahreddine and M. Belić, “Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional nonlinear Schrödinger equation with linear potential,’’ Phys. Scr. 83, 065001 (2011). 1.204 (35/84) M22

[11] S. Xu, N. Z. Petrović and M. Belić, “Vortex solitons in the (2+1)-dimensional nonlinear Schrödinger equation with variable diffraction and nonlinearity coefficients,’’ Phys. Scr. 87, 045401 (2013). IF 1.296 (40/78) M22

[12] M. R. Belić, M. S. Petrović, D. M. Jović, A. I. Strinić, D. D. Arsenović, S. Prvanović,
R. D. Jovanović, N. Z. Petrović, “Dancing Light: Counterpropagating, Beams in Photorefractive Crystals,’’ Acta Physica Polonica A 212, 729 (2007). M33 (M23 as a journal)

[13] N. Z. Petrović and H. Zahreddine, “Exact traveling wave solutions to coupled generalized nonlinear Schrödinger equations,’’ Phys. Scr. T149, 014039 (2012). M33 (IF 1.032 (48/83) M22 as a journal)

[14] W. P. Zhong, L. Chen, M. Belić, N. Petrović, “Controllable parabolic-cylinder optical rogue wave,” Phys. Rev. E 90 (4), 043201 (2014). IF=2.288 (5/54) SNIP=1.14, M21a

[15] S. L. Xu, N. Petrović, M. R. Belić, “Exact solutions of the (2+ 1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients,” Nonlinear Dynamics 80 (1-2), 583-589 (2015). IF=3.000 (8/135) SNIP=1.47, M21a

[16] S. L. Xu, N. Petrović, M. R. Belić, W. Deng, “Exact solutions for the quintic nonlinear Schrödinger equation with time and space,” Nonlinear Dynamics 84 (1), 251-259 (2016). IF=3.464 (8/133) SNIP=1.54, M21a

[17] S. L. Xu, N. Petrović, M. R. Belić, Z. L. Hu, “Light bullet supported by parity-time symmetric potential with power-law nonlinearity,” Nonlinear Dynamics 84 (4), 1877-1882 (2016) IF=3.464 (8/133) SNIP=1.54, M21a

[18] N. Z. Petrović, “Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrodinger equation with single-and dual-power law nonlinearity,” Nonlinear Dynamics 93 (4), 2389-2397 (2018) IF=4.339 (8/134) SNIP=1.75, M21a

[19] N. Z. Petrović, N.B. Aleksić, M. Belić, “Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach,” Optics Express 23 (8), 10616-10630 (2015) IF=3.148 (14/90) SNIP=1.67, M21

[20] S. L. Xu, Y. Zhao, N. Z. Petrović, M. R. Belić, “Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities,” EPL (Europhysics Letters) 115 (1), 14006 (2016) IF=1.957 (23/79) SNIP=0.60, M21

[21] S. L. Xu, N. Petrović, M. R. Belić, “Two-dimensional dark solitons in diffusive nonlocal nonlinear media,” Journal of Optics 44 (2), 172-177 (2015). IF=1.847 (36/90) SNIP=0.87, M22

[22] S. L. Xu, G. P. Zhou, N. Petrović, M. R. Belić, “Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential,” Journal of Optics 17 (10), 105605 (2015). IF=1.847 (36/90) SNIP=0.87, M22

[23] N. Z. Petrović, M. Bohra, “General Jacobi elliptic function expansion method applied to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation,” Optical and Quantum Electronics 48 (4), 268 (2016). IF=1.055 (70/92) SNIP=0.61, M23

РАДОВИ ПУБЛИКОВАНИ ПОСЛЕ ПРЕТХОДНОГ ИЗБОРА У ЗВАЊЕ

Радови у међународним часописима изузетних вредности M21a

[24] N. Petrović, M. Belić, W. Krolikowski, ’’Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions,’’ Chaos, Solitons & Fractals: X 13, 100121 (2024). IF=5.3 (2/57) SNIP=1.8, M21a

Радови у истакнутим међународним часописима M22

[25] N. Z. Petrović, ’’Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method,’’ Optical and Quantum Electronics 52 (6), 319 (2020). IF=2.084 (57/99) SNIP=0.78, M22

[26] N. Petrović, ’’Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method,’’ Optical and Quantum Electronics 54 (10), 644 (2022). IF=3.0 (42/100) SNIP=0.87, M22

Рад у националном часопису међународног значаја M24

[27] N. Petrović, "Solitary and traveling wave solutions to nematic liquid crystal equations with cubic-quintic nonlinearity using the Jacobi elliptic function expansion method,’’ Facta Universatis: Electronics and Energetics (2025) (рад прихваћен у часопису), IF=0.6, M24. Овај часопис је добио Clarivate (WOS) фактор током 2024 (о чему прилажемо доказ). Међутим како то још увек није укључено у базу КОБСОН овај часопис смо рангирали као M24 јер је то најбоља категоризација (Матични одбор за телекомуникација) коју има у одборима Министарства за науку и технолошки развој.

Предавања по позиву са међународних скупова штампана у изводу M32

[28] N. Petrović, ’’Solitary and traveling wave solutions to equations governing nematic liquid crystals using the Jacobi elliptic function expansion method,’’ International Congress and Expo on Optics, Photonics and Lasers (EUROPL2023), p.19 (2023)

Саопштења са међународних скупова штампана у целини M33

[29] N. Petrović, ''Exact traveling and solitary wave solutions to the generalized Gross-Pitaevskii equation with cylindrical potential,'' Proceedings of the 2nd Conference on Nonlinearity, Serbian Academy of Nonlinear Science, p. 259 (2021)

Саопштења са међународних скупова штампана у изводу М34

[30] N. Petrović,’’Spatio-temporal solitary and traveling wace solutions to the Kundu-Mukherjee-Naskar equation’’, VIII International School and Conference on Photonics (PHOTONICA2021), p.70 (2021)

[31] N. Petrović,’’Solitary and traveling two-component wave solutions in menatic liquid crystals using the Jacobi Elliptic function expansion method.’’ 3rd Conference on Nonlinearity, Serbian Academy of Nonlinear Science, (2023)

[32] N. Petrović, ’’Solutions to nematic liquid crystal systems with cubic-quintic and septic nonlinearities using the Jacobi elliptic function expansion method,’’ IX International School and Conference on Photonics (PHOTONICA2023), p. 53 (2023)

[33] N. Petrović, “Solitary and traveling wave solutions to the Nonlinear Schrödinger equation describing quantum droplets”, XI-th International Conference "Solitons, Collapses and Turbulence: Achievements, Developments and Perspectives," p. 60 (SCT2024) (2024)

8. Подаци о цитираности радова

Никола З Петровић

Приложене су странице са прегледом цитираности кандидата у бази WOS.

English Products

Web of Science™ Search Sign In Register

Researcher Search > Author Records > Author Profile > Citation Report: Nikola Petrović (Author)

Citation Report

MENU

Nikola Petrović (Author)

Analyze Results Create Alert

Export Full Report

Publications

26
Total

From 1985 to 2025

Citing Articles

414 Analyze
Total

391 Analyze
Without self-citations

Times Cited

604
Total

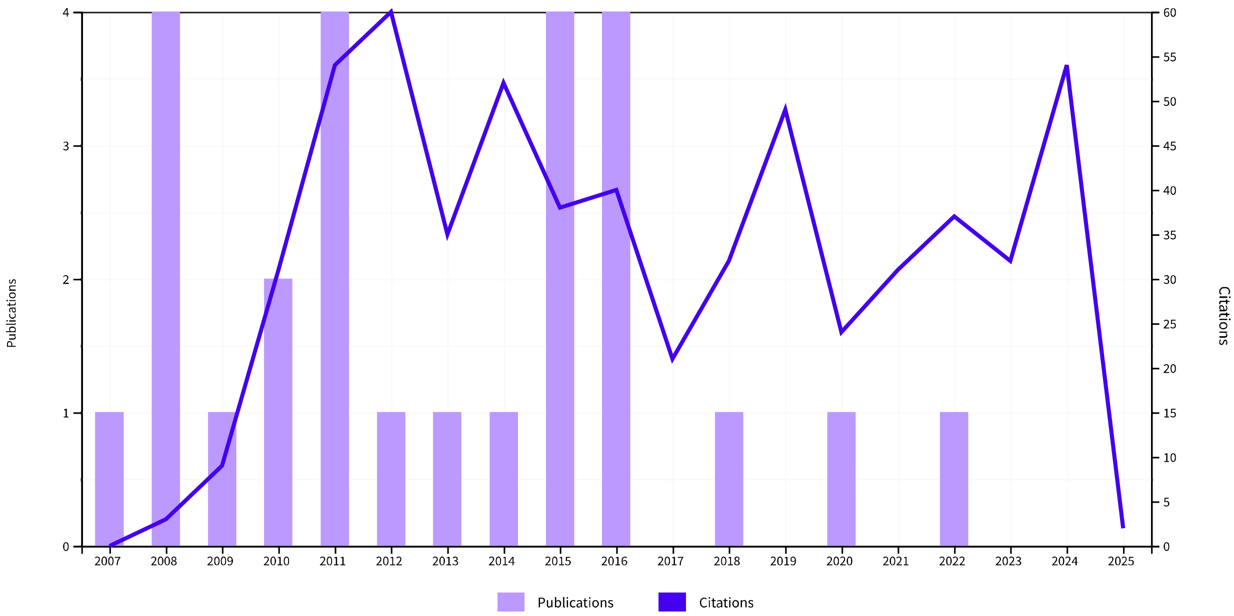
23.23
Average per item

537
Without self-citations

11
H-Index

Times Cited and Publications Over Time

DOWNLOAD



26 Publications Citations: highest first 1 of 1

Total

Citation Reports

We've refreshed and enhanced the Citation Report. Let us walk you through it.

1 Analytical light bullet solutions to the generalized (3+1)-dimensional nonlinear schrodinger equation Belic, M; Petrovic, N; (...); Chen, G Sep 19 2008 | PHYSICAL REVIEW LETTERS 101 (12)	0	0	0	0	6	9.89	178
2 Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients Zhong, WP; Xie, RH; (...); Chen, G Aug 2008 | PHYSICAL REVIEW A 78 (2)	0	0	0	3	3	6.11	110
3 Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrodinger equation for both normal and anomalous dispersion Petrovic, NZ; Belic, M; (...); Chen, G May 15 2009 | OPTICS LETTERS 34 (10) , pp.1609-1611	0	0	0	0	0	2.29	39
4 Controllable parabolic-cylinder optical rogue wave Zhong, WP; Chen, L; (...); Petrovic, N Oct 6 2014 | PHYSICAL REVIEW E 90 (4)	0	0	0	0	0	3	36
5 Exact traveling-wave and spatiotemporal soliton solutions to the generalized (3+1)-dimensional Schrodinger equation with polynomial nonlinearity of arbitrary order Petrovic, NZ; Belic, M and Zhong, WP Feb 28 2011 | PHYSICAL REVIEW E 83 (2)	0	0	0	0	0	2.2	33
6 Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross-Pitaevskii equation Petrovic, NZ; Belic, M and Zhong, WP Jan 2010 | PHYSICAL REVIEW E 81 (1)	0	0	0	0	0	2	32

7	Two-dimensional dark solitons in diffusive nonlocal nonlinear media Xu, SL; Petrovic, N and Belic, MR Jun 2015 | JOURNAL OF OPTICS-INDIA 44 (2) , pp.172-177	0	0	0	0	0	2.36	26
8	General Jacobi elliptic function expansion method applied to the generalized (3+1)-dimensional nonlinear Schrodinger equation Petrovic, NZ and Bohra, M Apr 2016 | OPTICAL AND QUANTUM ELECTRONICS 48 (4)	0	0	0	0	0	2.3	23
9	Exact solutions of the (2+1)-dimensional quintic nonlinear Schrdinger equation with variable coefficients Xu, SL; Petrovic, N and Belic, MR Apr 2015 | NONLINEAR DYNAMICS 80 (1-2) , pp.583-589	0	0	0	0	0	2.09	23
10	Light bullet supported by parity-time symmetric potential with power-law nonlinearity Xu, SL; Petrovic, N; (...); Hu, ZL Jun 2016 | NONLINEAR DYNAMICS 84 (4) , pp.1877-1882	0	0	0	0	0	2.1	21
11	Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential Petrovic, NZ; Aleksic, NB; (...); Belic, MR Mar 30 2011 | PHYSICAL REVIEW E 83 (3)	0	0	0	0	0	0.93	14
12	Exact solutions for the quintic nonlinear Schrodinger equation with time and space Xu, SL; Petrovic, N; (...); Deng, WW Apr 2016 | NONLINEAR DYNAMICS 84 (1) , pp.251-259	0	0	0	0	0	1.1	11

13	Analytical chirped solutions to the (3+1)-dimensional Gross-Pitaevskii equation for various diffraction and potential functions Al Bastami, A; Belic, MR; (...); Petrovic, NZ Jul 22 2011 | PHYSICAL REVIEW E 84 (1)	0	0	0	0	0	0.67	10
14	Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities Xu, SL; Zhao, Y; (...); Belic, MR Jul 2016 | EPL 115 (1)	0	0	0	0	0	0.8	8
15	Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional nonlinear Schrodinger equation with linear potential Petrovic, NZ; Zahreddine, H and Belic, MR Jun 2011 | PHYSICA SCRIPTA 83 (6)	0	0	0	0	0	0.53	8
16	Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential Xu, SL; Zhou, GP; (...); Belic, MR Oct 2015 | JOURNAL OF OPTICS 17 (10)	0	0	0	0	0	0.55	6
17	Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrodinger equation and the Gross-Pitaevskii equation using a variational approach Petrovic, NZ; Aleksic, NB and Belic, M Apr 20 2015 | OPTICS EXPRESS 23 (8) , pp.10616-10630	0	0	0	0	0	0.55	6
18	Vortex solitons in the (2+1)-dimensional nonlinear Schrodinger equation with variable diffraction and nonlinearity coefficients Xu, SL; Petrovic, NZ and Belic, MR Apr 2013 | PHYSICA SCRIPTA 87 (4)	0	0	0	0	0	0.31	4
19	Chirped solitary and traveling wave solutions for the Kundu-Mukherjee-Naskar equation using the Jacobi elliptic function expansion method Petrovic, N							

Citation Reports

We've refreshed and enhanced the Citation Report. Let us walk you through it.

	Oct 2022 | OPTICAL AND QUANTUM ELECTRONICS 54 (10) Enriched Cited References							
20	Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrodinger equation with single- and dual-power law nonlinearity Petrovic, NZ Sep 2018 | NONLINEAR DYNAMICS 93 (4) , pp.2389-2397	0	0	0	0	0	0.38	3
21	Exact traveling wave solutions to coupled generalized nonlinear Schrodinger equations Petrovic, N and Zahreddine, H 3rd International School and Conference on Photonics Apr 2012 | PHYSICA SCRIPTA T149	0	0	0	0	0	0.21	3
22	SPECIAL SOLUTIONS OF THE RICCATI EQUATION WITH APPLICATIONS TO THE GROSS-PITAEVSKII NONLINEAR PDE Al Bastami, A; Belic, MR and Petrovic, NZ May 8 2010 | ELECTRONIC JOURNAL OF DIFFERENTIAL EQUATIONS	0	0	0	0	0	0.19	3
23	Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrodinger equation with distributed coefficients (vol 78, art no 023821, 2008) Zhong, WP; Xie, RH; (...); Chen, G Sep 2008 | PHYSICAL REVIEW A 78 (3)	0	0	0	0	0	0.17	3
24	Dancing light: Counterpropagating beams in photorefractive crystals Belic, MR; Petrovic, MS; (...); Petrovic, NZ International School and Conference on Optics and Optical Materials Nov 2007 | ACTA PHYSICA POLONICA A 112 (5) , pp.729-736	0	0	0	0	0	0.05	1
2	Solitary and traveling wave solutions for the Davey-Stewartson							

Citation Reports

We've refreshed and enhanced the Citation Report. Let us walk you through it.

NZ Petrovic

Institute of Physics
, University of Belgrade

TITLE	CITED BY	YEAR
Analytical Light Bullet Solutions to the Generalized $(3+1)$-Dimensional <? format ?>Nonlinear Schrödinger Equation M Belić, N Petrović, WP Zhong, RH Xie, G Chen Physical Review Letters 101 (12), 123904	206	2008
The IMO compendium: a collection of problems suggested for the International Mathematical Olympiads: 1959-2004 D Djukić, V Janković, I Matić, N Petrović Springer	184	2006
Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients WP Zhong, RH Xie, M Belić, N Petrović, G Chen, L Yi Physical Review A—Atomic, Molecular, and Optical Physics 78 (2), 023821	153	2008
Special solutions of the Riccati equation with applications to the Gross-Pitaevskii nonlinear PDE. A Al Bastami, MR Belić, NZ Petrović Electronic Journal of Differential Equations (EJDE)[electronic only] 2010 …	50	2010
Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Schrödinger equation for both normal and anomalous dispersion NZ Petrović, M Belić, WP Zhong, RH Xie, G Chen Optics letters 34 (10), 1609-1611	44	2009
Spatiotemporal wave and soliton solutions to the generalized $(3+1)$-dimensional Gross-Pitaevskii equation NZ Petrović, M Belić, WP Zhong Physical Review E—Statistical, Nonlinear, and Soft Matter Physics 81 (1 …	42	2010
Controllable parabolic-cylinder optical rogue wave WP Zhong, L Chen, M Belić, N Petrović Physical Review E 90 (4), 043201	41	2014

TITLE	CITED BY	YEAR
Exact traveling-wave and spatiotemporal soliton solutions to the generalized $(3+1)$-dimensional Schrödinger equation with polynomial nonlinearity of arbitrary order NZ Petrović, M Belić, WP Zhong Physical Review E—Statistical, Nonlinear, and Soft Matter Physics 83 (2 …	38	2011
General Jacobi elliptic function expansion method applied to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation NZ Petrović, M Bohra Optical and Quantum Electronics 48, 1-8	31	2016
Exact solutions of the (2+ 1)-dimensional quintic nonlinear Schrödinger equation with variable coefficients SL Xu, N Petrović, MR Belić Nonlinear Dynamics 80, 583-589	27	2015
Two-dimensional dark solitons in diffusive nonlocal nonlinear media SL Xu, N Petrović, MR Belić Journal of Optics 44, 172-177	25	2015
Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential NZ Petrović, NB Aleksić, AA Bastami, MR Belić Physical Review E—Statistical, Nonlinear, and Soft Matter Physics 83 (3 …	24	2011
Analytical chirped solutions to the $(3+1)$-dimensional Gross-Pitaevskii equation for various diffraction and potential functions A Al Bastami, MR Belić, D Milović, NZ Petrović Physical Review E—Statistical, Nonlinear, and Soft Matter Physics 84 (1 …	22	2011
Light bullet supported by parity-time symmetric potential with power-law nonlinearity SL Xu, N Petrović, MR Belić, ZL Hu Nonlinear Dynamics 84, 1877-1882	21	2016
Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation with linear potential NZ Petrović, H Zahreddine, MR Belić Physica Scripta 83 (6), 065001	14	2011
Spatiotemporal soliton supported by parity-time symmetric potential with competing nonlinearities SL Xu, Y Zhao, NZ Petrović, MR Belić Europhysics Letters 115 (1), 14006	12	2016
Exact solutions for the quintic nonlinear Schrödinger equation with time and space SL Xu, N Petrović, MR Belić, W Deng Nonlinear Dynamics 84, 251-259	10	2016

TITLE	CITED BY	YEAR
Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross-Pitaevskii equation using a variational approach NZ Petrović, NB Aleksić, M Belić Optics Express 23 (8), 10616-10630	9	2015
Vortex solitons in the (2+ 1)-dimensional nonlinear Schrödinger equation with variable diffraction and nonlinearity coefficients S Xu, NZ Petrović, MR Belić Physica Scripta 87 (4), 045401	9	2013
Nonautonomous vector matter waves in two-component Bose-Einstein condensates with combined time-dependent harmonic-lattice potential SL Xu, GP Zhou, N Petrović, MR Belić Journal of Optics 17 (10), 105605	7	2015
Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method N Petrović Optical and Quantum Electronics 54 (10), 644	6	2022
Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrödinger equation with single-and dual-power law nonlinearity NZ Petrović Nonlinear Dynamics 93 (4), 2389-2397	5	2018
Exact traveling wave solutions to coupled generalized nonlinear Schrödinger equations N Petrović, H Zahreddine Physica Scripta 2012 (T149), 014039	5	2012
Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method NZ Petrović Optical and Quantum Electronics 52 (6), 319	1	2020
Publisher's Note: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients [Phys. Rev. A 78, 023821 (2008)] WP Zhong, RH Xie, M Belić, N Petrović, G Chen Physical Review A—Atomic, Molecular, and Optical Physics 78 (2), 029905	1	2008
Dancing light: Counterpropagating beams in photorefractive crystals M Belić, M Petrović, D Jović, A Strinić, D Arsenović, S Prvanović, ... Acta Physica Polonica A 112 (5), 729-736	1	2007
Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions N Petrović, M Belić, W Krolikowski Chaos, Solitons & Fractals: X 13, 100121		2024

TITLE	CITED BY	YEAR
THE NOT-SO-NOBLE SAVAGE: BALKANISM IN MARY EDITH DURHAM'S THROUGH THE LANDS OF THE SERB M Živković, N Petrović Facta Universitatis, Series: Linguistics and Literature, 053-065		2023
Exact traveling and solitary wave solutions to the generalized Gross-Pitaevskii equation with cylindrical potential NZ Petrović Book of abstracts, 75		2017
Spatio-temporal general Jacobi elliptic function expansion method applied to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation N Petrović, M Bohra, MR Belić Photonica2015: V International School and Conference and Photonics, 80		2016
Tačna talasna i solitonska rešenja generalisane nelinearne Šredingerove jednačine NZ Petrović Универзитет у Београду		2013
Exact Spatiotemporal Traveling and Solitary Wave Solutions for the Generalized Nonlinear Schrödinger Equation NZ Petrović PQDT-Global		2013
Exact spatio-temporal soliton solutions to the two-component co-and counter-propagating beams in Kerr-like media N Petrovic, H Zahreddine, M Belic International School and Conference on Photonics, 97		2011
Analytical Light Bullet Solutions to the Generalized š3 ž 1Ž-Dimensional Nonlinear Schrödinger Equation M Belic, N Petrovic, WP Zhong, RH Xie, G Chen		2008
Basic Concepts and Facts D Djukić, V Janković, I Matić, N Petrović The IMO Compendium: A Collection of Problems Suggested for the International …		2006
Solutions D Djukić, V Janković, I Matić, N Petrović The IMO Compendium: A Collection of Problems Suggested for the International …		2006
Constructing an Infinite Class of Perfect Codes NZ Petrović Massachusetts Institute of Technology, Department of Physics; and,(SB …		2003
Solutions to nematic liquid crystals systems with cubic-quintic and septic nonlinearities using the Jacobi elliptic function expansion method N Petrović Book of abstracts, 57		

Никола З Петровић

9. За кандидате који се бирају у звање научни сарадник или се први пут бирају у звање у Србији потребно је приложити и докторску диплому.

Ово није први избор у звање! Кандидат је докторирао у Србији

10. Копије објављених радова и других публикација након претходног избора у звање (верзије из часописа, зборника апстраката, итд.).

приложене су копије радова публиковане после претходне одлуке научног већа/избора у звања

11. Решење о претходном избору у звање (за кандидате који већ имају научно звање приликом избора у више звање или реизбора).

приложено

Chaos, Solitons & Fractals: X 13 (2024) 100121

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals: X

journal homepage: www.elsevier.com/locate/csfx

Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions

Nikola Petrović [a,b,*], Milivoj Belić [c], Wieslaw Krolikowski [d]

[a] Institute of Physics, Pregrevica 118, 11080 Belgrade, University of Belgrade, Serbia
[b] Division of Arts and Sciences, Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar
[c] College of Science and Engineering, Hamad Bin Khalifa University, P.O. Box 23874, Doha, Qatar
[d] Department of Quantum Science and Technology, Research School of Physics, Australian National University, Canberra, ACT 2601, Australia

ARTICLE INFO

Keywords:
Nonlinear
Schrödinger
Nematicon
Liquid
Crystal

ABSTRACT

In our paper we apply the Jacobi elliptic function (JEF) expansion method to obtain exact solutions to the system of equations governing nematic liquid crystals, a system of high importance in nonlinear optics with numerous physical applications. We obtain solutions that are second-order polynomials in terms of JEFs for both the wave function and the tilt angle of molecular orientation. The solutions differ from previously obtained solutions in including both traveling and solitary wave solutions, with and without chirp. They also include the longitudinal dependence of coefficients in the equations, allowing for the management of both the dispersion and diffraction. Only two parameters of the differential equation need to be defined in terms of other coefficients, providing a wide range of flexibility when it comes to constructing solutions.

1. Introduction

Nematic liquid crystals (NLCs) are important systems in nonlinear optics that allow the study of many nonlinear phenomena at low power, due to a very large nonlocal nonlinear response via the light-induced reorientation of NLC molecules [1]. Nonlinear dynamical phenomena such as strange attractors, bifurcations, quasi-periodic behavior and the emergence of chaotic regimes can be observed in NLCs [2,3]. Spatial solitons, which are known as nematicons [4,5], can also be easily observed in NLCs [6] and have been shown to be remarkably stable in the two transverse dimensions [7]. NLCs are generally described by a pair of coupled nonlinear differential equations governing the behavior of the wave function of light and the angular tilt of molecules in the crystal [8].

Various methods have been proposed to find optical solutions to this system of partial differential equations (PDEs), including the $\tan(\phi/2)$-expansion method [9], the generalized exponential rational function method [10], the simple equation method [11,12], the variational method for finding approximate solutions [13–15], and others [8,16]. Among the approaches, a basic ansatz involving JEFs had been used in [17], but only the boundary cases that produce solitary waves were considered.

Recently, a lot of progress in finding new solutions to PDEs was made using the JEF expansion method [18]. The JEF expansion method has been used to find solutions for various forms of the nonlinear Schrödinger equation (NLSE) [19–22], the Gross–Pitaevskii equation (GP) [23,24], and others [25–27].

There are numerous advantages to using this method over the more conventional function expansion methods such as the hyperbolic tangent method [18]. First, the JEF itself has a flexible form, encompassing both the elliptic and hyperbolic trigonometric functions and allowing both solitary and traveling wave solutions, depending on the parameter M [18,19]. Second, one can find both chirped and unchirped solutions to the considered equations [20–27]. Here, the term 'chirp' refers to the quadratic dependence of the phase with respect to the transverse variables [19]. Finally, the form of equations in these papers sometimes contains distributed coefficients rather than constants, which allows for the management of both the dispersion [28] and diffraction [29]. The management in this sense means a careful longitudinal control of dispersion and diffraction, so that the resulting solutions can be effectively tailored for various applications. The solutions obtained in [20–24] were found to be in most cases modulationally stable, either absolutely or under the regime of diffraction/dispersion management [24,30].

In this work, we generalize the JEF expansion method, developed in [21,22], and, expanding upon the results in [17], find exact solutions to the NLC system of equations (NLCSOE), including the effects of both the chirp and dispersion/diffraction management. We apply the principle of harmonic balance to both the wave function and the angular tilt, and apply matching conditions to obtain the forms of

* Corresponding author at: Institute of Physics, Pregrevica 118, 11080 Belgrade, University of Belgrade, Serbia.
E-mail address: nzpetr@ipb.ac.rs (N. Petrović).

https://doi.org/10.1016/j.csfx.2024.100121
Received 21 March 2024; Received in revised form 18 August 2024; Accepted 13 September 2024
Available online 18 September 2024

these two functions in terms of JEFs. These forms depend on the degree of nonlinearity and other coefficients inside the liquid crystal. In this paper we will consider the solutions for the lowest-order (Kerr) reorientational nonlinearity. Unlike most of other models, the form of the wave function used here also includes a quadratic dependence on the transverse variables in the phase, which is known as the chirp [21]. We obtain solitary and traveling wave solutions to the NLCSOE, both with and without the chirp.

2. The method

The NLCSOE with Kerr-like nonlinearity was first introduced in [31] and further developed in [32]. We will use the general form as given in [8,9]:

$$iu_t + \frac{\beta(t)}{2}\nabla^2 u + \chi(t)pu = 0, \tag{1}$$

$$c(t)\nabla^2 p + l(t)p + \alpha(t)|u|^2 = 0, \tag{2}$$

where u is the optical amplitude wave function and t is the longitudinal evolution variable. In some cases, the z direction is given as the longitudinal variable instead [7] and all our solutions also translate to this case. The subscript t in Eqs. (1)–(2) denotes the partial time derivative and $\nabla^2 = \sum_{j=1}^{s} \frac{\partial^2}{(\partial x_j)^2}$ denotes the transverse Laplacian, where s stands for the number of transverse dimensions and x_j, $j = 1, \ldots, s$ are the labels for the transverse coordinates. Further, p is the angle function describing the tilt of the molecular director of the NLC, and $\chi(t)$, $c(t)$, $l(t)$ and $\alpha(t)$ are all real functions that specify the relative strengths of different terms in Eqs. (1)–(2). These coefficients, as mentioned before, may contain a longitudinal dependence, which allows for more general solutions that include both the dispersion and diffraction management [28,29].

What follows next is a rather technical presentation of the method, which nonetheless is necessary for a more complete understanding of how the method works. Let n and m be the degrees of u and p in terms of F, the selected JEF in the expansion. By balancing the largest degree of F in each equation, so that one excludes the trivial solutions, we obtain $n+2 = n+m$ and $m+2 = 2n$, from which it follows that $m = n = 2$. This result is also derived in other methods, such as the one in [9]. We will thus, following [21], assume the following ansatz for our solutions:

$$u(x,t) = A(x,t)e^{iB(x,t)}, \tag{3}$$

$$A = f_2(t)F(\theta)^2 + f_0(t) + f_{-2}(t)F(\theta)^{-2}, \tag{4}$$

$$\theta = \sum_{j=1}^{s} k_j(t)x_j + \omega(t), \tag{5}$$

$$B = a(t)x^2 + b(t)\left(\sum_{j=1}^{s} x_j\right) + e(t), \tag{6}$$

$$p(x,t) = g_2(t)F(\theta)^2 + g_0(t) + g_{-2}(t)F(\theta)^{-2}, \tag{7}$$

where $x = (x_1, \ldots, x_s)$ is the shorthand for the transverse position vector of s components x_j $(j = 1, \ldots, s)$, $x^2 = \sum_{j=1}^{s} x_j^2$ and, analogously, k_j $(j = 1, \ldots, s)$ are the components of the wave vector k. Furthermore, a, b, e, ω, f_i and g_i $(i = 2, 0, -2)$ are all functions of t, and F is the JEF satisfying:

$$\frac{d^2F}{d\theta^2} = c_0 + c_2F^2 + c_4F^4, \tag{8}$$

where c_0, c_2 and c_4 are coefficients that depend on the JEF parameter M and the chosen JEF. For example, when $F = \mathrm{dn}$ we have $c_0 = M-1$, $c_2 = 2-M$ and $c_4 = -1$, and when $F = \mathrm{sn}$ we have $c_0 = 1$, $c_2 = -(1+M)$ and $c_4 = M$. Note the dependence of θ on x and t, which is reminiscent of a phase variable $k \cdot x + \omega(t)$, but here it also serves as the argument of JEFs, normally an elliptic integral of the first kind. We do not include the odd degrees of F in our formulas for A and p, because that would lead to 8 new equations for only 4 new functions.

Further, following [21], we plug the ansatz into equations, separating Eq. (1) into the real and imaginary parts. We obtain a polynomial in terms of F and $\partial_\theta F$ and, setting each coefficient to 0, we obtain a system of algebraic and ordinary differential equations (ODEs). Thus, the original system of coupled PDEs is transferred into a system of algebraic and ODEs that are more amenable to exact treatment.

For functions f_i, $i = -2, 0, 2$, k_j, $j = 1, \ldots, s$, ω, a and b, one obtains the following equations:

$$f_{it} + s\beta a f_i = 0, \; i = 2, 0, -2, \tag{9}$$

$$k_{jt} + 2\beta a k_j = 0, \; j = 1, \ldots, s \tag{10}$$

$$\omega_t + \beta b k = 0, \tag{11}$$

$$a_t + 2\beta a^2 = 0, \tag{12}$$

$$b_t + 2\beta a b = 0. \tag{13}$$

For function e one obtains multiple equations:

$$-e_t f_0 - \frac{s}{2}b^2\beta f_0 + \chi f_0 g_0 + \chi f_2 g_{-2} + \chi f_{-2} g_2 + \beta\left(c_0 f_2 + c_4 f_{-2}\right)k^2 = 0, \tag{14}$$

$$-e_t f_i - \frac{s}{2}b^2\beta f_i + \chi f_i g_0 + \chi f_0 g_i + 2\beta c_2 f_i k^2 = 0, \; i = 2, -2, \tag{15}$$

where $k^2 = \sum_{j=1}^{s} k_j^2$ is the square of the wave vector. We thus need to employ matching conditions, to ensure Eqs. (14)–(15) are consistent. The remaining equations one obtains are as follows:

$$\chi f_i g_i + 3\beta c_{2+i} f_i k^2 = 0, \; i = 2, -2, \tag{16}$$

$$g_0 l + \alpha f_0^2 + 2\alpha f_2 f_{-2} + 2c(c_0 g_2 + c_4 g_{-2})k^2 = 0, \tag{17}$$

$$\alpha f_i^2 + 6c c_{2+i} g_i k^2 = 0, \; i = 2, -2, \tag{18}$$

$$2\alpha f_0 f_i + 4c c_2 g_i k^2 + g_i l = 0, \; i = 2, -2. \tag{19}$$

These are the integrability conditions for the system.

3. Results

We now proceed to solve Eqs. (9)–(19). Eqs. (9)–(13) are easily solved using the basic techniques, to obtain:

$$f_i = f_{i0}\eta^{\frac{s}{2}}, \; i = 2, 0, -2, \tag{20}$$

$$k_j = k_{j0}\eta, \; j = 1, \ldots, s \tag{21}$$

$$\omega = \omega_0 - b_0\left(\sum_{j=1}^{s} k_{j0}\right)\eta \int_0^t \beta(t)dt, \tag{22}$$

$$a = a_0\eta, \tag{23}$$

$$b = b_0\eta, \tag{24}$$

where:

$$\eta = \frac{1}{1 + 2a_0\int_0^t \beta(t)dt} \tag{25}$$

is the chirp function [21] and the '0' subscript indicates the value of the given function at $t = 0$. Without loss of generality, one can assume $f_{20} \neq 0$. We now consider two different cases.

3.1. Case 1: $f_{-20} = 0$

In the first case one can assume $f_{-20} = 0$ and therefore $f_{-2} = 0$. From Eq. (18) for $i = -2$ one obtains:

$$g_{-2} = 0. \tag{26}$$

Eqs. (15), (16), (18) and (19) for $i = -2$ are trivially satisfied. The matching condition for Eqs. (14)–(15) gives us:

$$\frac{f_{20}}{f_{00}} = r, \text{ where } r = \frac{c_2 + \epsilon_r\sqrt{c_2^2 - 3c_0c_4}}{c_0}, \; \epsilon_r = \pm 1, \text{ for } c_0 \neq 0, \text{ and} \tag{27}$$

$$r = \frac{3c_4}{2c_2}, \text{ for } c_0 = 0. \tag{28}$$

Note that the solution for r with $\epsilon_r = -1$ for $c_0 \neq 0$ converges to the solution for $c_0 = 0$ as c_0 converges to 0. Solving algebraically the remaining equations among Eqs. (16)–(19), one now obtains:

$$g_0 = \frac{3\beta c_4 k^2 (c_0 r^2 - 3c_4)}{2\chi r(3c_4 - c_2 r)}, \tag{29}$$

$$g_2 = \frac{-3\beta c_4 k^2}{\chi}, \tag{30}$$

$$\alpha = \frac{18\beta c c_4^2 k^4}{\chi f_2^2}, \tag{31}$$

$$l = 4ck^2\left(\frac{3c_4}{r} - c_2\right). \tag{32}$$

It is worth noting that unlike in [21–24], here the parameter χ (representing the strength of the nonlinearity) can be completely arbitrary. Parameters α, l and c contain integrability conditions imposed on them in Eqs. (31)–(32), wherein α and l are expressed as functions of c. Reformulations of Eqs. (31)–(32) are possible, in which it is α or l that defines the other two parameters in the set $\{c, \alpha, l\}$. Finally, from Eq. (14) one can obtain:

$$e = e_0 + \frac{\eta}{2}\left(k_0^2\left(2c_0 r + \frac{3c_4(c_0 r^2 - 3c_4)}{r(3c_4 - c_2 r)}\right) - sb_0^2\right)\int_0^t \beta(t)dt, \tag{33}$$

where $k_0^2 = \sum_{j=1}^{s} k_{j0}^2$ is the square of the initial wave vector.

3.2. Case 2: $f_{-20} \neq 0$

Now, we assume $f_{-20} \neq 0$ and therefore $f_{-2} \neq 0$. It follows from Eq. (18) that $c_0 \neq 0$ and $g_{-2} \neq 0$. Comparing the pairs of expressions in two equations in (16) and (18) one obtains

$$\frac{f_{20}}{f_{-20}} = \frac{g_{20}}{g_{-20}} = \frac{c_4}{c_0}. \tag{34}$$

The corresponding condition for Eqs. (18)–(19) gives us:

$$\frac{f_{20}}{f_{00}} = r, \text{ where } r = \frac{-c_2 + \epsilon_r\sqrt{c_2^2 + 12c_0c_4}}{4c_0}, \ \epsilon_r = \pm 1. \tag{35}$$

Note that the equation for r changes due to the presence of extra terms changing the coefficient next to r^2. Solving algebraically the remaining equations among Eqs. (16)–(19), one finds:

$$g_0 = -\frac{3\beta c_4 k^2(4c_0 r^2 + 3c_4)}{2\chi r(3c_4 - c_2 r)}, \tag{36}$$

while the formulas for g_2, α and l are the same as for Case 1, given in Eqs. (30)–(32). Finally, from Eq. (14) one obtains:

$$e = e_0 + \frac{\eta}{2}\left(k_0^2\left(4c_2 - \frac{6c_4}{r} - \frac{3c_4(4c_0 r^2 + 3c_4)}{r(3c_4 - c_2 r)}\right) - sb_0^2\right)\int_0^t \beta(t)dt. \tag{37}$$

4. Solutions

We now present the obtained solutions for some realistic values of the parameters. The two regimes we will study is the regime of dispersion/diffraction management and the regime with constant dispersion.

In the first case, we study the $(1+1)$-dimensional case with diffraction management. For this case we will have $s = 1$ and, simply, $x_1 = x$ and $k_1 = k$. Since in this case the diffraction periodically changes sign, it is usually modeled with the diffraction being a trigonometric function, e.g. $\beta(t) = \sin(\Omega t)$. Unless otherwise indicated, we will take the $f_{-2} = 0$ case.

In Fig. 1, one can see typical solution profiles of a solitary wave for $F = \text{dn}$ and $M = 1$. Due to the combination of the background signal from f_0 and from f_2, one can obtain a dark solitary wave. The fact that the amplitude of f_2 does not fully match f_0 for the parameters given, results in a small elevation in the middle of the wave, giving the wave a 'double dip' shape. Same characteristic shape can be found in [9,10]. In Figs. 1(b) and (c) one can see the effect of the chirp on the form of functions, with the background signal also being altered by the chirp. The larger the value of chirp parameter the bigger the oscillations of the background wave. At $a_0 = 0.5$, singularities occur, given that $\beta_0 = 1$. The form of p that would allow this form of the solution for u to exist also seems to indicate alternating orientations of crystals, with the general amplitude of p also increasing with the chirp.

In Fig. 2, we present the traveling wave solutions obtained for $F = \text{dn}$ and $M = 0.8$. Unlike the solitary wave case for $F = \text{dn}$, the traveling wave case gives two distinct values of r for $F = \text{dn}$, since c_0 is no longer 0. One can notice more easily that the chirp has the effect of not just altering the amplitude of the function, but also stretching the wave front in the transverse direction, depending on the phase of the traveling wave. As for the function p, we see periodicity in both directions in the case without chirp and the corresponding stretching effect in the transverse direction.

In Fig. 3, we analyze the solutions for $F = sn$. When the backgrounds of f_2 and f_0 cancel each other out, one obtains a typical bright solitary wave. We see that the chirp produces breather-type solutions, in addition to deforming the profile of the wave. Given that parameter ω controls the central position of the wave and given its form in Eq. (22), solutions where the waves travel in a straight line can be obtained by setting $b_0 = 0$ and in this case the solitary waves take on a shape of classical breathers. Unlike in the case of the dark solitary wave, the p function is zero away from the wave signal indicating a self-guiding effect.

In Fig. 4(a), we look at the effect of changing the sign for ϵ_r. One can see in Fig. 4(a), for which $\epsilon_r = -1$, that the periodicity of the solutions is affected in comparison with Fig. 2(a), where $\epsilon_r = 1$. Since the solution for $\epsilon_r = -1$ converges to the solution for $c_0 = 0$ as M converges to 1, one can note that the connection between Fig. 4(a) and Fig. 1(a) is the characteristic double-dip solution. The solution becomes a periodic wave with alternating amplitudes.

In Fig. 4(b) and (c), we look at the solutions for case 2 and compare them with Fig. 2(a) and Fig. 4(a). One can see that the inclusion of the inverse function has doubled the periodicity of our solutions. We have purposely selected an example where $c_0 \neq 0$ and the inverse of F does not produce singularities, which would not be the case for $F = \text{sn}$ or $F = \text{cn}$.

Finally, in Fig. 5 we examine the $(2+1)$-dimensional case, i.e. with $s = 2$, where the coefficients β, α and χ are constant. For this case we set the variables to be $t = z$, $x_1 = x$ and $x_2 = y$. In accordance with the system given in [13], we have $\beta = 1$, $\alpha = 2$ and $\chi = 2$. For $s = 2$ one obtains from Eqs. (31)–(32):

$$c = \frac{2f_{20}^2}{9c_4^2 k_0^4 \eta^2}, \tag{38}$$

$$l = \frac{8f_{20}^2}{9c_4^2 k_0^2}\left(\frac{3c_4}{r} - c_2\right), \tag{39}$$

where $k_0^2 = k_{10}^2 + k_{20}^2$. We note that due to f_{20} being proportional to the chirp function in Eq. (20) for $s = 2$, only for this value of s does the chirp cancel out with the chirp from k_j in Eq. (32) when c is plugged in. Therefore, only for $s = 2$ will l be a constant, i.e. not dependent on the longitudinal variable, as given in Eq. (39). In the absence of chirp, c will also be constant.

Following the notation used in earlier papers [21], we will denote in Fig. 5 parameters k_{10} and k_{20} as k_0 and l_0 (not to be confused with the earlier definition of k_0). Since β is constant, one obtains a wave profile traveling with constant speed, as depicted in Fig. 5(a), i.e. along a straight line. The value of $b_0 = -1$ was chosen for the solutions in Fig. 5 to be in the direction of the viewing perspective. The figure in Fig. 5(a) represents a dark solitary wave solutions with the accompanying angle function p in Fig. 5(d). We see that the NLCs act as a waveguide for the solitary wave. One can also observe a periodic pattern in Fig. 5(b) and (e). Finally, we see the effect of chirp in Fig. 5(c) and (f). In the presence of chirp, the solutions and the background decay. As demonstrated

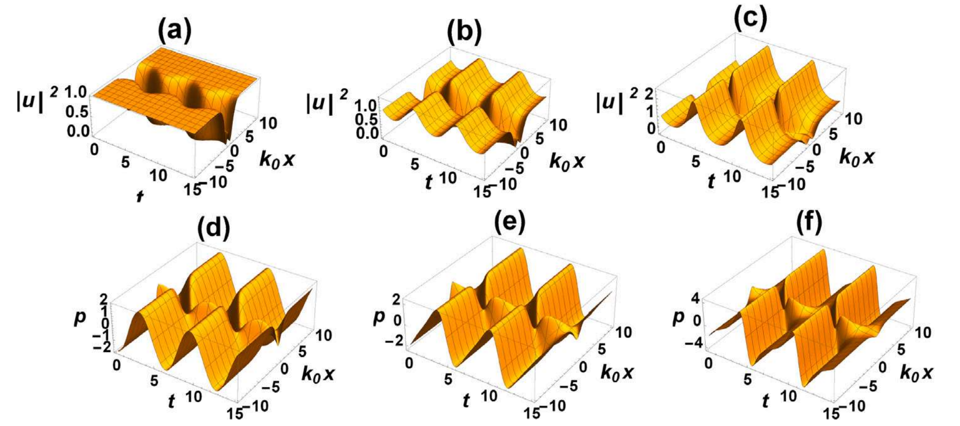


Fig. 1. Solitary wave solutions for $F = \text{dn}$, $M = 1$ and $f_{-20} = 0$ (Case 1) as functions of time and k_0x. Intensity $|u|^2$ for (a) $a_0 = 0$, (b) $a_0 = 0.15$ and (c) $a_0 = 0.3$ and the angle function p for (d) $a_0 = 0$, (e) $a_0 = 0.15$ and (f) $a_0 = 0.3$ are presented as functions of k_0x and t, for the form of $\beta(t)$ given as $\beta(t) = \beta_0 \cos \Omega t$. The values of other coefficients are: $b_0 = 1$, $e_0 = 0$, $k_{10} = 1$, $f_{00} = 1$, $\omega_0 = 0$, $\beta_0 = 1$, $\Omega = 1$, while ϵ_r is undefined because $c_0 = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

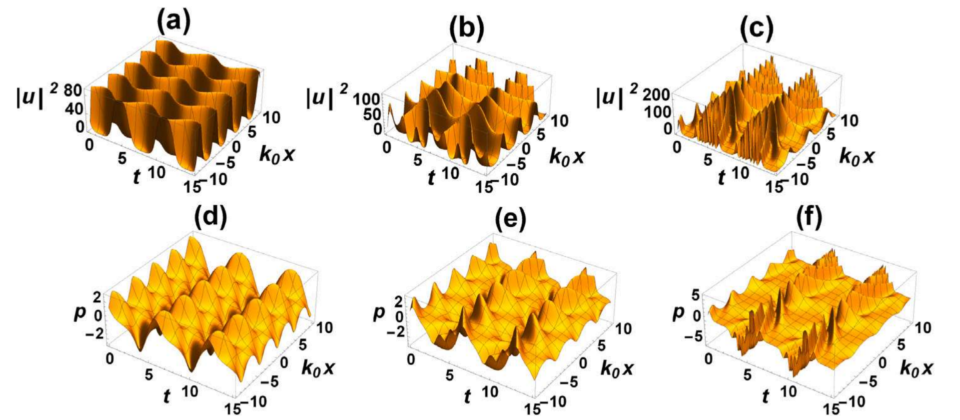


Fig. 2. Traveling wave solutions as functions of k_0x and t. The parameters are the same as in Fig. 1 except $M = 0.8$ and $\epsilon_r = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

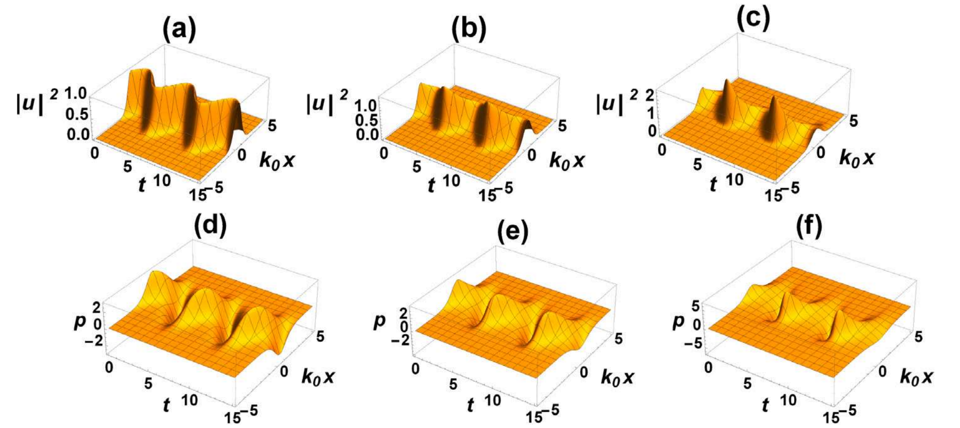


Fig. 3. Bright solitary wave solutions as functions of k_0x and t. The parameters are the same as in Fig. 1 except $F = \text{sn}$ and $\epsilon_r = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in [23], such a signal can artificially be maintained with the addition of an external power source, by including the gain/loss term γu on the RHS of Eq. (1); albeit γ has to be proportional to $\frac{1}{z}$, where (in this case) z is the longitudinal variable.

We now compare the solutions we obtained with those in some of the other papers. In [9], the $\tan(\phi/2)$-expansion method to obtain a variety of solutions to the NLCSOE with various forms of nonlinearity, including Kerr nonlinearity. It also concludes that the solutions to the NLCSOE are second order polynomials of a function satisfying an appropriate differential equation, which is given in Eqs. (5) and (6) of Ref. [9]. The paper analyzed the so-called 'periodic' solutions obtained from the tan function and the dark-soliton solutions obtained

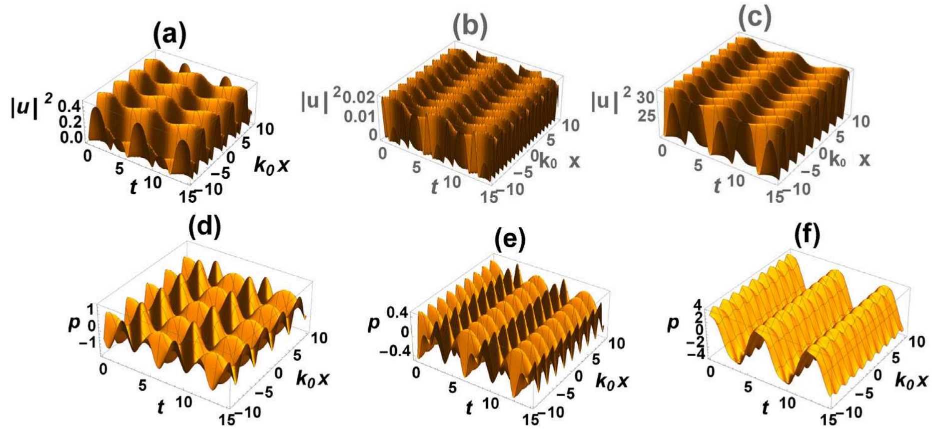


Fig. 4. Traveling wave solutions as functions of k_0x and t. The parameters are the same as in Fig. 2(a) except (a) $\epsilon_r = -1$ (b) $f_{-20} \neq 0$ (Case 2), $\epsilon_r = 1$ and (c) $f_{-20} \neq 0$, $\epsilon_r = -1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

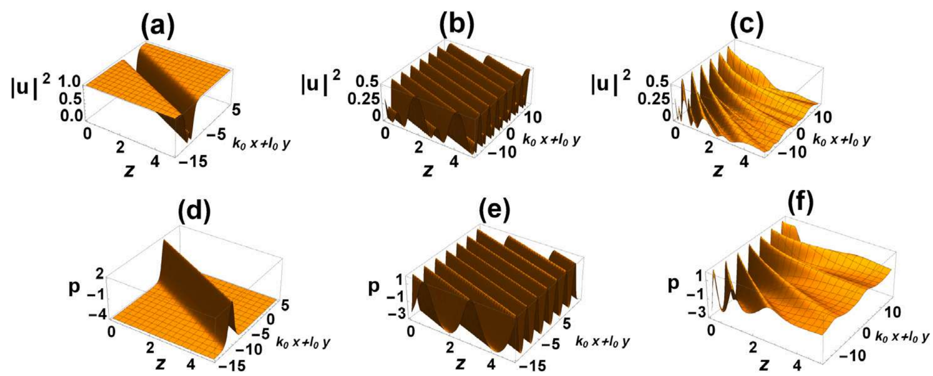


Fig. 5. Two-dimensional solutions for $\beta = 1$, $\alpha = 2$, $\chi = 2$ and $s = 2$ as functions of z (the longitudinal variable) and $k_0x + l_0y$ for (a) $M = 1$, $a_0 = 0$, (b) $M = 0.8$, $a_0 = 0$ and (c) $M = 0.8$, $a_0 = 0.2$. We also selected $b_0 = -1$ and $k_{20} = 1$. All other parameters are the same as in Fig. 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

from the tanh function. The periodic solutions contain singularities due to the nature of the tan function and we do not study such functions in our paper. For the dark soliton case, solutions qualitatively similar to Fig. 1(a) are obtained. Since β (the parameter a in [9]) is constant in the paper, the wave travels in a straight line.

Paper [10] also obtains qualitatively the solutions from Figs. 1(a) and 3(a), but covers neither traveling wave solutions nor the solutions with chirp. Paper [17] also covers these cases, except that 'p' is replaced by 'p^2' in Eq. (1). For this system we have $n = 1$ [17]. Paper [12] obtains the solution based on the step function and some traveling wave solutions.

The advantages of the solutions obtained in this paper are significant. None of the other papers consider a quadratic dependence of the phase on the transverse chirp, a necessary condition for chirp to arise. Given that chirped soliton solutions have applications in the design of pulse compression and amplification [19], we believe that the study of chirped solutions to NLCSOE will have important applications. The approach in our paper, in addition, has the advantage that it is very flexible and in general works for any function F which satisfies Eq. (8). In addition, it allows for the efficient management of dispersion/diffraction by considering longitudinally changing coefficients, which is also generally not included in other papers.

5. Conclusion

In this paper, we have applied the JEF expansion method to obtain different classes of novel exact solutions to the system of equations describing the propagation of light beams in nematic liquid crystals with distributed coefficients. The solutions allow for the free choice of all coefficients except two, providing a wide range of flexibility. Solutions both with and without chirp are obtained. Solutions for two transverse dimensions are obtained when all the coefficients are constant in the case without chirp and only one is non-constant in the case with chirp. Our method also allows for the general management of dispersion and diffraction, which represents a significant new development in the treatment of solitons in NLCs.

CRediT authorship contribution statement

Nikola Petrović: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis. **Milivoj Belić:** Writing – review & editing, Validation, Supervision. **Wieslaw Krolikowski:** Writing – review & editing, Validation, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no conflict of interest.

Data availability

Data availability is not applicable to this article as no new data were created or analyzed in this study.

Acknowledgments

Work at the Institute of Physics is supported by the project OI 171006 of the Serbian Ministry of Education and Science. Work at the Texas A&M University in Qatar was supported by the project NPRP13S-0121-200126 with the Qatar National Research Fund (a member of the Qatar Foundation.)

References

[1] Assanto G, Fratalocchi A, Peccianti M. Spatial solitons in nematic liquid crystals: from bulk to discrete. Opt Express 2007;15(8):5248–59. http://dx.doi.org/10.1364/OE.15.005248.
[2] Cipparrone G, Carbone V, Versace C, Umeton C, Bartolino R, Simoni F. Optically induced chaotic behavior in nematic liquid-crystal films. Phys Rev E 1993;47:3741–4. http://dx.doi.org/10.1103/PhysRevE.47.3741.
[3] Demeter G, Krimer DO. Light-induced dynamics in nematic liquid crystals — a fascinating world of complex nonlinear phenomena. Phys Rep 2007;448:133–62. http://dx.doi.org/10.1016/j.physrep.2007.02.015.
[4] Assanto, G, Peccianti M, Conti C. Nematicons: optical spatial solitons in nematic liquid crystals. Opt Photonics News 2003;14(2):44–8. http://dx.doi.org/10.1364/OPN.14.2.000044.
[5] Assanto G. Nematicons: reorientational solitons from optics to photonics. Liq Cryst Rev 2018;6:170–94. http://dx.doi.org/10.1080/21680396.2019.1588797.
[6] Laudyn UA, Kwasny M, MA. Karpierz. Nematicons in chiral nematic liquid crystals. Appl Phys Lett 2009;94(9):1–3. http://dx.doi.org/10.1063/1.3093498, 091110.
[7] Peccianti M, Assanto G. Nematicons. Phys Rep 2012;516:147–208.
[8] Savescu M, Johnson S, Sanchez P, Zhou Q, Mahmood MF, Zerrad E, et al. Nematicons in liquid crystals. J Comput Theor Nanosci 2015;12:4667–73. http://dx.doi.org/10.1166/jctn.2015.4417.
[9] Ilhan OA, Manafian J, Alizadeh AA, Baskonus HM. New exact solutions for nematicons in liquid crystals by the $\tan(\phi/2)$-expansion method arising in fluid mechanics. Eur Phys J Plus 2020;135:1–19. http://dx.doi.org/10.1140/epjp/s13360-020-00296-w.
[10] Ismael HF, Bulut H, Baskonus HM. W-shaped surfaces to the nematic liquid crystals with three nonlinearity laws. Soft Comput 2021;25:4513–24. http://dx.doi.org/10.1007/s00500-020-05459-6.
[11] Arnous AH, Ullah MZ, Asma M, Moshokoa SP, Mirzazadeh M, Biswas A, et al. Nematicons in liquid crystals by modified simple equation method. Nonlinear Dynam 2017;88:2863–72. http://dx.doi.org/10.1007/s11071-017-3416-2.
[12] Altawallbeh Z, Az-Zo'bi E, Alleddawi AO, Şenol M, Akinyemi L. Novel liquid crystals model and its nematicons. Opt Quantum Electron 2022;54:1–17. http://dx.doi.org/10.1007/s11082-022-04279-2.
[13] MacNeil JML, Smyth NF, Assanto G. Exact and approximate solutions for optical solitary waves in nematic liquid crystals. Phys D 2014;284:1–15. http://dx.doi.org/10.1016/j.physd.2014.06.005.
[14] Reimbert CGarcía, Minzoni AA, Smyth NF, Worthy AL. Large-amplitude nematicon propagation in a liquid crystal with local response. J Opt Soc Am B 2006;23:2551–8. http://dx.doi.org/10.1364/JOSAB.23.002551.
[15] Minzoni AA, Smyth NF, Worthy AL. Modulation solutions for nematicon propagation in nonlocal liquid crystals. J Opt Soc Am B 2007;24:1549–56. http://dx.doi.org/10.1364/JOSAB.24.001549.
[16] Duran S, Karabulut B. Nematicons in liquid crystals with Kerr law by sub-equation method. Alexa Eng J 2022;61:1695–700. http://dx.doi.org/10.1016/j.aej.2021.06.077.
[17] Kavitha L, Venkatesh M, Dhamayanthi S, Parasuraman E, Gopi D. Optically induced switching of nematic deformations. Phys Scr 2013;88:1–9. http://dx.doi.org/10.1088/0031-8949/88/06/065015, 065015.
[18] Liu S, Fu Z, Liu S, Q. Zhao. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. Phys Lett A 2001;289:69–74. http://dx.doi.org/10.1016/S0375-9601(01)00580-1.
[19] Kruglov VI, Peacock AC, Harvey JD. Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. Phys Rev Lett 2003;90:1–4. http://dx.doi.org/10.1103/PhysRevLett.90.113902, 113902.
[20] Zhong WP, Xie RH, Belić M, Petrović N, Chen G, Yi L. Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients. Phys Rev A 2008;78:1–6. http://dx.doi.org/10.1103/PhysRevA.78.023821, 023821.
[21] Belić M, Petrović N, Zhong WP, Xie RH, Chen G. Analytical light bullet solutions to the generalized (3+1)-dimensional nonlinear Schrödinger equation. Phys Rev Lett 2008;101:1–4. http://dx.doi.org/10.1103/PhysRevLett.101.123904, 0123904.
[22] Petrović NZ, Belić M, Zhong WP, Xie RH, Chen G. Exact spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Schrödinger equation for both normal and anomalous dispersion. Opt Lett 2009;34:1609–11.
[23] Petrović NZ, Belić M, Zhong WP. Spatiotemporal wave and soliton solutions to the generalized (3+1)-dimensional Gross–Pitaevskii equation. Phys Rev E 2010;81:1–5. http://dx.doi.org/10.1103/PhysRevE.81.016610, 016610.
[24] Petrović NZ, Aleksić NB, Bastami AA, Belić MR. Analytical traveling-wave and solitary solutions to the generalized Gross–Pitaevskii equation with sinusoidal time-varying diffraction and potential. Phys Rev E 2011;83:1–5. http://dx.doi.org/10.1103/PhysRevE.83.036609, 036609.
[25] Petrović NZ. Spatiotemporal traveling and solitary wave solutions to the generalized nonlinear Schrödinger equation with single-and dual-power law nonlinearity. Nonlinear Dynam 2018;93:2389–97. http://dx.doi.org/10.1007/s11071-018-4331-x.
[26] Petrović NZ. Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method. Opt Quantum Electron 2020;52:1–10. http://dx.doi.org/10.1007/s11082-020-02385-7.
[27] Petrović N. Chirped solitary and traveling wave solutions for the Kundu-Mukherjee-Naskar equation using the Jacobi elliptic function expansion method. Opt Quantum Electron 2022;54:1–8. http://dx.doi.org/10.1007/s11082-022-04024-9.
[28] Eiermann B, Treutlein P, Anker T, Albiez M, Taglieber M, Marzlin KP, et al. Dispersion management for atomic matter waves. Phys Rev Lett 2003;91:1–4. http://dx.doi.org/10.1103/PhysRevLett.91.060402, 060402.
[29] Eisenberg HS, Silberberg Y, Morandotti R, Aitchison JS. Diffraction management. Phys Rev Lett 2000;85:1863–6.
[30] Petrović NZ, Aleksić NB, Belić M. Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross–Pitaevskii equation using a variational approach. Opt Exp 2015;23:1–15. http://dx.doi.org/10.1364/OE.23.010616, 10616.
[31] Conti C, Peccianti M, Assanto G. Route to nonlocality and observation of accessible solitons. Phys Rev Lett 2003;91:073901. http://dx.doi.org/10.1103/PhysRevLett.91.073901.
[32] Assanto G, Skuse BD, Smyth NF. Optical path control of solitary waves in dye-doped nematic liquid crystals. Photonics Lett Pol 2009;1:154–6. http://dx.doi.org/10.4302/plp.2009.4.04.

Optical and Quantum Electronics (2022) 54:644
https://doi.org/10.1007/s11082-022-04024-9

Chirped solitary and traveling wave solutions for the Kundu–Mukherjee–Naskar equation using the Jacobi elliptic function expansion method

Nikola Petrović[1]

Received: 22 November 2021 / Accepted: 19 July 2022

Abstract
In our paper we apply the Jacobi elliptic function expansion method to obtain solutions to the Kundu–Mukherjee–Naskar equation, which is asymmetric in the two transverse directions. We obtain that the solutions contain a quadratic dependency in the phase, i.e. chirp, in one of the two directions. Unlike in previous applications of the method, the chirp does not affect the amplitude of the solutions.

Keywords Chirp · Solitary wave · Traveling wave · Kundu–Mukherjee–Naskar equation · Nonlinear Schrodinger equation

1 Introduction

The Kundu–Mukherjee–Naskar (KMN) differential equation was first introduced in fluid dynamics for the study of the evolution of three-dimensional wave packets in water of finite depth (Kundu et al. 2014). In Kundu et al. (2014), a form of the Nonlinear Schrödinger equation (NLSE) was proposed which would have a spatial asymmetry between the two transverse directions, as well as modulational instability, in order to produce 2-dimensional (2D) rogue waves.

Various solutions have been found for the KMN equation. In Rezazadeh et al. (2021), the functional variable method was used to find basic trigonometric solutions. In Biswas et al. (2020) and Cimpoiasu et al. (2021), solitary wave solutions are introduced and using Lie symmetry methods conserved quantities are identified and new and more complex solutions are generated. In Qiu et al. (2016) rogue wave solutions are obtained and discussed. In Sulaiman and Bulut (2019) ratios of trigonometric expansions are used to obtain complicated solutions. In Ren et al. (2021), a basic traveling wave ansatz is considered and a bifurcation analysis is performed, leading to many solutions based on the inverses of elliptic integrals. In Ekici et al. (2019) a trial function method similar to that in Ren et al. (2021) is used to obtain a range of solutions which all feature trigonometric and elliptic

Nikola Petrović
nzpetr@ipb.ac.rs

[1] Institute of Physics, University of Belgrade, Pregrevica 118, Belgrade 11080, Serbia

functions in the denominator of a fractional expression. A similar idea is used in Bilal et al. (2021) where the Φ_6 method is used to find solutions related to the JEFs, except satisfying a sixth-order differential equation instead of the fourth-order differential equation satisfied by JEFs. In He (2020) the variational method is used to obtain another trigonometric-based solution. In Zayed et al. (2022) a basic ansatz was applied in combination with Kudryashov's techniques to find solutions to a coupled (2+1)-dimensional ((2+1)D) system. Finally, in Mukherjee (2020) solutions have been found which can change their curvature in space and time based on an arbitrarily chosen function.

All of the mentioned papers only contain solutions where the phase contains a linear dependence on the transverse variables, except in Cimpoiasu et al. (2021) and Mukherjee (2020) where a function affecting the phase is directly tied to a function affecting the amplitude, in Cimpoiasu et al. (2021) quadratically and in Mukherjee (2020) linearly affecting the characteristic variable of the elliptic function. Solution where the dependence of the phase with respect to the transverse variables is not linear are called chirped solutions. Chirped solutions have extensive application in the construction of various amplifiers, pulse compressors and soliton-based communications links due to their ability to alter the character of the propagating soliton pulse (Bouzida et al. 2017). In Desaix et al. (2002), it is established that at a certain level of chirp a two-soliton breather will split into two solitons or degrade through dispersive radiation. In Kruglov et al. (2003), the authors found solutions to the NLSE with the appropriate boundary conditions. They found that a pulse can be compressed to any degree while maintaining its shape until higher order nonlinearity terms become important and concluded that a chirped soliton propagation could be an alternative to loss management systems. The study of chirped solutions is therefore highly important for the development of optical systems.

Recently, a lot of work was done using the Jacobi elliptic function expansion method to find both unchirped and chirped solutions for various forms of the Nonlinear Schrödinger Equation (NLSE) (Kruglov et al. 2003; Zhong et al. 2008; Belić et al. 2008; Petrović et al. 2009) and the Gross–Pitaevskii equation (GPE) (Petrović et al. 2010, 2011). The forms obtained in these papers use distributed coefficients which allow us to consider both dispersion (Eiermann et al. 2003) and diffraction management (Eisenberg et al. 2000). A further advantage was that the solutions obtained in Zhong et al. (2008), Belić et al. (2008), Petrović et al. (2009, 2010, 2011) were found to be in most cases modulationally stable, either absolutely or under the regime of diffraction/dispersion management (Petrović et al. 2015, 2011). Finally, all the solutions in Zhong et al. (2008), Belić et al. (2008), Petrović et al. (2009, 2010, 2011) also include a quadratic dependence with regards to the transverse variable which we will henceforth refer to as chirp.

The form of the solutions of JEF expansion method extremely suitable for the KMN equation as the KMN equation ultimately features a nonlinearity of the third order. In this paper we will thus use the JEF expansion method as described in Belić et al. (2008) to obtain new solutions to the KMN equation that contain chirp. As will be demonstrated, the asymmetry in the transverse directions will lead to solutions in which the quadratic dependence occurs in only one of them.

2 Methods

The Kundu–Mukherje–Naskar (KMN) equation has the following general form:

$$iu_t + \frac{\beta(t)}{2}u_{xy} + i\chi(t)\left(uu_x^* - u^*u_x\right) = 0, \tag{1}$$

where u is the wave function, t is time, x and y are transverse variables, indices are partial derivatives, $\beta(t)$ is the diffraction coefficient and $\chi(t)$ is the strength of nonlinearity. As in Belić et al. (2008), we propose the following solution for the KMN:

$$u = Ae^{iB}, \tag{2}$$

where A and B are real functions of x, y and t denoting the amplitude and the phase of the solution. Following Belić et al. (2008) and Petrović et al. (2009) and accounting for the broken symmetry between x and y directions, we assume the following forms for A and B:

$$u = Ae^{iB}, \tag{3}$$

$$A = f_1(t)F(\theta) + f_0(t) + f_{-1}(t)F(\theta)^{-1}, \tag{4}$$

$$\theta = k(t)x + l(t)y + \omega(t), \tag{5}$$

$$B = a_1(t)x^2 + a_2(t)y^2 + a_3(t)xy + b_1(t)x + b_2(t)y + e(t). \tag{6}$$

where F is a Jacobi elliptic function satisfying the differential equation:

$$\left(\frac{dF}{d\theta}\right)^2 = c_0 + c_2F^2 + c_4F^4. \tag{7}$$

Here, c_0, c_2 and c_4 are coefficients which depend on the choice of the JEF and M, the parameter of the JEF.

Plugging in Eqs. 3–6 into Eq. 1 and matching each coefficient next to terms involving F or $\frac{dF}{d\theta}$, we obtain the following expressions and ordinary differential equations for parameters k, l, f_i $(i = 1, 0, -1)$, a, b and ω:

$$f_{it} + \frac{\beta}{2}a_3f_i = 0,\ \ i = 1, 0, -1, \tag{8}$$

$$k_t + \frac{\beta}{2}(a_3k + 2a_1l) = 0, \tag{9}$$

$$l_t + \frac{\beta}{2}(a_3l + 2a_2k) = 0, \tag{10}$$

$$\omega_t + \frac{\beta}{2}(b_2k + b_1l) = 0, \tag{11}$$

$$a_1 = a_3 = f_0 = 0, \tag{12}$$

$$a_{2t} = b_{1t} = 0, \tag{13}$$

$$b_{2t} + a_2 b_1 \beta = 0, \tag{14}$$

$$e_t + \frac{\beta}{2} b_1 b_2 + 6 b_1 \chi f_1 f_{-1} + \frac{\beta}{2} c_2 k l = 0. \tag{15}$$

We also obtain the following set of integrability conditions for the nonlinearity parameter χ:

$$f_1 (2 b_1 \chi f_1^2 + \beta c_4 k l) = 0, \tag{16}$$

$$f_{-1} (2 b_1 \chi f_{-1}^2 + \beta c_0 k l) = 0. \tag{17}$$

These equations can be readily solved using elementary mathematical techniques.

3 Results

We now proceed to solve Eqs. 8–17. One obtains the following formulas for the defined parameters:

$$a_1 = a_3 = f_0 = 0, \tag{18}$$

$$f_{-1} = \epsilon f_1 \sqrt{\frac{c_0}{c_4}} = const., \epsilon = 0, \pm 1, \tag{19}$$

$$k, b_1, a_2 = const., \tag{20}$$

$$l = l_0 - a_2 k \int_0^t \beta(t') dt', \tag{21}$$

$$b_2 = b_{20} - a_2 b_1 \int_0^t \beta(t') dt', \tag{22}$$

$$\omega = \omega_0 - \omega_1 \int_0^t \beta(t') dt' + \omega_2 \int_0^t \left(\beta(t') \int_0^{t'} \beta(t'') dt'' \right) dt', \tag{23}$$

$$e = e_0 - e_1 \int_0^t \beta(t') dt' + e_2 \int_0^t \left(\beta(t') \int_0^{t'} \beta(t'') dt'' \right) dt', \tag{24}$$

$$\chi = -\frac{\beta c_4 k (l_0 - a_2 k \int_0^t \beta(t') dt')}{2 b_1 f_1^2}, \tag{25}$$

where the coefficients ω_1, ω_2, e_1 and e_2 are defined as:

$$\omega_1 = \frac{b_{20}k + b_1 l_0}{2}, \tag{26}$$

$$\omega_2 = a_2 b_1 k, \tag{27}$$

$$e_1 = \frac{b_{20}b_1}{2} + kl_0\left(3\epsilon\sqrt{c_0c_4} - \frac{c_2}{2}\right), \tag{28}$$

$$e_2 = \frac{a_2 b_1^2}{2} + k^2 a_2\left(3\epsilon\sqrt{c_0c_4} - \frac{c_2}{2}\right). \tag{29}$$

We see that as opposed to Belić et al. (2008) where the solutions depended on a chirp function, here the solutions depend on the first and second integral of β. This is fundamentally due to a change in the nonlinearity term which forced the chirp function to be constant.

4 Solutions

We now analyze the obtained solutions. One notices that the method obtains a more restrictive set of solutions than in Belić et al. (2008). In particular, certain parameters are constrained to be either 0 or a constant, and the chirp no longer affects the amplitude of the solution making it impossible to create soliton breathers (Belić et al. 2008; Petrović et al. 2009). The fact that $a_1 = 0$ means that chirp can only occur in the y-transverse direction. Still, the solutions allow a range of forms, both solitary and traveling wave.

In Fig. 1a we see the standard solitary wave solution. This solution occurs when there is no chirp. When $a_2 = 0$ we see from the formulas that the symmetry between the two transverse variables is retained, which is why the plot is identical for the two transverse variables. When $a_2 = 1$, we see that the symmetry between the two transverse variables is broken. In the x direction (Fig. 1b) we have a combination of two sinusoidal functions, whereas in the y direction (Fig. 1c) we have the characteristic 'crested' forms of the solution. If one looks carefully, one sees the gradual narrowing and waning of the crests indicating that they are not infinite.

In Fig. 2 we compare these with the traveling wave solutions. We see the solitary wave solutions repeated periodically in the x direction whereas in the y direction the effect of stretching is noticed, similar to the chirped solutions in Belić et al. (2008).

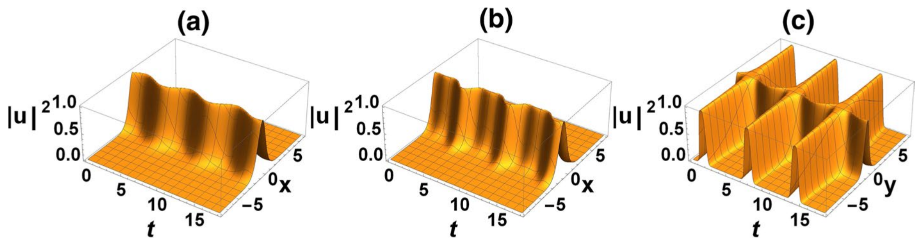


Fig. 1 (Color online) Solitary wave solutions for $F =$ dn and $M = 1$ as functions of time. Intensity $|u|^2$ for **a** $a_2 = 0$ and **b**, **c** and $a_2 = 1$ are presented as a function of either x for **a**, **b** or y **c** and t for $\beta(z) = \beta_0 \cos \Omega t$. Coefficients: $b_{20} = 0$, $e_0 = 0$, $k = 1$, $l_0 = 1$, $b_1 = 1$, $\omega_0 = 0$, $\beta_0 = 1$, $\Omega = 1$, $f_1 = 1$, $\epsilon = 0$

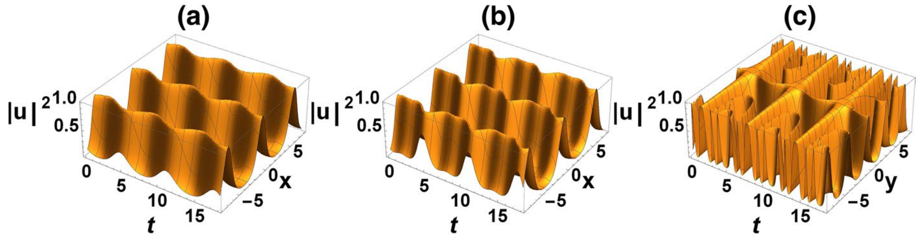


Fig. 2 (Color online) Traveling wave solutions as functions of time. The parameters are the same as in Fig. 1 except $M = 0.97$

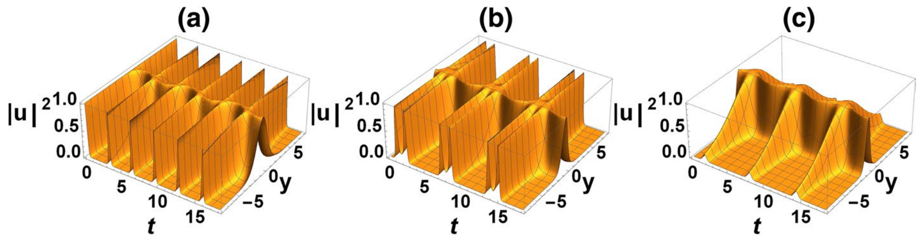


Fig. 3 (Color online) Traveling wave solutions as functions of time. The parameters are the same as in Fig. 1c except **a** $l_0 = 0$, **b** $l_0 = 0.8$ and **c** $l_0 = 1.2$

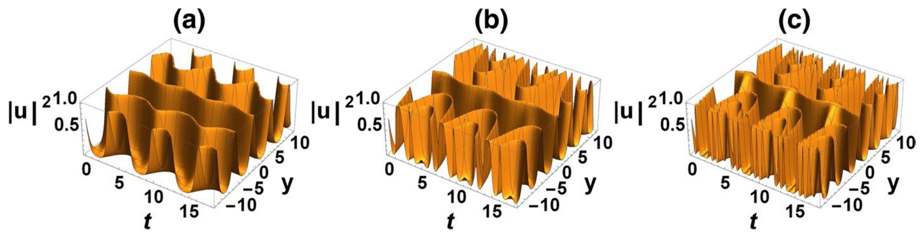


Fig. 4 (Color online) Traveling wave solutions as functions of time. The parameters are the same as in Fig. 2c except **a** $a_2 = 0.2$, **b** $a_2 = 0.5$ and **c** $a_2 = 0.8$

We now restrict our attention to solitary waves. We want to see how the solutions vary in terms of given parameters. In Fig. 3 we see what happens when we vary l_0 and thus control the behavior of the function in the y transverse direction. We see that at $l_0 = 0$ we have a uniform sinusoidal wave and distribution of crests and that as l_0 approaches 1 the two pairs of adjacent crests approach each other (Fig. 3c) and merge for $l_0 = 1$ in Fig. 1c. As for higher values of l_0 we see the crests decay rapidly (Fig. 3c).

Finally, in Fig. 4 we see the gradual development of chirp in traveling wave solutions and the slow transformation from Fig. 2a–c. We notice the gradual appearance of the crest forms indicating that the chirp is the strongest along these values of t.

We will now compare the obtained solutions with solutions from previous papers. In Rezazadeh et al. (2021) and Zayed et al. (2022) one obtains in both papers chirpless solutions using various hyperbolic and exponential functions, in the case of Zayed et al. (2022) for a coupled system. Neither chirp nor JEFs are considered in either paper. In Bilal et al.

(2021) an interesting form of the solutions is obtain by considering a sixth-order differential equation rather than the fourth order one. This allows us to find solutions which are of the form $\frac{f}{\sqrt{af^2+b}}$, where a and b are constants and f is a JEF. Again, chirp is not studied in this paper. In Cimpoiasu et al. (2021), one obtains a solution where the chirp function is related to the square of the amplitude, whereas in our solutions the two are unrelated to each other, i.e. one can set an arbitrary value of chirp given the amplitude. In Mukherjee (2020) a function affecting both the amplitude and the phase is introduced which can curve the soliton solutions in an arbitrary way. This allows for a large class of various curved solutions to be found using Jacobi Elliptic functions. However, as in Cimpoiasu et al. (2021), the amplitude and the phase are not independent of each other. Thus, the solutions in this paper allow for a higher degree of control of chirp.

5 Conclusion

To sum up, we analyzed the Kundu–Mukherjee–Naskar equation and obtained large new classes of solitary and traveling wave solutions, both with and without chirp, using the Jacobi elliptic function expansion method. Since the KMN equation is important in the area of researching rogue waves, there is a good possibility of practical application for these solutions. In particular, the chirped solutions will have many potential applications in soliton-based communications.

Acknowledgements Work at the Institute of Physics is supported by project OI 171006 of the Serbian Ministry of Education and Science.

Funding Work at the Institute of Physics was supported by project OI 171006 of the Serbian Ministry of Education and Science.

Declarations

Conflict of interest The author has no relevant financial or non-financial interests to disclose.

References

Belić, M., et al.: Analytical light bullet solutions to the generalized (3+ 1)-dimensional nonlinear Schrödinger equation. Phys. Rev. Lett. **101**, 0123904 (2008)
Bilal, M., et al.: Investigation of optical solitons and modulation instability analysis to the Kundu–Mukherjee–Naskar model. Opt. Quant. Electron. **53**(5), 1–22 (2021)
Biswas, A., et al.: Optical dromions, domain walls and conservation laws with Kundu–Mukherjee–Naskar equation via traveling waves and Lie symmetry. Res. Phys. **16**, 102850 (2020)
Bouzida, A., et al.: Chirped optical solitons in nano optical fibers with dual-power law nonlinearity. Optik **142**, 77–81 (2017)
Cimpoiasu, E., et al.: Symmetry reductions and invariant-group solutions for a two-dimensional Kundu–Mukherjee–Naskar model. Res. Phys. **28**, 104583 (2021)
Desaix, M., et al.: Propagation properties of chirped soliton pulses in optical nonlinear Kerr media. Phys. Rev. E **65**, 056602 (2002)
Eiermann, B., et al.: Dispersion management for atomic matter waves. Phys. Rev. Lett. **91**(060402), 1–4 (2003)
Eisenberg, S., et al.: Diffraction management. Phys. Rev. Lett. **85**, 1863–1866 (2000)
Ekici, M., et al.: Optical solitons in (2+1)-Dimensions with Kundu–Mukherjee–Naskar equation by extended trial function scheme. Chin. J. Phys. **57**, 72–77 (2019)

He, J.: Variational principle and periodic solution of the Kundu–Mukherjee–Naskar equation. Res. Phys. **17**, 103031 (2020)
Kruglov, V., et al.: Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. Lett. **90**, 113902 (2003)
Kundu, A., et al.: Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents. Proc. R. Soc. A **470**, 20130576 (2014)
Mukherjee, A.: Novel curved lump and topological solitons of integrable (21) dimensional KMN equation. Optik **219**, 165194 (2020)
Petrović, N., et al.: Exact spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Schrödinger equation for both normal and anomalous dispersion. Opt. Lett. **34**, 1609–1611 (2009)
Petrović, N., et al.: Spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Gross–Pitaevskii equation. Phys. Rev. E **81**(016610), 1–5 (2010)
Petrović, N., et al.: Analytical traveling-wave and solitary solutions to the generalized Gross–Pitaevskii equation with sinusoidal time-varying diffraction and potential. Phys. Rev. E **83**(036609), 1–5 (2011)
Petrović, N., et al.: Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrödinger equation and the Gross–Pitaevskii equation using a variational approach. Opt. Exp. **23**, 10616–10630 (2015)
Qiu, D., et al.: The rogue wave solutions of a new (2+1)-dimensional equation. Commun. Nonlinear Sci. Numer. Simul. **30**, 307–315 (2016)
Ren, M., et al.: Traveling wave solutions for the generalized (2+1)-dimensional Kundu–Mukherjee–Naskar equation. arXiv:2105.08538 (2021)
Rezazadeh, H., et al.: Wave behaviors of Kundu–Mukherjee–Naskar model arising in optical fiber communication systems with complex structure. Opt. Quant. Electron. **53**(6), 317 (2021)
Sulaiman, T.A., Bulut, H.: The new extended rational SGEEM for construction of optical solitons to the (2+1)-dimensional Kundu–Mukherjee–Naskar model. Appl. Math. Nonlinear Sci. **4**, 513–522 (2019)
Zayed, E.M., et al.: Optical solitons in Bragg gratings fibers for the nonlinear (2+1)-dimensional Kundu–Mukherjee–Naskar equation using two integration schemes. Opt. Quant. Electron. **54**(1), 16 (2022)
Zhong, W., et al.: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients. Phys. Rev. A **78**(023821), 1–6 (2008)

Optical and Quantum Electronics (2020) 52:319
https://doi.org/10.1007/s11082-020-02385-7

Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method

Nikola Zoran Petrović[1]

Received: 20 November 2019 / Accepted: 30 April 2020 / Published online: 9 June 2020

Abstract
In our paper we modify the Jacobi elliptic function expansion method to obtain solutions to the Davey–Stewartson system of equations. Two categories of nonsingular solutions are obtained for both traveling and solitary waves and both with and without chirp. In both cases there is an arbitrary term in the mean flow field, meaning one can obtain solutions for arbitrary forms of the mean flow field.

Keywords Davey–Stewartson equation · Jacobi elliptic function · Expansion method

1 Introduction

The Davey–Stewartson (DS) system of nonlinear partial differential equations, henceforth abbreviated as the DS system, was first introduced in fluid dynamics for the study of the evolution of three-dimensional wave packets in water of finite depth (Davey and Stewartson 1974). It has since found application in numerous areas of physics, most notably nonlinear-optics (Newell and Moloney 1992) as well as related fields such as the study of Bose–Einstein condensates (Huang 2005) and the study of electro-magnetic (EM) waves in ferromagnets (Leblond 1999). A surprising property of the DS system is that it is one of the few multidimensional systems whose inverse scattering transform is known (Sung 1994a, b, c, 1995). Of considerable interest is also the fact that rogue waves have been shown to exist in DS systems (Ohta and Yang 2012, 2013).

Various techniques have been put forth to obtain solutions to the DS system. The earliest attempt was given in Anker and Freeman (1978) where the Zakharov–Shabat scheme (1974) was used to obtain one- and two-soliton solutions, as well as model some basic properties of interaction of multiple solitons. In Hieraninta and Hirota (1990) the Hirota

This article is part of the Topical Collection on Advanced Photonics Meets Machine Learning.

Guest edited by Goran Gligoric, Jelena Radovanovic and Aleksandra Maluckov.

Nikola Zoran Petrović
nzpetr@ipb.ac.rs

[1] Institute of Physics, University of Belgrade, Pregrevica 118, Belgrade 11080, Serbia

method (Hieraninta 1997) was used to construct a multi-dromion solution. Various other methods have been used to find new solutions to the DS system: the variable separation method (Lou and Lu 1996; Lou 2002; Wang and Huang 2010), the G'/G method (Ebadi and Biswas 2011), the first integral method (Jafari 2012) as well as many others (Deng and Qin 2006; Wazwaz 2008; Tian and Gao 1997; Yildirm 2012). Of particular interest for this paper is the work done by Yan (2003) in which Jacobi elliptic functions (JEFs) were used to construct solutions to a system of equations resembling the DS system. In the paper, a basic expansion of the solution in terms of the twelve JEFs was used and solutions were obtained in the form of the first order polynomial (of the JEFs) for the basic wave, while the two auxiliary waves were represented with a second order polynomial.

Recently, work was done to find solutions using the JEF expansion method for various forms of the Nonlinear Schrödinger Equation (NLSE) (Zhong 2008; Belić 2008; Petrović 2009) and the Gross–Pitaevskii equation (GPE) (2010, 2011). These forms use distributed coefficients which allow the use of dispersion (Eiermann 2003) and diffraction management (Eisenberg 2000). The solutions obtained in Zhong (2008), Belić (2008) and Petrović (2009, 2010, 2011) were found to have either absolute modulational stability or modulational stability under diffraction/dispersion management (Petrović 2015, 2011).

The form of the solutions of JEF expansion method is well suited when all the nonlinearity in the problem is solely dependant on amplitude. In the DS system we have two fields: the wave-amplitude field which is complex and the mean-field which is real. As will be shown, it emerges from the DS system that for the matching conditions to work it is natural to consider the mean-flow field to be second order with respect to the wave-amplitude field. Therefore the DS system is highly suitable for the JEF expansion method. In this paper we will apply the JEF expansion method and the ideas developed in Belić (2008) to solving the DS system.

2 Method

The Davey–Stewartson (DS) system of equations has the following general form:

$$iu_t + \frac{\beta(t)}{2}(ru_{xx} + su_{yy}) + \chi(t)|u|^2u + \alpha(t)un = 0, \tag{1}$$

$$n_{xx} + qn_{yy} + \delta\left(|u|^2\right)_{xx} = 0, \tag{2}$$

where u is the wave-amplitude field (WAF), n is the mean-flow field (MFF), t is time, x and y are transverse variables, indices are partial derivatives, $\beta(t)$ is the diffraction coefficient, $\chi(t)$ is the strength of nonlinearity, $\alpha(t)$ is the coupling function and r, s, q and δ are nonzero real parameters. As in Belić (2008), we propose the following solution for the WAF:

$$u = Ae^{iB}, \tag{3}$$

where A and B are real functions of x, y and t denoting the amplitude and the phase of the solution. Following Belić (2008) and Petrović (2009) we assume the following forms for A and B:

$$A = f_1(t)F(\theta) + f_0(t) + f_{-1}(t)F(\theta)^{-1}, \tag{4}$$

$$\theta = k(t)x + l(t)y + \omega(t), \tag{5}$$

$$B = a(t)(x^2/r + y^2/s) + b(t)(x + y) + e(t), \tag{6}$$

where F is a JEF satisfying the differential equation:

$$\left(\frac{dF}{d\theta}\right)^2 = c_0 + c_2F^2 + c_4F^4. \tag{7}$$

Here, c_0, c_2 and c_4 are coefficients which depend on the choice of the JEF and M, the parameter of the JEF. We will assume that at most one of c_0, c_2 and c_4 is 0. For the MFF we take the following form to ensure matching conditions for the top-order terms with respect to F:

$$n = g_2(t)F(\theta)^2 + g_1(t)F(\theta) + g_0(t) + g_{-1}(t)F(\theta)^{-1} + g_{-2}(t)F(\theta)^{-2}. \tag{8}$$

We cannot have all of g_2, g_1, g_{-1}, g_{-2} be zero as n would have no dependence on the transverse spatial coordinates and Eq. (2) would be trivially satisfied.

Plugging in Eqs. (4)–(8) into Eqs. (1)–(2) we obtain the following equations for parameters k, l, f_i $(i = 1, 0, -1)$, a, b and ω:

$$f_{it} + 2a\beta f_i = 0,\ i = 1, 0, -1, \tag{9}$$

$$k_t + 2a\beta k = 0, \tag{10}$$

$$l_t + 2a\beta l = 0, \tag{11}$$

$$\omega_t + \beta b(rk + ls) = 0, \tag{12}$$

$$a_t + 2a^2\beta = 0, \tag{13}$$

$$b_t + 2a\beta b = 0. \tag{14}$$

We also obtain the following set of integrability conditions:

$$k^2(2\delta f_0 f_i + g_i) + g_i l^2 q = 0,\ i = \pm 1, \tag{15}$$

$$k^2(\delta f_i^2 + g_{2i}) + g_{2i} l^2 q = 0,\ i = \pm 1, \tag{16}$$

$$3\chi f_i^2 f_0 + \alpha f_i g_i + \alpha f_0 g_{2i} = 0,\ i = \pm 1, \tag{17}$$

$$\chi f_i^3 + \alpha f_i g_{2i} + \beta c_{2+2i}(rk^2 + sl^2) = 0,\ i = \pm 1. \tag{18}$$

and the following equations for parameter e:

$$f_0\left(-e_t - \frac{b^2\beta}{2}(1+s) + \chi f_0^2 + 6\chi f_1 f_{-1}\right) + \alpha(f_0 g_0 + f_1 g_{-1} + f_{-1} g_1) = 0, \tag{19}$$

$$f_i\left(-e_t - \frac{b^2\beta}{2}(1+s) + 3\chi f_0^2 + 3\chi f_1 f_{-1} + \frac{\beta c_2}{2}(k^2 r + l^2 s)\right) + \alpha(f_i g_0 + f_0 g_i + f_{-i} g_{2i}) = 0,\ i = \pm 1. \tag{20}$$

We note that while the general set-up is similar to that of Belić (2008), there are several key differences. First, due to the presence of the MFF, we obtain four pairs of integrability conditions instead of one, albeit with several new parameters to work with. Note that the function g_0 only appears in the equations for e. Second, the presence of the MFF in Eq. (1) affects Eqs. (19)–(20). In particular, one can no longer trivially discard Eq. (19) by assuming $f_0 = 0$. We shall see that the obtained constraints on the parameters are quite different from those in Belić (2008).

3 Results

We now proceed to solve Eqs. (9)–(20). Solving Eqs. (9)–(14) we obtain:

$$f_i = f_{i0} p,\ i = 1, 0, -1, \tag{21}$$

$$k = k_0 p, \tag{22}$$

$$l = l_0 p, \tag{23}$$

$$a = a_0 p, \tag{24}$$

$$b = b_0 p, \tag{25}$$

$$\omega = \omega_0 - b(k_0 + l_0 s) p \int_0^t \beta(t) dt, \tag{26}$$

Where p is the chirp function given by:

$$p = \frac{1}{1 + 2a_0 \int_0^t \beta(t) dt}. \tag{27}$$

We now distinguish between two cases: $f_0 \neq 0$ and $f_0 = 0$.

3.1 Case $f_0 \neq 0$

We first cover the most general case, i.e. the case when f_0 is non-zero. First we assume that f_1 and f_{-1} are also non-zero. We also assume $k_0^2 + q l_0^2 \neq 0$, as from assuming otherwise it quickly follows that $f_1, f_{-1} = 0$. Solving Eqs. (15)–(16), we obtain the following equations:

$$g_i = 2\epsilon f_0 f_i,\ i = \pm 1, \tag{28}$$

$$g_{2i} = \epsilon f_i^2,\ i = \pm 1, \tag{29}$$

where the parameter ϵ is given by the formula:

$$\epsilon = -\frac{\delta k_0^2}{k_0^2 + q l_0^2}. \tag{30}$$

Equations (28)–(29) coincide with Eq. (14) in Ebadi and Biswas (2011) for $n = 2$ in the special case of $f_0 = f_{-1} = 0$. Plugging the results in Eqs. (17) we obtain a matching condition:

$$\chi = -\epsilon\alpha. \tag{31}$$

Finally, plugging in this condition into Eqs. (18), one obtains the constraint:

$$r k_0^2 + s l_0^2 = 0. \tag{32}$$

This constraint doesn't occur in the previous systems studied in Belić (2008) and Petrović (2010). Given these conditions one obtains that Eqs. (19)–(20) are automatically matched with each other, i.e. equivalent. A surprising result emerges in that there are no constraints on function $g_0(t)$. In other words, for every form of $g_0(t)$ one can find a form for the free term of the phase $e(t)$ for which give us a solution to the DS system. Thus, we truly obtain a wide range of solutions and the ability to study many different forms of the DS system of equations. It is also worth noting that unlike in Belić (2008) the nonlinearity χ as an integrability condition no longer has to follow the form of f and that there is no longer any imposed relationship between f_{10} and f_{-10}. Additionally, since χ is free to be of arbitrary form, there is no longer a simple formula for e, but e is highly dependent on the choice of χ and g_0.

Assuming $f_{-1} = 0$ and $f_1 \neq 0$ one obtains:

$$g_1 = 2\epsilon f_0 f_1,\ i = \pm 1, \tag{33}$$

$$g_2 = \epsilon f_1^2,\ i = \pm 1, \tag{34}$$

$$g_{-1} = g_{-2} = 0, \tag{35}$$

$$c_4(r k_0^2 + s l_0^2) = 0. \tag{36}$$

Similarly, assuming $f_1 = 0$ and $f_{-1} \neq 0$ one obtains:

$$g_{-1} = 2\epsilon f_0 f_{-1},\ i = \pm 1, \tag{37}$$

$$g_{-2} = \epsilon f_{-1}^2,\ i = \pm 1, \tag{38}$$

$$g_1 = g_2 = 0, \tag{39}$$

$$c_0(r k_0^2 + s l_0^2) = 0. \tag{40}$$

In both cases, Eq. (31) holds and $g_0(t)$ is arbitrary.

3.2 Case $f_0 = 0$

We now assume $f_0 = 0$ and, without loss of generality, $f_1 \neq 0$. As in the previous section $k_0^2 + ql_0^2 \neq 0$. From Eqs. (15)–(16), we obtain:

$$g_i = 0, \; i = \pm 1, \tag{41}$$

$$g_{2i} = \epsilon f_i^2, \; i = \pm 1. \tag{42}$$

It follows that Eqs. (17) are automatically satisfied. In order for Eqs. (18) to be satisfied we must either have Eqs. (31)–(32) or:

$$f_{-1} = \eta \sqrt{\frac{c_0}{c_4}} f_1, \; \eta = 0, \pm 1, \tag{43}$$

$$\chi = -\alpha\epsilon - \frac{\beta c_4(k^2 + l^2 s)}{f_1^2}. \tag{44}$$

We note that for the special case of $\alpha = 0$, coinciding with the system in Belić (2008), we obtain the matching condition from Belić (2008). Finally, given these conditions, Eq. (19) is trivially satisfied, while Eq. (20) are automatically matched with each other. In this case, we no longer have the constraint given in Eq. (32).

4 Solutions

We now analyze the obtained solutions. We note that the condition (32) largely restricts us to r and s being the opposite sign. By default we take $F =$ dn which is the most convenient function as both it and its inverse are free from singularities, though one can obtain similar solutions in many cases with other choices for F. We note that for all cases where $g_0 = 0$ we have that n is qualitatively similar to $|u|^2$ and therefore only $|u|^2$ will be shown.

We take $M = 0.97$, describing so-called traveling wave solutions. In Fig. 1a we see the most basic form of the solution for $|u|^2$. Since k and l are of equal sign they cancel out in 12 leading to no time dependence in θ in the absence of chirp. In Fig. 1b we see the results when k_0 and l_0 are of opposite sign. For $M = 1$, a solitary wave solutions is obtained as shown in Fig. 1c. In Fig. 1d–f we see the effects of chirp on our solutions. We note the loss of periodicity in the traveling wave solutions and the stretching effect present in Fig. 1d away from the center, whereas in Fig. 1e this pattern is shifted away from the center. We also note the oscillation in amplitude in all three cases, especially in Fig. 1f, where the solution corresponds to a breather solitary wave.

In Fig. 2 we see the effects of combining several terms in the solution. We see the inverse function dominate in Fig. 2a with respect to Fig. 1a. The presence of $f_{00} = 1$ shifts the function upward in the regime without chirp.

Finally in Fig. 3 we only cover cases not applicable under Case 1, i.e. we see the solutions for $r = s = 1$ which was inadmissable under Case 1. In Fig. 3a we take $\eta = 0$, in Fig. 3b $\eta = 1$, while in Fig. 3c we look at dark soliton solutions by taking $F =$ sn.

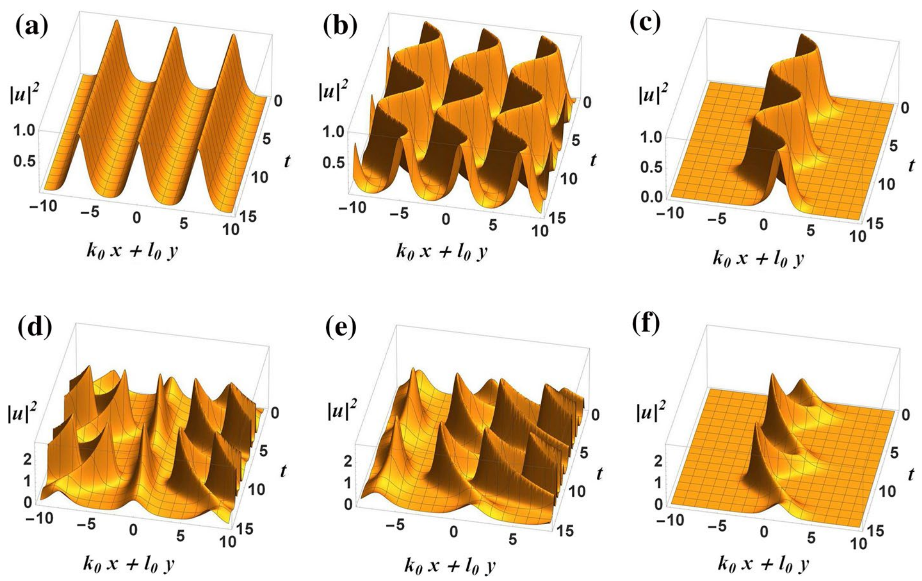


Fig. 1 (Color online) Solitary and traveling wave solutions for $F=$ dn as functions of time. Intensity $|u|^2$ for $a_0=0$ (**a–c**) and $a_0=0.2$ (**d–f**) are presented as a function of k_0x+l_0y and t for $p=-3$, $\beta(z)=\beta_0\cos\Omega t$ and **a**, **d** $M=0.97$, $l_0=1$ **b**, **e** $M=0.97$, $l_0=-1$ and **c**, **f** $M=1$, $l_0=-1$. Coefficients: $b_0=0$, $e_0=0$, $k_0=1$, $\omega_0=0$, $\beta_0=1$, $f_{10}=1$, $f_{00}=f_{-10}=0$, $r=1$, $s=-1$, $q=1$ and $\delta=-1$

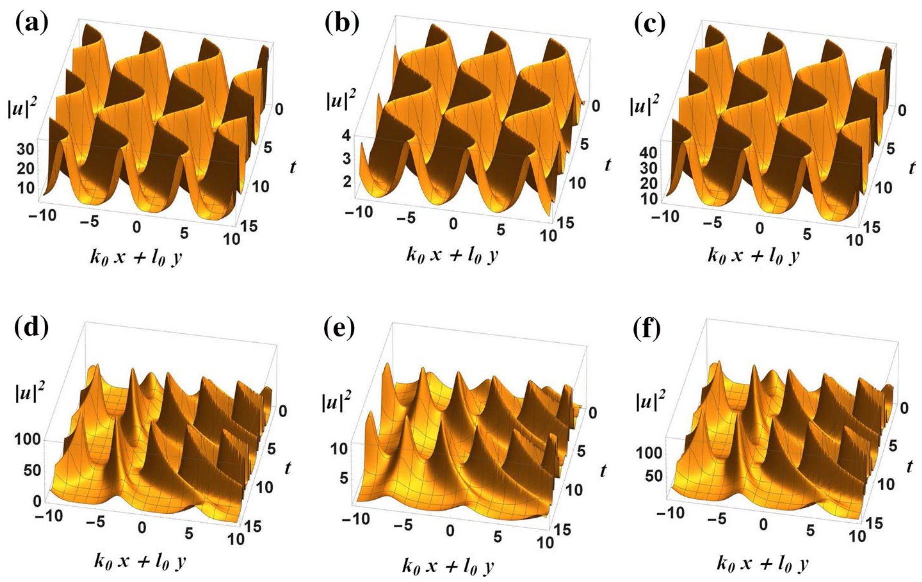


Fig. 2 (Color online) Traveling wave solutions as functions of time. The parameters are the same as in Fig. 1b except $a_0=0.2$ in (**d–f**) and **a**, **d** $f_{10}=f_{-10}=1$, $f_{00}=0$ **b**, **e** $f_{10}=f_{00}=1$, $f_{-10}=0$ and **c**, **f** $f_{10}=f_{-10}=f_{00}=1$

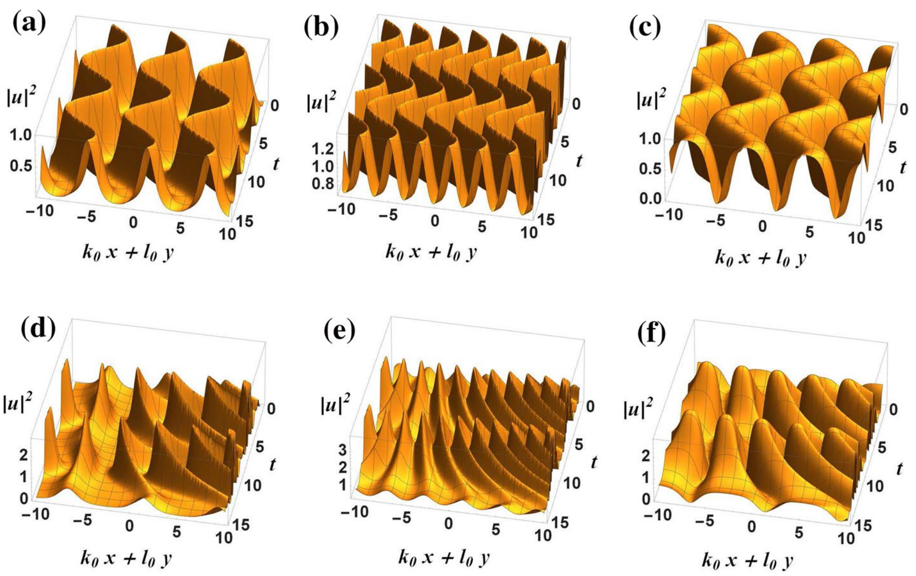


Fig. 3 (Color online) Traveling wave solutions as functions of time for Case 2. The parameters are the same as in Fig. 1b except s = 1, $a_0 = 0.2$ in (**d**–**f**) and **a**, **d** $\eta = 0$, **b**, **e** $\eta = 1$ and **c**, **f** $\eta = 0$ and $F =$ sn

In all of these solutions, the novelty comes from the presence of chirp. The previous papers dealing with solutions using expansion methods or related methods, such as Refs. Ebadi and Biswas (2011), Jafari (2012), Yildirm (2012) and Yan (2003) all utilize a linear dependence of the phase on the transverse variables. In addition, we have demonstrated that any function satisfying Eq. (7) can be used to construct solutions to the DS system of equations.

In all these solutions we've set $g_0 = 0$. However, you can add an arbitrary function of time to g_0 and as a consequence to n. The only restriction is that there is no dependence on the transverse variable. Thus, a large range of possible forms for n is possible.

5 Conclusion

To sum up, we analyzed the Davey–Stewartson system and obtained large new classes of solitary and traveling wave solutions using the JEF expansion method. We obtained large classes of new solutions, both solitary and traveling wave solutions and both with and without chirp. Since the DS system appears in many areas of physics, there is a good possibility of practical application for these solutions.

Acknowledgements Work at the Institute of Physics is supported by project OI 171006 of the Serbian Ministry of Education and Science. Work in Qatar was done under the Qatar National Research Fund (QNRF) Project: NPRP 8-028-1-001.

References

Anker, D., Freeman, N.: On the soliton solutions of the Davey–Stewartson equation for long waves. Proc. R. Soc. Lond. **360**, 1703–1740 (1978)

Belić, M., et al.: Analytical light bullet solutions to the generalized (3 + 1)-dimensional nonlinear Schrdinger equation. Phys. Rev. Lett. **101**, 0123904 1–0123904 4 (2008)

Davey, A., Stewartson, K.: On three-dimensional packets of surface waves. Proc. R. Soc. Lond. **338**, 101–110 (1974)

Deng, S., Qin, Z.: Darboux and Bäcklund transformations for the nonisospectral KP equation. Phys. Lett. A **357**, 467–474 (2006)

Ebadi, G., Biswas, A.: The G′/G method and 1-soliton solution of the Davey–Stewartson equation. Math. Comput. Model. **53**, 694–698 (2011)

Eisenberg, S., et al.: Diffraction management. Phys. Rev. Lett. **85**, 1863–1866 (2000)

Eiermann, B., et al.: Dispersion management for atomic matter waves. Phys. Rev. Lett. **91**(060402), 1–4 (2003)

Hieraninta, J.: Introduction to the Hirota Bilinear Method, vol. 95. Springer, Berlin (1997)

Hieraninta, J., Hirota, R.: Multidromion solutions to the Davey–Stewartson equation. Phys. Lett. A **145**, 237–244 (1990)

Huang, G., et al.: Davey–Stewartson description of two-dimensional nonlinear excitations in Bose–Einstein condensates. Phys. Rev. E **72**(036621), 1–8 (2005)

Jafari, H., et al.: The first integral method and traveling wave solutions to Davey–Stewartson equation. Nonlinear Anal. Model. Control **17**, 182–193 (2012)

Leblond, H.: Electromagnetic waves in ferromagnets: a Davey–Stewartson-type model. J. Phys A **32**, 7907–7932 (1999)

Lou, S.: Dromions, dromion lattice, breathers and instantons of the Davey–Stewartson equation. Phys. Scr. **65**, 7–112 (2002)

Lou, S., Lu, J.: Special solutions from the variable separation approach: the Davey–Stewartson equation. J. Phys. A **29**, 4209–4215 (1996)

Newell, A., Moloney, J.: Nonlinear Optics. Addison-Wesley, Reading (1992)

Ohta, Y., Yang, J.: Rogue waves in the Davey–Stewartson I equation. Phys. Rev. E **86**(036604), 1–8 (2012)

Ohta, Y., Yang, J.: Dynamics of rogue waves in the Davey–Stewartson II equation. J. Phys. A **46**(105202), 1–19 (2013)

Petrović, N., et al.: Exact spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional Schrdinger equation for both normal and anomalous dispersion. Opt. Lett. **34**, 1609–1611 (2009)

Petrović, N., et al.: Spatiotemporal wave and soliton solutions to the generalized (3 + 1)-dimensional Gross–Pitaevskii equation. Phys. Rev. E **81**(016610), 1–5 (2010)

Petrović, N., et al.: Analytical traveling-wave and solitary solutions to the generalized Gross–Pitaevskii equation with sinusoidal time-varying diffraction and potential. Phys. Rev. E **83**(036609), 1–5 (2011)

Petrović, N., et al.: Modulation stability analysis of exact multidimensional solutions to the generalized nonlinear Schrdinger equation and the Gross–Pitaevskii equation using a variational approach. Opt. Exp. **23**, 10616–10630 (2015)

Sung, L.: An inverse scattering transform for the Davey–Stewartson-II equations, I. J. Math. Anal. Appl. **183**, 121–154 (1994a)

Sung, L.: An inverse scattering transform for the Davey–Stewartson-II equations, II. J. Math. Anal. Appl. **183**, 289–325 (1994b)

Sung, L.: An inverse scattering transform for the Davey–Stewartson-II equations, III. J. Math. Anal. Appl. **183**, 477–494 (1994c)

Sung, L.: Long-time decay of the solutions of the Davey–Stewartson II equations. J. Nonlinear Sci. **5**, 433–452 (1995)

Tian, B., Gao, Y.: Solutions of a variable-coefficient Kadomtsev–Petviashvili equation via computer algebra. Appl. Math. Comput. **84**, 125–130 (1997)

Wang, R., Huang, Y.: Exact solutions and excitations for the Davey–Stewartson equations with nonlinear and gain terms. Eur. Phys. J. D **57**, 395–401 (2010)

Wazwaz, A.: The Hirotas bilinear method and the tanhcoth method for multiple-soliton solutions of the Sawada–Kotera–Kadomtsev–Petviashvili equation. Appl. Math. Comput. **200**, 160–166 (2008)

Yan, Z.: Abundant families of Jacobi elliptic function solutions of the (2 + 1)-dimensional integrable Davey–Stewartson-type equation via a new method. Chaos Solitons Fractals **18**, 299–309 (2003)

Yildirm, A., et al.: New exact traveling wave solutions for DS-I and DS-II equations. Nonlinear Analy. Model. Control **17**, 369–378 (2012)

Zakharov, V., Shabat, A.: A scheme for integrating the nonlinear equations of mathematical physics by the method of the inverse scattering problem. I. Funct. Anal. Appl. **8**, 226–235 (1974)

Zhong, W., et al.: Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrdinger equation with distributed coefficients. Phys. Rev. A **78**(023821), 1–6 (2008)

FACTA UNIVERSITATIS
Series: **Electronics and Energetics** Vol. 38, Nº 1, March 2025, pp. 1 - 11
https://doi.org/10.2298/FUEE...

Original scientific paper

SOLITARY AND TRAVELING WAVE SOLUTIONS TO NEMATIC LIQUID CRYSTAL EQUATIONS WITH CUBIC-QUINTIC NONLINEARITY USING THE JACOBI ELLIPTIC FUNCTION EXPANSION METHOD

Nikola Z. Petrović

Institute of Physics, Belgrade, Serbia
Texas A&M University at Qatar, Doha, Qatar

ORCID iDs: Nikola Z. Petrović https://orcid.org/0000-0002-1297-3163

Abstract. *In this paper, the Jacobi elliptic function (JEF) expansion method is applied to the system of equations governing nematic liquid crystals with a cubic-quintic nonlinearity. Solutions that are first order polynomials of the JEFs for the wave function and second order for the angle function are obtained. The solutions impose constraints on only two parameters and include a wide range of functions. Both solitary and traveling wave solutions are possible, as well as solutions both with and without chirp.*

Key words: *Jacobi, nematic, liquid, crystal, photonics*

1. INTRODUCTION

Nonlinearities in optics are studied perhaps more than most other nonlinear system as there is a pressing need to support application in optical communications [1]. In particular, nonlinear behavior may be well controlled and defined by different kinds of optical materials such as nematic liquid crystals (NLCs) that have been recently produced and studied [1, 2]. Nematic liquid crystals are extremely versatile materials with a large range of practical uses in modern photonics [1]. They are an important system in nonlinear optics as they allow the study of many nonlinear phenomena at low power due to a very large nonlinear response via the light-induced reorientation of the NLC molecules [2], in particular the study of spatial solitons, which when propagating through NLCs are also known as nematicons [3]. The study and modeling of the behavior of nematicons, in particular finding the exact solutions describing their form, has numerous potential practical applications, such as optical information processing [4], molding of optical waveguides [5], beaming and control of the so-called random lasers [6] and many others [7,8].

NLCs are generally described by a pair of interconnected nonlinear differential equations describing the time evolution of the wave function of light and the angular function which describes the tilt of the molecules of the crystal [9]. There are several forms of nonlinearity which can occur in the second equation determining the angular function. The most common form of nonlinearity studied is the third-order nonlinearity, also known as the Kerr nonlinearity [9]. Several papers have produced solutions for the NLC system of equations (NLCSOE) with Kerr nonlinearity and basic solitary wave solutions have been obtained [10-12]. This paper will focus on the NLCSOE with the so-called cubic-quintic nonlinearity.

Cubic quintic nonlinearity is a form of nonlinearity where the third and fifth order nonlinearities compete against each other [13]. It has emerged as an important topic of study in nonlinear optics due to the possibility of stabilizing solitary wave solutions with multiple transverse dimensions due to the competing signs of nonlinearities [14]. Several papers have used various techniques, such as the trial equation method [15], the sinh-Gordon expansion method [16] and others [9, 17-19] to find solutions for the NLCSOE with a cubic-quintic nonlinearity, often referred to in the papers as the parabolic law [17].

Recently, there has been a lot of progress in applying the JEF expansion method to find solutions to the Nonlinear Schrodinger equation with various forms of nonlinearity [20-22], as well as the Gross-Pitaevskii equation [23-24]. The method has also successfully been applied to two-component systems such as the Davey-Stewartson equation [25] and the two-component NLSE [26]. The first application of the JEF expansion method on NLCs was made in [27] where solutions were found for the NLC system of equations with a third-order nonlinearity.

In this work, we generalize the Jacobi elliptic function (JEF) expansion method that was developed in [22] and [27] to find exact solutions to the NLC system of equations (NLCSOE) for the cubic-quintic (CQ) nonlinearity. As in [27], we apply the principle of harmonic balance to both the wave function and the angular tilt and apply matching conditions to obtain the polynomial degrees of these two functions in terms of the JEF. These degrees will depend on the degree of the nonlinearity inside the liquid crystal and it turns out will differ from the degrees obtained in [27].

2. METHOD

The NLCSOE for the CQ nonlinearity has the general form as follows [9]:

$$iu_t + \frac{\beta}{2}\, u_{xx} + \chi p\, u = 0, \tag{1}$$

$$cp_{xx} + lp + \alpha_1|u|^2 + \alpha_2|u|^4 = 0, \tag{2}$$

where u is the wave function, p is the angle function determined by the orientation of NLCs, β is the diffraction parameter, χ is the coupling parameter, c and l are parameters describing the strength of the non-local response of the NLCs and α_1 and α_2 are parameters determining the strength of the nonlinear response to the propagating light. In the special case where the parameter c is equal to 0, the system of equations reduces to the standard cubic-quintic NLSE.

Following [22], the function u is split into the real and imaginary parts:

$$u = Ae^{iB}, \tag{3}$$

where A is the amplitude and B is the phase of the solution. Plugging in the equations and splitting the real and imaginary parts we obtain:

$$A_t + \frac{\beta}{2}(2A_xB_x + AB_{xx}) = 0, \tag{4}$$

$$-AB_t + \frac{\beta}{2}(2A_{xx} + AB_x^2) + \chi AP = 0, \tag{5}$$

$$cp_{xx} + lp + \alpha_1 A^2 + \alpha_2 A^4 = 0. \tag{6}$$

We now assume the following forms for A and B:

$$A = f_1(t)F(\theta) + f_{-1}(t)F^{-1}(\theta), \tag{7}$$

$$\theta = k(t)x + \omega(t), \tag{8}$$

$$B = a(t)x^2 + b(t)x + e(t), \tag{9}$$

where F is a Jacobi elliptic function satisfying the following differential equations:

$$\frac{dF}{d\theta} = \sqrt{c_0 + c_2F^2 + c_4F^4} \text{ and } \frac{d^2F}{d\theta^2} = c_2F + 2c_4F^3, \tag{10}$$

where c_0, c_2 and c_4 are coefficients that depend on the choice of the Jacobi elliptic function and the so-called JEF parameter M. For F=dn we have $c_0=M-1$, $c_2=2-M$ and $c_4=-1$, while for F=sn we have $c_0=1$, $c_2=-(1+M)$, $c_4=M$. The remaining parameters f_1, f_{-1}, k, ω, a, b and e are functions of time to be determined. We note that the phase contains the quadratic term a with respect to the transverse variable that is known as the chirp [20].

We now apply the matching principle to find the needed degree of F in p. Since in Eq. (7), the highest degree of F is 3 in the term A_{xx}, the matching conditions indicate that the degree of AP should also be 3 and, therefore, the angle function p should be a second order function of F:

$$p = g_2(t)F^2 + g_0(t) + g_{-2}(t)F^{-2}. \tag{11}$$

The terms of odd degree are omitted because they add too many new equations without any benefit. It is worth noting that for the ordinary Kerr nonlinearity the matching conditions imposed second degree functions in F for both A and p [27].

We now plug Eqs. (7-9) and Eq. (11) into Eqs. (4-6) to obtain a polynomial function of F. Taking care to equate each coefficient of the polynomial to 0, we obtain a series of algebraic and ordinary differential equations:

$$f_{it} + a\beta f_i = 0, \;\; i = 1, -1 \tag{12}$$

$$a_t + 2a^2\beta = 0, \tag{13}$$

$$b_t + 2ab\beta = 0, \tag{14}$$

$$k_t + 2ak\beta = 0, \tag{15}$$

$$\omega_t + bk\beta = 0. \tag{16}$$

For the parameter e, we obtain a pair of equations that will have to be equivalent, i.e. matched, for the solution to be valid:

$$-e_t f_i - \frac{1}{2}\beta b^2 f_i + \chi f_i g_0 + \chi f_{-i} g_{2i} + \frac{1}{2}\beta c_2 f_i k^2 = 0, \; i = 1, -1. \tag{17}$$

Finally, we obtain several additional constraits between parameters which can be thought of as integrability conditions:

$$\chi f_i g_{2i} + \beta c_{2+i} f_i k^2 = 0, \; i = 1, -1, \tag{18}$$

$$\alpha_2 f_i^4 + 6cc_{2+i} g_{2i} k^2 = 0, \; i = 1, -1, \tag{19}$$

$$\alpha_1 f_i^2 + 4\alpha_2 f_i^3 f_{-i} + 4cc_2 g_{2i} k^2 + g_{2i} l = 0, \; i = 1, -1, \tag{20}$$

$$2\alpha_1 f_1 f_{-1} + 6\alpha_2 f_1^2 f_{-1}^2 + 2cc_0 g_2 k^2 + 2cc_4 g_{-2} k^2 + g_0 l = 0. \tag{21}$$

We now proceed to solve Eqs (12-21). Solutions to Eqs. (12-16) are obtained using standard techniques and are as follows:

$$f_i = f_{i0}\eta^{\frac{1}{2}}, \tag{22}$$

$$a = a_0\eta, \tag{23}$$

$$b = b_0\eta, \tag{24}$$

$$k = k_0\eta, \tag{25}$$

$$\omega = \omega_0 - b_0 k_0 \eta \int_0^t \beta dt, \tag{26}$$

where $\eta = \frac{1}{1+2a_0\int_0^t \beta\, dt}$ is the so-called chirp function [17]. In the absence of chirp, i.e. $a_0 = 0$, we have $\eta = 1$.

Without loss of generality, we can now assume $f_1 \neq 0$. For $f_{-1} = 0$, we obtain $g_{-2} = 0$ from Eq. (17) for $i = -1$. From solving Eqs. (18)-(21), we obtain:

$$g_2 = \frac{-\beta c_4 k^2}{\chi}, \tag{27}$$

$$\alpha_2 = \frac{6\beta c c_4 k^4}{\chi f_1^4}, \tag{28}$$

$$\alpha_1 = \frac{\beta c_4 k^2}{\chi f_1^2}(4cc_2k^2 + l), \tag{29}$$

$$g_0 = \frac{2\beta c c_0 c_4 k^4}{\chi l}. \tag{30}$$

For $f_{-1} \neq 0$, from matching the two equations for e in Eq. (17), we obtain:

$$\frac{f_1 g_{-2}}{f_{-1}} = \frac{f_{-1} g_2}{f_1} \tag{31}$$

and therefore since:

$$g_2 = \frac{-\beta c_4 k^2}{\chi}, \tag{32}$$

$$g_{-2} = \frac{-\beta c_0 k^2}{\chi}, \tag{33}$$

we obtain:

$$\frac{f_{-1}}{f_1} = \epsilon\sqrt{\frac{c_0}{c_4}}, \quad \epsilon = \pm 1. \tag{34}$$

The formula for α_2 is the same as in Eq. (28). The remaining formulas are:

$$\alpha_1 = \frac{\beta c_4 k^2}{\chi f_1^2}\left(4ck^2\left(c_2 - 6\epsilon\sqrt{c_0 c_4}\right) + l\right), \tag{35}$$

$$g_0 = \frac{2\beta c_4 k^2}{\chi l}\left(ck^2\left(c_0(1+7\epsilon^2) - 4\epsilon c_2\sqrt{\frac{c_0}{c_4}}\right) - \epsilon l\sqrt{\frac{c_0}{c_4}}\right). \tag{36}$$

As can be seen, the solutions impose constraints on only two parameters, α_1 and α_2, while the remaining parameters β, χ, c and l, are completely arbitrary. This allows for a wide range of flexibility in constructing our solutions. Finally, the formulas for e in both cases will be complicated and dependent on the form of β, χ, c and l chosen.

3. RESULTS

We now present the solutions we obtained with this method. We will first select F=dn for our Jacobi elliptic function. This function is convenient because the reciprocal function F=nd doesn't contain singularities, thus allowing us to obtain novel nonsingular solutions for non-zero ϵ.

In Fig. 1a we see a standard bright solitary wave solution. The position can be altered by changing ω_0 and the extent of oscillations can be controlled by changing b_0. We see in Fig. 1c that the NLC acts as a wave guide for the signal. In Figs 1b and d we see the effects of the chirp function. The chirp function will deform the solution in the transverse direction and introduce oscillations in the amplitude. Solitary waves with such oscillations in amplitude are often called breathers. Since $c_0=0$ for $M=1$, from Eq. (34)

we have $f_{-1}=0$ and therefore we do not have any solutions for $M=1$ which combine F=dn and F=nd.

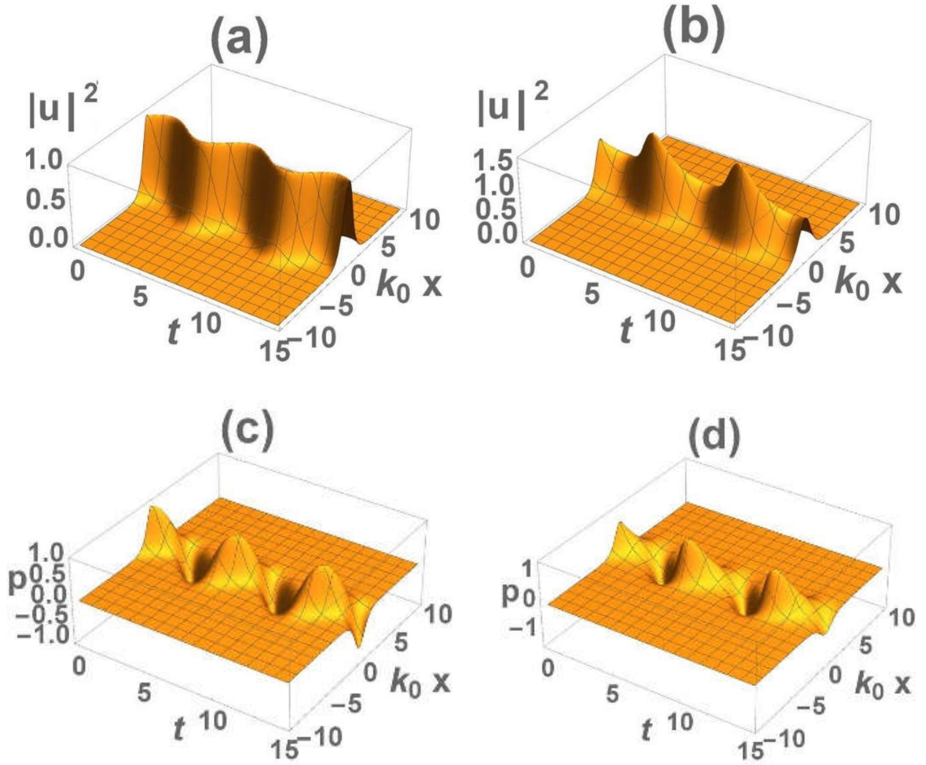


Fig. 1 Bright solitary wave solutions to the NLCSOE as a function of k_0x and t for F=dn, $\boldsymbol{\beta(t) = \beta_0 \cos(\Omega t)}$ and M=1. Graphs (a), (b) depict the square of the angle function $|\boldsymbol{u}|^2$ and graphs (c), (d) depict the angle function p. The values of the parameters are: $\boldsymbol{\beta_0 = \Omega = k_0 = b_0 = f_{10} = l = c = \chi = 1}$, $\boldsymbol{\omega_0 = e_0 = \epsilon = 0}$ and (a),(c): $\boldsymbol{a_0 = 0}$, (b),(d): $\boldsymbol{a_0 = 0.2}$.

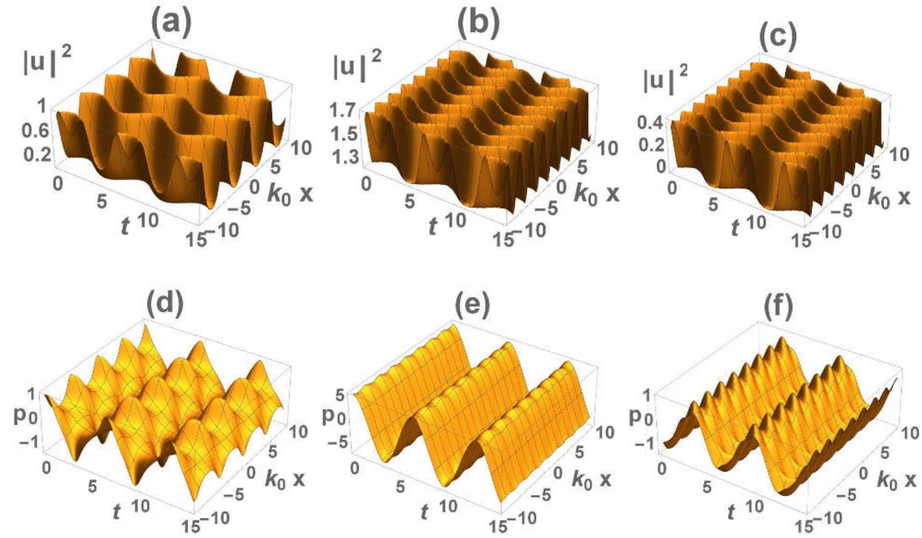


Fig. 2 Traveling wave solutions to the NLCSOE as a function of k_0x and t for M=0.9, $\boldsymbol{a_0 = 0}$. Graphs (a), (b), (c) depict the square of the angle function $|\boldsymbol{u}|^{\mathbf{2}}$ and graphs (d), (e), (f) depict the angle function p. We have for (a),(d): $\boldsymbol{\epsilon = 0}$, (b),(e): $\epsilon=1$, (c),(f): $\epsilon=-1$. All the other parameters are the same as in Fig. 1.

In Fig. 2, we see the periodic, so-called traveling wave, solutions to the NLCSOE. For $M<1$, the JEF no longer produces a solitary wave but a periodic wave structure. We see that both the wave and the angle functions (Figs 2a and 2d) become periodic in the transverse direction. In Figs 2b, c, e and f we see the effects of a non-zero value of ϵ. We see that the overall effect of combining F=dn and F=nd is to double the periodicity of the solutions. The variation of the angle functions also becomes more prominent in the longitudinal direction. Solutions in Fig 2b and Fig 2c are qualitatively alike except for the shift in the overall background amplitude due to the sign of ϵ. The forms of the angle function p are, however, far more complicated and the two solutions in Fig 2e and Fig 2f are quite different from each other.

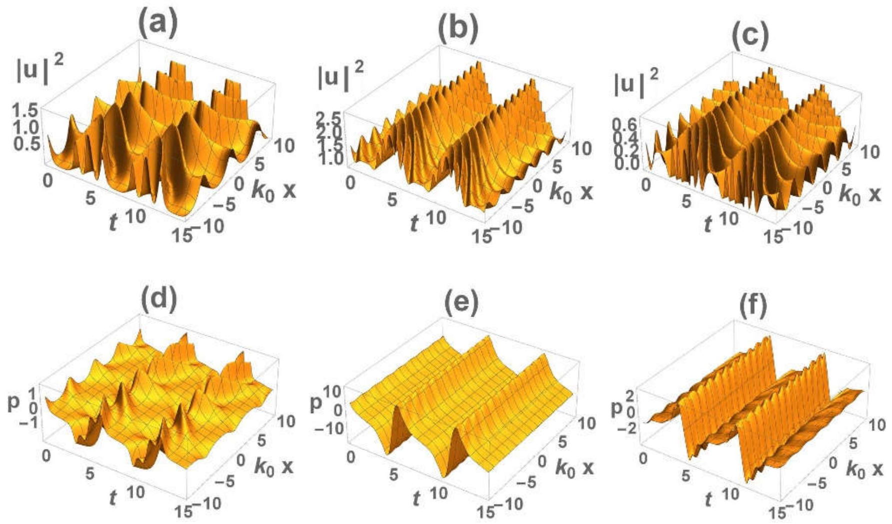


Fig. 3 Traveling wave solutions to the NLCSOE with chirp as a function of k_0x and t for M=0.9, $\boldsymbol{a_0 = 0.2}$. All the other parameters are the same as in Fig. 2.

In Fig. 3 we see the effects of chirp on the traveling wave solutions. The wave fronts in Figs 3a, b and c are stretched out in the transverse direction and no longer periodic. The more one deviates from an equilibrium point which is near the axis, the more extreme the stretching of the wave front. We also note that the solutions in Fig 3b and Fig 3c are no longer qualitatively alike due to the disruption of symmetry caused by the chirp function. In Fig 3d we see the effect of chirp on the angle function. One can clearly see the orientation of the wave crests change with the change in the transverse variable. This is less noticeable in Figs 3e and f where there is a strong background component coming from g_0.

Finally, in Fig 4, we see the dark soliton solutions to the NLCSOE where we have used the JEF *F*=sn. The standard dark solitary wave solution is shown in Fig 4a. We note the periodic structure of the angle function in Fig 4b in the presence of the background with almost a small deviation from it that produces the dark solitary wave. In Fig 4b we see that the background of the solution is affected by the chirp. There is a similar effect on the angle function in Fig 4d of pushing the wave to one side as in Figs 3d, e and f. Lastly, we see an example of the traveling wave solution for the dark solitary wave in Figs 4c and f. We see, unlike *F*=dn in Fig 2a, that *F*=sn reaches 0 and therefore including non-zero ϵ would produce singularities. We see the structure in Fig 2d that produced a dark solitary wave now repeated periodically in Fig 2f.

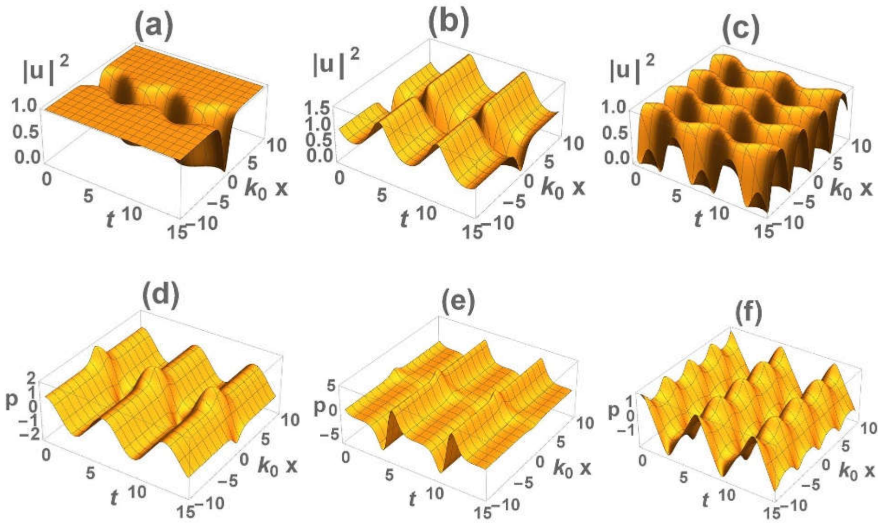


Fig. 4 Dark solitary wave solutions to the NLCSOE as a function of k_0x and t for F=sn. All the other parameters for (a), (b), (d), (e) are respectively the same as in Fig. 1(a), (b), (c) and (d). All the other parameters for (c) and (f) are respectively the same as in Fig 2(a) and (d).

4. DISCUSSION

We now compare the presently obtained results with the previous results obtained in other papers. In [9], the authors apply a simple ansatz to obtain the basic A=sech solution for the amplitude corresponding to the bright solitary wave for $M=1$. In [16], the sinh-Gordon expansion method is applied and various forms of hyperbolic trigonometric functions are obtained for the wave function, although it has to be mentioned that the form for the angle function is not necessarily related to the wave function. In [17], the $exp(-\varphi)$ method is applied and a couple of solutions are obtained, usually in the form of a fraction with a complicated denominator. In [18], the so-called simple equation method is applied. Solutions for the amplitude of the form tanh and coth are obtained as well as various ratios of exponential functions. In [19], two solutions are obtained based on the F=tan and the F=tanh function.

In [28], the Lie point symmetry method is applied to the dual-power law nonlinearity which reduces to the cubic-quintic nonlinearity for $n=1$ and several solutions related to the csch, sec and cos functions are obtained. In [29], the W-shaped solutions, which occur in the case of Kerr nonlinearity [27], are studied using the general exponential rational function method and some solutions for the case of the parabolic nonlinearity are obtained. The exponential rational functions that are used to construct both the wave and angle functions are ratios of either exponential or trigonometric functions. In [30], the Kudryashov's approach and the tanh–coth technique are used to construct solutions for various nonlinearities, including parabolic nonlinearity. Finally, in [15] large classes of solutions are obtained although typically they also involve complicated expressions in the denominator, usually some form of a function plus constant terms. Several solutions involving hyperbolic trigonometric functions are obtained and there is even one mention of the Jacobi sn function squared, albeit in one such denominator.

There are several advantages to the methods in this paper. First, the method is conceptually simple and doesn't require many complicated parameters like in the other approaches. Second, the method works for arbitrary functions of β, χ, c, l, α_1 and α_2 with respect to time, whereas the previous papers treat these functions as constant parameters. This allows greater flexibility in the study of various NLC systems, especially those that employ dispersion management. We note that our method is completely applicable to the case where β, χ, c, l, α_1 and α_2 are arbitrary constants in the case of no chirp. The only difference in this case is that constraints are then imposed on k and f_1 via Eqs (28)-(29). Third, the JEFs are extremely flexible functions, containing both solitary and periodic waves. By varying the choice of the JEF and the parameter M, many different qualitative forms of solutions can be obtained. Finally, no previous paper covers solutions with chirp, which is an important phenomenon in understanding pulse propagation.

Here we will briefly discuss the limitations of the method. First, the method is limited by the forms of functions that satisfy Eq. (10). The functions satisfying (10) either have a solitary wave or are periodic. Modeling multiple, but finite, number of waves is difficult with this method. Second, as mentioned before, the method is not applicable for a completely arbitrary set of parameters, but does have two constraints. Finally, the form of the free parameter e in the phase is complicated and needs to be calculated for each set of functions individually because the form of the differential equations in (17) will greatly differ based on the forms of β, χ, c and l. Nevertheless, for most practical applications only the amplitude of u is needed.

5. CONCLUSION

We applied the Jacobi elliptic function expansion method to the NLC system of equations with a cubic quintic nonlinearity and obtained abundant classes of solutions to the system. Both solitary and traveling wave solutions were obtained, as well as solutions that contain chirp. In particular, the second derivative of the angle function allowed the wave function to have a better match with the degree of nonlinearity in the system. In addition, the fact that there are only two constraints in the system allow systems of NLCs to be flexibly tuned to allow the propagation of the wave function through them. This could potentially have many applications in the fields of photonics and nonlinear optics.

There are many potential systems to which this method can further be applied, including two-component NLC systems and NLC systems with different forms of nonlinearity, especially the so-called septic (seventh order) nonlinearity.

Acknowledgement: *Work at the Institute of Physics is supported by the project OI 171006 of the Serbian Ministry of Education and Science. Work at the Texas A&M University in Qatar (now the Hamad Bin Khalifa University College of Science and Engineering) was supported by the project NPRP13S-0121-200126 with the Qatar National Research Fund (a member of the Qatar Foundation.)*

REFERENCES

[1] A. d'Alessandro and R. Asquini, "Light propagation in confined nematic liquid crystals and device applications." *Applied Sciences,* vol. 11.18, pp. 8713, 2021.

[2] G. Assanto, A. Fratalocchi and M. Peccianti, "Spatial solitons in nematic liquid crystals: from bulk to discrete," *Opt Express*, vol. 15.8, pp. 5248-5259, 2007.

[3] G. Assanto, M. Peccianti and C. Conti C. "Nematicons: optical spatial solitons in nematic liquid crystals," *Optics and Photonics News*, vol. 14.2, pp. 44-48, 2003.

[4] G. Assanto, "Nematicons: reorientational solitons from optics to photonics," *Liq. Cryst. Rev.* vol. 6(2), pp. 170–194, 2018.

[5] N. Karim, et al., "Molding optical waveguides with nematicons," *Advanced Optical Materials*, vol. 5.14, pp. 1700199, 2017.

[6] S. Perumbilavil, et al., "Beaming random lasers with soliton control," *Nature Communications*, vol. 9.1, pp. 3863, 2018.

[7] S. Zhang, et al., "Deflecting and routing nematicons via orientation programmable liquid crystal array," *Optics Express*, vol. 31.13, pp. 20955-20964, 2023.

[8] U. A. Laudyn, M. Kwasny and M. A. Karpierz, "Nematicons in chiral nematic liquid crystals." *Applied Physics Letters*, vol. 94.9, pp. 091110, 2009.

[9] M. Savescu, et al., "Nematicons in liquid crystals." *Journal of Computational and Theoretical Nanoscience*, vol. 12.11, pp. 4667-4673, 2015.

[10] L. Kavitha, et al. "Optically induced switching of nematic deformations," *Physica Scripta*, vol. 88.6, pp. 065015, 2013.

[11] S. Duran and B. Karabulut B, "Nematicons in liquid crystals with Kerr Law by sub-equation method," *Alexandria Engineering Journal* vol. 61.2, pp. 1695-1700, 2022.

[12] J. M. L. MacNeil, N. F. Smyth and G. Assanto, "Exact and approximate solutions for optical solitary waves in nematic liquid crystals," *Physica D*, vol. 284, pp. 1-15, 2014.

[13] A. Desyatnikov, A. Maimistov and B. Malomed, "Three-dimensional spinning solitons in dispersive media with the cubic-quintic nonlinearity," *Phys Rev E*, vol. 61.3, pp. 3107, 2000.

[14] F. Ndzana II, A. Mohamadou and T. C. Kofané, "Modulational instability in the cubic–quintic nonlinear Schrödinger equation through the variational approach," *Optics Communications*, vol. 275.2, pp. 421-428, 2007.

[15] M. Ekici, et. al., "Nematicons in liquid crystals by extended trial equation method," *Journal of Nonlinear Optical Physics & Materials,* vol. 26, No. 1, pp. 1750005, 2017.

[16] D. Kumar, et. al., "Investigation of dynamics of nematicons in liquid crystals by extended sinh-Gordon equation expansion method," *Optical and Quantum Electronics*, vol. 51, pp. 212, 2019.

[17] N. Raza, et al., "Optical solitons in nematic liquid crystals with Kerr and parabolic law nonlinearities," *Optical and Quantum Electronics*, vol. 51, pp. 107, 2019.

[18] A. H. Arnous, et. al., "Nematicons in liquid crystals by modified simple equation method." *Nonlinear Dyn* vol. 88, pp. 2863–2872, 2017.

[19] O. A. Ilhan, et al., "New exact solutions for nematicons in liquid crystals by the tan (φ/2) -expansion method arising in fluid mechanics," *The European Physical Journal Plus* vol. 135.3, pp. 1-19, 2020.

[20] V. I. Kruglov, A. C. Peacock and J. D. Harvey, "Exact self-similar solutions of the generalized nonlinear Schrödinger equation with distributed coefficients," *Phys Rev Lett,* vol. 90, pp. 113902 1-4, 2003.

[21] W. Zhong, et al., "Exact spatial soliton solutions of the two-dimensional generalized nonlinear Schrödinger equation with distributed coefficients," *Phys Rev A* vol. 78.2, pp. 023821, 2008.

[22] M. Belić, et al., "Analytical Light Bullet Solutions to the Generalized (3+ 1)-Dimensional Nonlinear Schrödinger Equation," *Phys Rev Lett*, vol. 101.12, pp.123904, 2008.

[23] N. Z. Petrović, M. Belić, and W. Zhong, "Spatiotemporal wave and soliton solutions to the generalized (3+ 1)-dimensional Gross-Pitaevskii equation," *Phys Rev E*, vol. 81.1, pp. 016610, 2010.

[24] N. Z. Petrović, et al., "Analytical traveling-wave and solitary solutions to the generalized Gross-Pitaevskii equation with sinusoidal time-varying diffraction and potential," *Phys Rev E*, vol. 83.3, pp. 036609, 2011.
[25] N. Z. Petrović, "Solitary and traveling wave solutions for the Davey–Stewartson equation using the Jacobi elliptic function expansion method," *Optical and Quantum Electronics*, vol. 52.6, pp. 319, 2020.
[26] N. Petrović and H. Zahreddine, "Exact traveling wave solutions to coupled generalized nonlinear Schrödinger equations," *Physica Scripta*, vol. 2012.T149, pp. 014039, 2012.
[27] N. Petrović, M. Belić and W. Krolikowski, "Solitary and traveling wave solutions to nematic liquid crystal equations using Jacobi elliptic functions," *Chaos, Solitons & Fractals: X*, vol. 13, pp. 100121, 2024.
[28] R. Ramaswamy, A. H. Abdel Kader, and A. Elsonbaty, "New exact nematicon solutions of liquid crystal model with different types of nonlinearities." *IEEE Access*, vol. 9, pp. 107909-107916, 2021.
[29] H. F. Ismael, H. Bulut, and H. M. Baskonus, "W-shaped surfaces to the nematic liquid crystals with three nonlinearity laws," *Soft Computing*, vol. 25.6, pp. 4513-4524, 2021.
[30] Raza, Nauman, et al., "Investigation of new solitons in nematic liquid crystals with Kerr and non-Kerr law nonlinearities," *Journal of Nonlinear Optical Physics & Materials*, vol. 32.02. pp. 2350020, 2023.

ТАБЕЛА 9. КАТЕГОРИЗАЦИЈА ДОМАЋИХ НАУЧНИХ ЧАСОПИСИ ЗА ЕЛЕКТРОНИКУ, ТЕЛЕКОМУНИКАЦИЈЕ И ИНФОРМАЦИОНЕ ТЕХНОЛОГИЈЕ				
Бр.	**ISSN**	**Наслов часописа**	**Издавач**	**2024**
1	0353-3670	Facta Universitatis Series: Electronics and Energetics	Универзитет у Нишу	M24
2	0352-9665	Facta Universitatis Series: Mathematics and Informatics	Универзитет у Нишу	M24
3	1820-6530	Journal of the Serbian Society for Computational Mechanics Society for Computational Mechanics	Српско друштво за рачунску механику, Крагујевац, Факултет инжењерскх наука Универзитета у Крагујевцу, BioIRC - Bioengineering Research and Development Center, Крагујевац	M24
4	2217-2661	International Journal of Industrial Engineering and Management	Факултет техничкин наука, Департман за индустријско инжињерство и менаџмент, Универзитет Нови Сад	M24
5	2334-847X	International Journal of Cognitive Research in Science, Engineering and Education - IJCRSEE	Удружење за развој науке, инжењерства и образовања, Врање	M24
6	0354-0243	Yugoslav Journal of Operations Research YUJOR	Факултет организационих наука, Београд; Институт Михајло Пупин, Београд; Саобаћајни факултет, Београд; Математички институт САНУ, Београд; Економски факултет, Београд; Факултет техничких наука, Нови Сад; Академија техничко-уметничких струковних студија, Београд; Рударски институт, Београд	M24
7	0042-8469	Војнотехнички гласник	Војна академија, Универзитет одбране у Београду	M51
8	2560-4961	European Project Management Journal	Удружење за управљање пројектима Србије IPMA, Београд	M52
9	1450-5444	Novi Sad Journal of Mathematics	Природно-математички факултет - Департман за математику и информатику, Нови Сад	M52
10	1451-4869	Serbian Journal of Electrical Engineering	Факултет техничких наука, Чачак	M52

JCR Year
2023

Facta Universitatis-Series Electronics and Energetics

Open Access since 2004

ISSN
0353-3670

EISSN
2217-5997

JCR ABBREVIATION
FACTA UNIV-SER ELECT

ISO ABBREVIATION
Facta Univ.-Ser. Electron. Energ.

Journal information

EDITION
Emerging Sources Citation Index (ESCI)

CATEGORY
ENGINEERING, ELECTRICAL & ELECTRONIC

LANGUAGES	REGION	1ST ELECTRONIC JCR YEAR
English	SERBIA	2020

Publisher information

PUBLISHER	ADDRESS	PUBLICATION FREQUENCY
UNIV NIS	UNIVERZITETSKI TRG 2, PO BOX 123, NIS 18000, SERBIA	4 issues/year

Journal's performance

Journal Impact Factor

The Journal Impact Factor (JIF) is a journal-level metric calculated from data indexed in the Web of Science Core Collection. It should be used with careful attention to the many factors that influence citation rates, such as the volume of publication and citations characteristics of the subject area and type of journal. The Journal Impact Factor can complement expert opinion and informed peer review. In the case of academic evaluation for tenure, it is inappropriate to use a journal-level metric as a proxy measure for individual researchers, institutions, or articles. Learn more

2023 JOURNAL IMPACT FACTOR
0.6
View calculation

JOURNAL IMPACT FACTOR WITHOUT SELF CITATIONS
0.5
View calculation

Journal Impact Factor contributing items

Export

Citable items (80) Citing Sources (34)

TITLE	CITATION COUNT

Subject **[FUEE] Editor Decision**
From Danijel Dankovic via casopisi.junis.ni.ac.rs <casopisi@ni.ac.rs>
To Nikola Z. Petrović <nzpetr@ipb.ac.rs>
Reply-To Danijel Dankovic <danijel.dankovic@elfak.ni.ac.rs>
Date 2025-01-31 19:09

Nikola Z. Petrović:

We have reached a decision regarding your submission to Facta Universitatis, Series: Electronics and Energetics, "SOLITARY AND TRAVELING WAVE SOLUTIONS TO NEMATIC LIQUID CRYSTALS EQUATIONS WITH CUBIC-QUINTIC NONLINEARITY USING THE JACOBI ELLIPTIC FUNCTION EXPANSION METHOD".

Our decision is to: Accept Submission

Best regards,

Prof. Dr. Danijel Dankovic
University of Niš, Faculty of Electronic Engineering, Serbia
danijel.dankovic@elfak.ni.ac.rs

Reviewer A:
I am satisfied with the answers the author gave, as well as the improved version of the paper he sent after the review. So in this form I propose a paper for publication in this journal.

Reviewer B:

All suggestions were accepted and significant corrections were made.

The paper should be accepted.

Electronics and Energetics
http://casopisi.junis.ni.ac.rs/index.php/FUElectEnerg

JOINT CONFERENCE

International Congress and Expo on
Optics, Photonics and Lasers
&
International Congress and Expo on
Materials Science & Nanoscience

June 28-30, 2023 | Paris, France

CONTACT US:

+91 8520092691

contact@signatureconferences.com

Welcome to the Signature Conferences !

Welcome To EUROPL2023

Dear Colleagues,

We are delighted to announce and honored to invite you to the International Congress and Expo on Optics, Photonics and Lasers (EUROPL2023) on June 28-30, 2023 in Paris, France

The EUROPL2023 aim is to become the most prestigious forum for the exchange of new ideas, technologies, and novel findings in a broad spectrum of scales ranging from the optoelectronics, photonic materials, photodetectors, and laser science, as well as in basic research and applications.

The primary goal of the conference is to promote research and developmental activities in Optics, Photonics and Lasers and provide opportunities for the delegates to exchange new ideas and application experience face to face, to establish business or research relations and to find global partners for future collaboration.

The leading researchers, scholars and experts of the fields will be brought together to attend the international conference.

We warmly welcome the prospective authors who are interested in the sessions to submit abstract to EUROPL2023 to join in the conference.

We are looking forward to seeing you in Paris, France!

Sincerely,
Prof. Dieter Bimberg
Conference Chair
EUROPL2023
Executive Director "Bimberg Chinese-German Center for Green Photonics", CIOMP, CAS, China and "Founding Director "Center of NanoPhotonics", TU Berlin, Germany.

Welcome to the Signature Conferences !

Welcome To EUROMSN2023

Dear Colleagues,

We are delighted to announce and honored to invite you to the International Congress and Expo on Materials Science and Nanoscience (EUROMSN2023) on June 28-30, 2023 in Paris, France.

The EUROMSN2023 aim is to become the most prestigious forum for the exchange of new ideas, technologies, and novel findings in a broad spectrum of scales ranging from the electronic materials, biomaterials, energy materials, and nanotechnology, as well as in basic research and applications.

The primary goal of the conference is to promote research and developmental activities in Materials Science and Nanoscience and provide opportunities for the delegates to exchange new ideas and application experience face to face, to establish business or research relations and to find global partners for future collaboration.

The leading researchers, scholars and experts of the fields will be brought together to attend the international conference.

We warmly welcome the prospective authors who are interested in the sessions to submit abstract to EUROMSN2023 to join in the conference.

We are looking forward to seeing you in Paris, France !

Solitary and traveling wave solutions to equations governing nematic liquid crystals using the Jacobi elliptic function expansion method

Nikola Petrović[1,2]
[1]*Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia*
[2]*Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar*

Abstract
Nematic liquid crystals (NLCs) are an important system in nonlinear optics as they allow the study of many nonlinear phenomena at low power due to a very large nonlinear response via the electro-optic effect [1]. In particular, spatial solitons, also known as nematicons [2], can easily be observed in NLCs and have shown to be remarkably stable in the two transverse dimensions [3]. NLCs are generally described by a pair of interconnected nonlinear differential equations governing the behavior of the wave function of light and the angular tilt of the molecules of the crystal [4]. Various methods have been proposed to solve this system of differential equations, including the tan(φ/2)-expansion method [5], the modified simple equation method [6] and others [4,7].

In this work, we generalize the Jacobi elliptic function (JEF) expansion method, developed in [8] and [9], to find exact solutions to the NLC system of equations. We apply the principle of harmonic balance to both the wave function and the angular tilt and apply matching conditions to obtain the degrees of these two functions in terms of the JEF. These degrees depend on the form of the nonlinearity inside the liquid crystal. Unlike many other models, the form of the wave functions given in [8] and [9] also includes a quadratic dependence on the transverse variables in the phase which is known as the chirp. Solitary and travelling wave solutions to the NLC system of equations are obtained, both with and without chirp.

References
[1] G. Assanto et. al., Opt. Express 15, 5248-5259 (2007)
[2] G. Assanto et. al., Optics and Photonics News 14(2), 44-48 (2003)
[3] M. Peccianti, G. Assanto, Physics Reports 516, 147-208 (2012)
[4] M. Savescu et. al., J. Comput. Theor. Nanosci. 12, 4667–4673 (2015)
[5] O.A. Ilhan et. al., Eur. Phys. J. Plus 135, 313 (2020)
[6] A. H. Arnous et al. Nonlinear Dyn. 88, 2863-2872 (2017)
[7] S. Duran, B. Karabulut, Alexandria Engineering Journal 61, 1695–1700 (2022)
[8] M. Belić, N. Z. Petrović, et. al., Phys. Rev. Lett. 101, 123904 (2008)
[9] N. Z. Petrović et. al., Opt. Lett. 34, 1609 (2009)

Subject **We request the honour of your presence at EUROPL2023**

From EUROPL2023 <europl2023@signatureconferences.net>

To <nzpetr@ipb.ac.rs>

Date 2022-09-18 21:37

Priority Highest

Dear Dr. Nikola Z Petrović,

I am writing you on behalf of Conference chair Prof. Dieter Bimberg, we are pleased to announce and cordially invite you to be our **Invited Speaker** at the International Congress and Expo on Optics, Photonics and Lasers (EUROPL2023) on June 28-30, 2023 in Paris, France.

We are inviting leading scientists in the field as plenary and keynote speakers from all over the world to have insightful discussions and we would be delighted to include you in the list of guest speakers. I hope this will be an excellent opportunity to foster innovation, network with the brightest minds in academia and industry that are working in this field.

Meet our Plenary Speakers at EUROPL2023

PROF. STEVEN DENBAARS, Co-Director, Solid State Lighting & Energy Electronics Center, University of California, Santa Barbara, USA

PROF. DIETER BIMBERG, Changchun and Founding Director "Center of NanoPhotonics, TU Berlin, Germany

PROF. ALBERT P.H.J. SCHENNING, Eindhoven University of Technology, The Netherlands

PROF. VOLKER J. SORGER, Director of the Institute on AI & Photonics, George Washington University, USA

PROF. TAKASHIGE OMATSU, Chiba University, Japan

PROF. YASUHIKO ARAKAWA, Institute for Nano Quantum Information Electronics, The University of Tokyo, Japan

PROF. ALFRED FORCHEL, University of Würzburg, Germany

For more speakers and information about conference, please visit our website: https://bit.ly/3D7Dak7

Considering your outstanding academic achievements, we are inviting you to join our conference as a Invited speaker. I do hope you can make time in your busy schedule to become part of the conference and share your ideas on this topic on the panel!

More information will be provided later on your acceptance to join us as a speaker!

Thanks and regards,
Susan J

Conference Secretary
EUROPL2023
Signature Conferences
E: europl2023(at)signatureconferences.com

SERBIAN ACADEMY OF NONLINEAR SCIENCE

2nd CONFERENCE ON NONLINEARITY

Editors

B. Dragovich, Ž. Čupić

Belgrade, 2022
SERBIA

P r o c e e d i n g s

of the

2nd CONFERENCE ON NONLINEARITY

(October 18 – 22, 2021, Belgrade, Serbia)

Editors

B. Dragovich, Ž. Čupić

Publisher

Serbian Academy of Nonlinear Sciences

Belgrade, 2022

SERBIA

Naslov: 2nd CONFERENCE ON NONLINEARITY
(Druga konferencija o nelinearnosti) Autor: Grupa autora
Izdavač: Srpska akademija nelinearnih nauka, Beograd, Srbija
Godina izdanja: 2022
Štampa: Skripta internacional, Beograd
Tiraž: 120
ISBN: 978-86-905633-7-1
Urednici: Dragović Branko, Čupić Željko
Nelinearne nauke-Zbornici

CIP - Каталогизација у публикацији Народна библиотека Србије, Београд

530.182(082)
51-7:53(082)

CONFERENCE on Nonlinearity (2 ; 2021 ; Beograd)
Proceedings of the 2nd Conference on Nonlinearity, (October 18 – 22, 2021, Belgrade, Serbia) / editors B. [Branko] Dragovich, Ž. [Željko] Čupić ; [organizer] Serbian Academy of Nonlinear Sciences [(SANS)], Belgrade. - Belgrade : Serbian Academy of Nonlinear Sciences = Beograd : Srpska akademija nelinearnih nauka,
2022 (Beograd : Skripta internacional). - VIII, 300 str. : ilustr. ; 25 cm

Nasl. u kolofonu: Druga konferencija o nelinearnosti. - Str. V-VI: Preface / Branko Dragovich, Zeljko Cupic. - Napomene uz radove. - Bibliografija uz većinu radova. - Registar.

ISBN 978-86-905633-7-1

a) Нелинеарне теорије -- Зборници b) Математичка физика -- Зборници

COBISS.SR-ID 81001225

2nd Conference on Nonlinearity

Belgrade, October 18 - 22, 2021

Organizer

- **Serbian Academy of Nonlinear Sciences (SANS), Belgrade**

Coorganizers

- **Mathematical Institute, Serbian Academy of Sciences and Arts, Belgrade,**
- **Faculty of Mathematics - University of Belgrade,**
- **Institute of Chemistry, Technology and Metallurgy - University of Belgrade,**

and

- **Faculty of Sciences and Mathematics - University of Niš**

Sponsor

- **Ministry of Education, Science and Technological Development, Republic of Serbia**

INTERNATIONAL ADVISORY COMMITTEE

- Andrew Adamatzky (Bristol, UK)
- Irina Ya. Arefeva (Moscow, Russia)
- Milan S. Dimitrijević (Belgrade, Serbia)
- Ćemal Dolićanin (Novi Pazar, Serbia)
- Nenad Filipović (Kragujevac, Serbia)
- Dezso Horvath (Szeged, Hungary)
- Zoran Jakšić (Belgrade, Serbia)
- Dušan Jovanović (Belgrade, Serbia)
- Nenad Mladenovic (UAE)
- Sergei D. Odintsov (Barcelona, Spain)
- Zoran Ognjanović (Belgrade, Serbia)
- Zoran Rakić (Belgrade, Serbia)
- Branislav Sazdović (Belgrade, Serbia)
- Ryo Yoshida (Tokyo, Japan)
- Anatoly Zagorodny (Kiev, Ukraine)

INTERNATIONAL PROGRAM COMMITTEE

- Milivoj Belić (Doha, Qatar)
- Livija Cvetićanin (Novi Sad, Serbia)
- Željko Čupić (Belgrade, Serbia)
- Vladan Djordjević (Belgrade, Serbia)
- Branko Dragovich (Belgrade, Serbia)
- Miloš Kojić (Houston, USA)
- Ljiljana Kolar Anić (Belgrade, Serbia)
- Miguel A. F. Sanjuan (Madrid, Spain)
- Igor V. Volovich (Moscow, Russia)

LOCAL ORGANIZING COMMITTEE

- Branko Dragovich , Chairman (SANS, Inst. Phys., Belgrade)
- Željko Čupić, Vice Chairman (SANS, IHTM, Belgrade)
- Ivana Atanasovska (SANS, MI SASA, Belgrade)
- Ivan Dimitrijević (Faculty of Mathematics, Belgrade)
- Goran Djordjević (SANS, SEENET, Faculty of Sciences, Niš)
- Božidar Jovanović (SANS, MI SASA, Belgrade)
- Predrag Jovanović (SANS, Astron. Observatory, Belgrade)
- Branko Urošević (SANS, School of Computiong, Belgrade)

PREFACE

This book contains some papers related to the talks presented at the *2nd Conference on Nonlinearity*, held online on October 18–22, 2021. The conference is organized by the Serbian Academy of Nonlinear Sciences (SANS) in cooperation with the Mathematical Institute (Serbian Academy of Sciences and Arts), Faculty of Mathematics (University of Belgrade), Institute of Chemistry, Technology and Metallurgy (University of Belgrade), and Faculty of Sciences and Mathematics (University of Niš).

It is well known that nonlinear phenomena and processes are present everywhere in nature – from fundamental interactions between elementary particles, via various terrestrial processes in fluids and optics, to the dynamics of celestial objects and the evolution the universe as a whole. Nonlinear methods, in particular nonlinear differential equations, are used in research of all sciences – from fundamental to applied. Contemporary comfortable human life largely depends on technological achievements based on nonlinear processes.

Serbian Academy of Nonlinear Sciences is a scientific society whose members are scientists that significantly contributed to developments of nonlinear sciences in Serbia. The main goal of SANS is a strong fruitful support to versatile developments of nonlinear sciences, particularly in Serbia. Organization of scientific meetings – colloquiums and conferences on nonlinearity – are among principal activities of SANS. SANS strives to connect as much as possible with scientists and related scientific activities throughout the world. More information on the Serbian Academy on Nonlinear Sciences is available at its website http://www.sann.kg.ac.rs/en/sans/.

About 70 scientists from 19 countries participated in this conference (Australia, Austria, France, Germany, Hungary, India, Israel, Japan, Poland, Qatar, Romania, Russia, Serbia, Slovenia, Spain, UAE, UK, Ukraine, USA). Lectures were given by 4 keynote speakers (45 min.), 9 invited speakers (35 min.) and 35 other participants (25 min.). Some details can be seen on the conference website http://www.nonlinearity2021.matf.bg.ac.rs/.

On behalf of the Serbian Academy of Nonlinear Sciences, we would like to express our gratitude to the Ministry of Education, Science and Technological Development of the Republic of Serbia for a financial support to publish these Proceedings. We are also thank the Coorganizers and the management of the journal Symmetry for a support of this conference. In particular, we are thankful to all speakers and the authors of contributions to the Proceedings. We hope very much that this collection of papers will be useful not only to participants of this conference but also to all others who are interested in nonlinearity.

The Serbian Academy of Nonlinear Sciences plans to continue with the organization of Conferences on Nonlinearity regularly with a period of two years. It is our great wish that next year there will be no problem with the Corona virus epidemic and that the third conference will be held in person. We will be happy to see all participants of the first two conferences again, as well as many new ones.

Belgrade, Summer 2022

E d i t o r s

Branko Dragovich
(President of SANS)

Zeljko Cupic
(General secretary of SANS)

CONTENTS

M. P. Cartmell
On the terrestrial measurement of frame-dragging 1

I. D. Atanasovska, D. B. Momcilovic
Analysis of contact fatigue coupled with geometry and material nonlinearities by Theory of critical distances 27

M. M. Filipovic, Lj. B. Kevac
The procedure of generating trajectory of motion of carrier conveyor for granular material in dependance of pod systems characteristics 48

F. Geogriades
Design of a mechanical system to admit similar rigid body nonlinear modes and the correlation with perpetual mechanics 63

K.-Y. Huang, Y. Zhao, L. Xue, J.-R. He, K.-W. Wang, S.-L. Xu, M. R. Belić, B. A. Malomed
Supporting quantum squeezing in two component solitons by means of electromagnetically-induced transparency 79

A. Obradović
Mass minimization of an AFG Timoshenko beam with coupled axial and bending vibrations 96

V. Skarka, M. Lekić
Self-organized bridge connecting THEORY to EXPERIMENTS 109

S. Zdravković, D. Ranković
W-potentials in nonlinear biophysics of microtubules 136

S. Paunović,
Modelling the Steel Fibre Reinforced Concrete material nonlinearity with ANSYS 150

Ž. Čupić, A. Ivanović-Šašić
Bray-Liebhavsky reaction: From monotonous to chaotic evolution 170

C. N. Babalic, R. Constantinescu
Auxiliary equations for solving nonlinear evolutionary equations 200

D. Jovanović, O. Alexandrova, M. Maksimović, M. Belić
Fluid theory of coherent magnetic vortices in high-β space plasmas 212

P. Jovanović, V. Borka Jovanović, D. Borka, A. F. Zakharov
Graviton mass and Yukawa-like nonlinear correction to the gravitational potential: constraints from stellar orbits around the Galactic Center 232

A. Maluckov, D. Leykam, S. Flach, A. Mančić, E. Smolina, D. Smirnova
Scanning band topology by the modulation instability 246

Nikola Petrović
Exact traveling and solitary wave solutions to the generalized Gross-Pitaevskii equation with cylindrical potential 259

R. Cimpoiasu, R. Constantinescu, G. Florian, A. Pauna
Reduction methods and travelling wave solutions for nonlinear evolutionary phenomena 268

B. Dragovich
On *p*-adic scalar matter 280

Talks not presented to the Proceedings 297

List of participants 299

Exact traveling and solitary wave solutions to the generalized Gross-Pitaevskii equation with cylindrical potential*

Nikola Petrović†
Institute of Physics, Pregrevica 118, 11080 Belgrade,
University of Belgrade, Serbia

Abstract

In our paper we modify the Jacobi Elliptic function (JEF) expansion method to obtain solutions to the Gross-Pitaevskii equation with a cylindrical potential, in which case the three transverse dimensions are no longer symmetric. The solutions end up combining the features of the solutions for the spherical potential and the solutions of the ordinary Nonlinear Schrödinger equation (NLSE). Solutions which have an oscillating amplitude and modulational stability can be found.

PACS numbers: 05.45.Yv, 42.65.Tg

1. Introduction

Gross-Pitaevskii equation (GPE) is of the extreme importance in the study of the Bose-Einstein condensates (BEC), where it describes the behavior of the condensate wavefunction [1]. It has been introduced independently by Gross [2] and Pitaevskii [3] for an unrelated problem, but has since been found of great use in the study of BEC. Solitary wave solutions [4] have been discovered in GPE. One of the main methods of finding approximate solutions to the GP equation and related equations such as the Ginzburg-Landau equation is using the variational approximation [5, 6, 7]. Such an approach uncovered various complicated forms of solutions such as vortices [5], dipoles [6], tripoles [6] and various oscillating structures [7]. In [8] exact analytic solutions for the (1-1)D solutions to the GPE were found. The gain function was used in the diffraction coefficient to produce stable oscillating solitary wave solutions. The paper introduces many novel ideas, such as the chirp function for which in order to obtain it one must solve a Ricatti

* Work at the Institute of Physics is supported by project OI 171006 of the Serbian Ministry of Education and Science. Work in Qatar was done under the Qatar National Research Fund (QNRF) project: NPRP 8-028-1-001.
† e-mail address: nzpetr@ipb.ac.rs

differential equation. The paper obtains stable oscillating solutions for constant potential, but a complicated form for the nonlinearity coefficient.

Recently, a new class of solutions based on the Jacobi Elliptic Function (JEF) expansion [9, 10, 11] for the Nonlinear Schrödinger Equation (NLSE) and the methods used have been generalized in [12, 13] for the GPE. Unlike in [8], in [9, 10, 11, 12, 13] the dispersion/diffraction coefficient is no longer a constant and the gain function is factored differently into the formula for the nonlinearity coefficient, among other differences in the parameters of the solution. Having a non-constant dispersion/diffraction coefficient allows the use of dispersion [14] and diffraction management [15]. The modulational stability of the solutions in [9, 10, 11, 12, 13] for both the NLSE and the GPE was analyzed in [16] and also [13] for both dark/bright and spatial/temporal solitary waves, under both normal and anomalous dispersion. It was determined that in all three possible combinations the solitary waves achieve either unconditional stability or stability under dispersion management.

All the obtained solutions for the GPE have so far been for a potential with spherical symmetry. However, other forms of the potential are possible including ones that use trigonometric functions [17]. For many practical situations, confining potentials with cylindrical symmetry are used. In other words, there is a confining potential only in two transverse dimensions, reminiscent of an infinitely long cylinder [18, 19]. The elongated cylindrical form of the potential allows experimental physicists to analyze the Bose-Einstein condensate in a regime close to the 1D case [20]. In this paper we extend the results in [12, 13] to the case of the cylindrical potential.

2. Method

We consider GPE in (3+1)D with distributed coefficients [1]:

$$i\partial_t u + \frac{\beta(t)}{2}\Delta u + \chi(t)|u|^2 u + \alpha(t)(x^2 + y^2)u = i\gamma(t)u. \tag{1}$$

Here t is time, $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the 3D Laplacian and $\alpha(t)$ stands for the strength of the quadratic potential as a function of time. The functions β, χ, and γ stand for the diffraction, nonlinearity, and gain coefficients, respectively. All coordinates in Eq. (1) are made dimensionless by the choice of coefficients. It is worth noting that the transverse variable z is no longer symmetric with respect to the other two transverse variables x and y.

As described in Ref. [12], we separate the real and imaginary part of u:

$$u(x, y, z, t) = A(x, y, z, t) \exp(iB(x, y, z, t)). \tag{2}$$

and, after plugging in Eq. (2) into Eq. (1), divide Eq. (1) into the real and imaginary part. We now assume the following form for A and B:

$$\begin{aligned}
A &= f_1(t)F(\theta) + f_2(t)F(\theta)^{-1}, &(3)\\
\theta &= k(t)x + l(t)y + m(t)z + \omega(t), &(4)\\
B &= a_1(t)(x^2+y^2) + a_2(t)z^2 + b_1(t)(x+y) + b_2(t)z + e(t), &(5)
\end{aligned}$$

where f_1, f_2, k, l, m, ω, a_1, a_2, b_1, b_2 and e are functions of t to be determined, and F is an arbitrary Jacobi elliptic function (JEF) satisfying:

$$\left(\frac{dF}{d\theta}\right)^2 = c_0 + c_2F^2 + c_4F^4, \tag{6}$$

where c_0, c_2 and c_4 are constants that depend on the form of the JEF and the parameter of the JEF M. For $F = cn$, the JEF we will be using in this paper, we have $c_0 = 1 - M$, $c_2 = 2M - 1$ and $c_4 = -M$. The JEFs are well suited as an ansatz for nonlinear partial differential equations with terms containing the third degree of the original function. Function f_2 may be set to 0. In contrast to Ref. [12], the functions next to the quadratic and linear terms in the phase have been split into two pairs of functions, a_1, a_2, b_1 and b_2 to account for the asymmetry between z and the other transverse variables.

Applying the F-expansion method and the principle of harmonic balance [9] we obtain the following system of algebraic and first order differential equations for f_i $(i = 1, 2)$, a_1, a_2, b_1, b_2, k, l, m and ω:

$$\begin{aligned}
\frac{df_j}{dt} + (2a_1 + a_2)\beta f_j - \gamma f_j &= 0, &(7)\\
\frac{dk}{dt} + 2ka_1\beta = 0, \quad \frac{dl}{dt} + 2la_1\beta = 0, \quad \frac{dm}{dt} + 2ma_2\beta &= 0, &(8)\\
\frac{da_1}{dt} + 2\beta a_1^2 - \alpha = 0, \quad \frac{da_2}{dt} + 2\beta a_2^2 &= 0, &(9)\\
\frac{db_1}{dt} + 2\beta a_1 b_1 = 0, \quad \frac{db_2}{dt} + 2\beta a_2 b_2 &= 0, &(10)\\
\frac{d\omega}{dt} + \beta((k+l)b_1 + mb_2) &= 0, &(11)\\
\frac{de}{dt} + \frac{\beta}{2}(2b_1^2 + b_2^2 - (k^2+l^2+m^2)c_2) - 3\chi f_1 f_2 &= 0, &(12)\\
f_1\left(\beta(k^2+l^2+m^2)c_4 + \chi f_1^2\right) &= 0, &(13)\\
f_2\left(\beta(k^2+l^2+m^2)c_0 + \chi f_2^2\right) &= 0, &(14)
\end{aligned}$$

3. Results

Now we need to classify solutions based on the forms of the functions α and β. For the spherically symmetric form of α, the case where α and β

are constant was covered in Ref. [12], while the case where α and β are sinusoidal was covered in Ref. [13].

Following the notation established in [13], we obtain the following most general results:

$$f_1 = f_{10} p_1 \sqrt{p_2} \exp\left(\int_0^t \gamma dt\right), \qquad f_2 = \epsilon \sqrt{\frac{c_0}{c_4}} f_1, \tag{15}$$

$$k = p_1 k_0, \quad l = p_1 l_0, \quad m = p_2 m_0, \tag{16}$$

$$\omega = \omega_0 - \left((k_0 + l_0) b_{10} q_1 + m_0 b_{20} q_2\right), \tag{17}$$

$$a_2 = p_2 a_{20}, \tag{18}$$

$$b_1 = p_1 b_{10}, \qquad b_2 = p_2 b_{20}, \tag{19}$$

$$e = e_0 + \frac{1}{2}\left((k_0^2 + l_0^2)(c_2 - 6\epsilon\sqrt{c_0 c_4}) - 2b_{10}^2\right) q_1 + \frac{1}{2}\left(m_0^2 (c_2 - 6\epsilon\sqrt{c_0 c_4}) - b_{20}^2\right) q_2, \tag{20}$$

with a different formula for a_1, p_1, p_2, q_1 and q_2 in each case. Here, $\epsilon = 0, \pm 1$ and p_2 corresponds to the chirp function in [10]. The index '0' represents the value of the given parameter at $t = 0$. The results in this section are not for an arbitrary χ, but one subject to an integrability condition:

$$\chi = -\beta(k^2 + l^2 + m^2) c_4 f_1^{-2}. \tag{21}$$

For the case where α and β are constants we obtain the following results:

$$a_1 = \sqrt{\frac{\alpha}{2\beta}} \frac{Ce^{pt} - 1}{Ce^{pt} + 1}, \tag{22}$$

$$p_1 = \frac{e^{pt/2}(1 + C)}{1 + Ce^{pt}}, \quad p_2 = \frac{1}{1 + 2a_{20}\beta t}, \tag{23}$$

$$q_1 = \frac{(1 + C)(e^{pt} - 1)}{p(1 + Ce^{pt})}, \tag{24}$$

$$q_2 = \frac{\beta t}{1 + 2a_{20}\beta t}, \tag{25}$$

where $C = (\sqrt{\frac{\alpha}{2\beta}} + a_{10})/(\sqrt{\frac{\alpha}{2\beta}} - a_{10})$ and $p = 2\sqrt{2\alpha\beta}$.

For the case where α and β are sinusoidal we obtain the following results, i.e. $\alpha(t) = \alpha_0 \cos(\Omega t)$, $\beta(t) = \beta_0 \cos(\Omega t)$ or $\alpha(t) = \alpha_0 \sin(\Omega t)$, $\beta(t) = \beta_0 \sin(\Omega t)$ (in this case, α_0 and β_0 stand for amplitudes, not initial values) we have the following results:

$$a_1 = \sqrt{\frac{\alpha_0}{2\beta_0}}\tanh(\tau(t)), \tag{26}$$

$$p_1 = \sqrt{\frac{\alpha_0}{\alpha_0 - 2a_{10}^2\beta_0}}\mathrm{sech}(\tau(t)), \quad p_2 = \frac{1}{1 + 2a_{20}\int_0^t \beta(t)dt}, \tag{27}$$

$$q_1 = \frac{\sqrt{\alpha_0\beta_0}}{\sqrt{2}(\alpha_0 - 2a_{10}^2\beta_0)}\tanh(\tau(t)) - \frac{a_0\beta_0}{\alpha_0 - 2a_{10}^2\beta_0}, \tag{28}$$

$$q_2 = \frac{\int_0^t \beta(t)dt}{1 + 2a_{20}\int_0^t \beta(t)dt}, \tag{29}$$

where:

$$\tau(t) = \mathrm{arctanh}\left(a_{10}\sqrt{\frac{2\beta_0}{\alpha_0}}\right) + \sqrt{\frac{2\alpha_0}{\beta_0}}\int_0^t \beta(t)dt. \tag{30}$$

For $\beta(t) = \beta_0\cos(\Omega t)$ we have $\int_0^t \beta(t)dt = \beta_0\dfrac{\sin(\Omega t)}{\Omega}$ and for $\beta(t) = \beta_0\sin(\Omega t)$ we have $\int_0^t \beta(t)dt = \beta_0\left(\dfrac{1-\cos(\Omega t)}{\Omega}\right)$.

For this case we can perform a stability analysis similar to one in Section 5 of [16]. The key difference is that now we analyze stability for solitary waves along the z direction as a separate case, and thus have two distinct cases: $k_0 = m_0 = 0$, $l_0 = 1$ and $k_0 = l_0 = 0$, $m_0 = 1$. We also must specify up front whether $k_0 = m_0 = 0$, $l_0 = 1$ or $k_0 = l_0 = 0$, $m_0 = 1$ due to the fact that the formula for χ now contains multiple terms, each with different form of the chirp function. In Eqs. (50) of [16] we must take $p = \sqrt{p_1^2 p_2}$ instead of $p^{3/2}$. For $k_0 = m_0 = 0$, $l_0 = 1$ we must take $p = p_1$ in Eqs. (51)-(54) of [16], while for $k_0 = l_0 = 0$, $m_0 = 1$ we must take $p = p_2$ in Eqs. (51)-(54) in order to obtain the form of the GPE with constant coefficients given in Eq. (55) of [16]. We have thus shown that the same stability analysis given in [16] can also be performed on the solutions in this paper. The detailed calculations pertaining to this analysis are beyond the scope of the paper, but one obtains similar conclusions to those in [16] for the NLSE and GPE with spherical potential (the latter also covered in [13]). In any case, it can be concluded that the solutions in this paper are either unconditionally stable or stable under the regime of dispersion management. Computer simulations were performed on the solutions to the NLSE in [10] and the solutions preserved their shape after long runs.

4. Solutions

In this section we analyze the forms of the obtained solutions. As in [12], the solutions cannot be made to be of stable amplitude unless an external

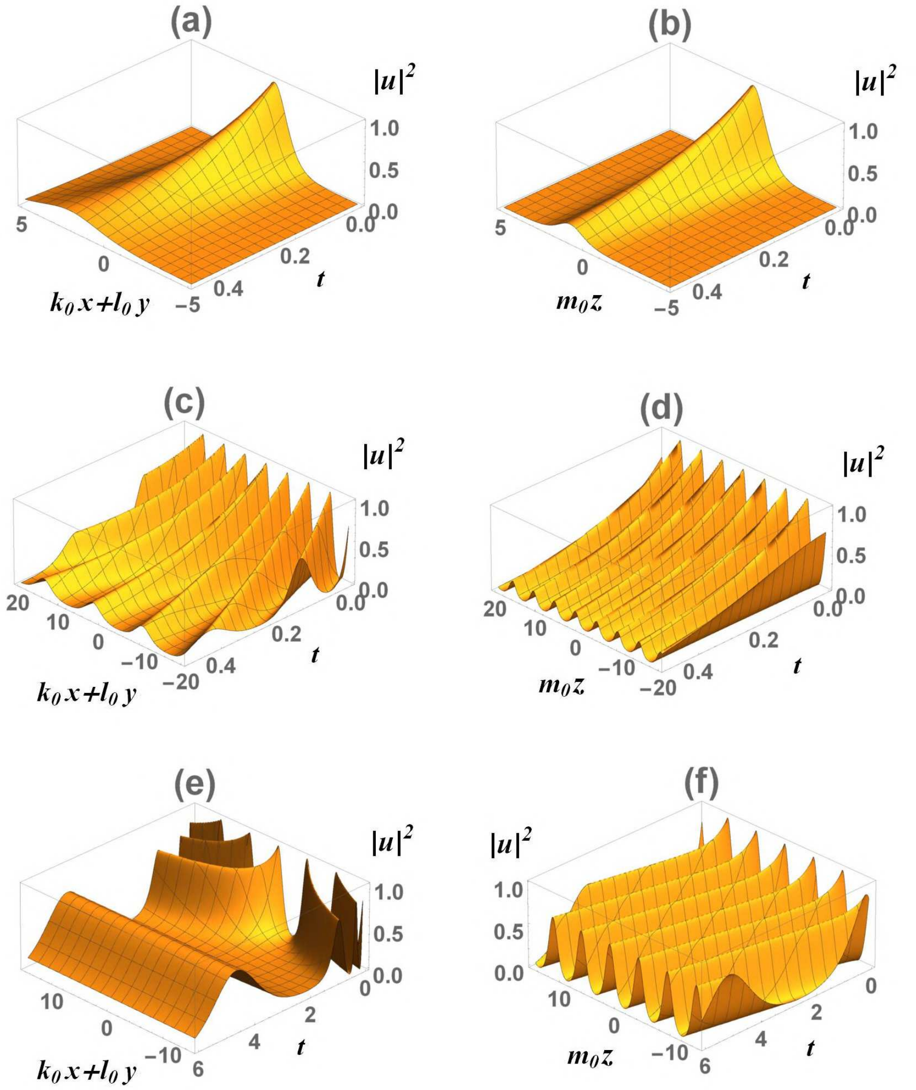


Figure 1: (Color online) Solitary and traveling wave solutions for $F = \mathrm{cn}$ as functions of time for $\alpha = \beta = 1$. Intensity $|u|^2$ is presented as a function of: $k_0x + l_0y$ for (a), (c) and (e) and m_0z for (b), (d) and (f). The main parameters are: (a), (b): $M = 1$, $\gamma = 0$; (c), (d): $M = 0.9$, $\gamma = 0$; (e), (f): $M = 0.9$, $\gamma = p/2$. The other parameters are: $k_0 = l_0 = m_0 = 1$, $a_{10} = 1$, $a_{20} = 0$, $b_{10} = b_{20} = 1$, $\epsilon = 0$ and $\omega_0 = 0$.

gain is added, even in the absence of chirp. Due to the specific form of the function, the dependencies of the function on $k_0x + l_0y$ and m_0z are no longer qualitatively the same, hence these cases must be analyzed and presented separately. The parameters p_1 and p_2 are qualitatively similar for the case of constant α and β, so there is not much of a physical difference apparent when both are present. A more noticeable difference between them occurs when $a_{20} = 0$.

In Fig. (1) we see the results for the case where α and β is constant. We see in Fig. (1)(a) and (b) the difference in the two solitary waves, where we have used $M = 1$ and $F = \text{cn}$. In both plots, the function decays, i.e. spreads out, relatively quickly. The slight rightward bend is due to parameters b_{10} and b_{20} being positive.

In Fig. (1)(c) and (d) we see the difference for traveling wave solutions, i.e. when $M < 1$. We see that in Fig. (1)(c) the wave front spreads out, as is the case in [12], while in Fig. (1)(d) the distance between the traveling waves remains the same.

Finally, in Fig. (1)(e) and (f), we see the effects of adding artificial gain. Unlike in [12], the value of γ for stable solutions is $\gamma = p/2$, due to the change in the number of transverse variables. In order for the stabilization to work, we must have $a_{20} = 0$. In Fig. (1)(e), we see the dramatically different effect in comparison with similar plots in [12]. Instead of a wave front of stable intensity growing wider, we have a radical divergence of initial waves, which are replaced by a giant oscillating wave whose first period can be seen on Fig. (1)(e) and which repeats infinitely. On the other hand, in Fig. (1)(f), we obtain a wave front that doesn't spread out and whose intensity converges to a certain value. The result in Fig. (1)(e) indicates that the artificial addition of gain might not lead to stable solutions in the case of the 2D potential.

In Fig. (2) we see the solutions for $\alpha = \alpha_0 \cos(\Omega t)$ and $\beta = \beta_0 \cos(\Omega t)$, where $\alpha_0 = \beta_0 = \Omega_0 = 1$. We see that the oscillatory solutions resemble those in [13] and, as in Ref. [13], the chirp functions modulate the intensity and the overall shape of the solutions. Here, each form of chirp affects both solutions. This suggest that as in the case of the 3D potential, the dispersion management might be a good approach towards finding stable solutions.

5. Conclusion

To sum up, we have analyzed the problem of the GP equation for the 2D potential, a scenario relevant for practical applications. We have determined that the factors corresponding to the NLSE and the GP equation interact with each other to produce novel and interesting solutions. We established that the artificial gain approach in [12] does not give the same effect as for the 3D potential. We also studied the solutions for the sinusoidal variation of α and β, in other words, the case under dispersion management, and found that as in [13] one obtains stable solutions. Finally, while the

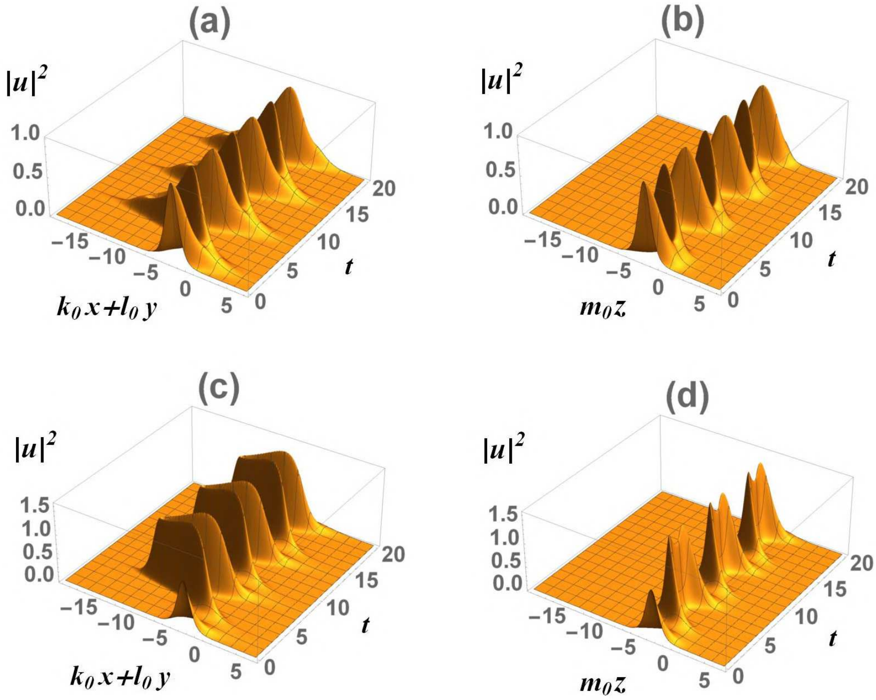


Figure 2: (Color online) Solitary wave solutions for $F = \mathrm{cn}$ as functions of time for $\alpha = \alpha_0 \cos(\Omega t)$ and $\beta = \beta_0 \cos(\Omega t)$, where $\alpha_0 = \beta_0 = \Omega_0 = 1$. Intensity $|u|^2$ is presented as a function of: $k_0 x + l_0 y$ for (a) and (c) and $m_0 z$ for (b) and (d). For (a), (b): $a_{10} = a_{20} = 0$, and for (c), (d): $a_{10} = a_{20} = 0.3$. Other parameters are the same as in Fig. (1)(a).

modulational-stability analysis of these solutions is beyond the scope of this paper, work done in [13] and [16] is strongly indicates that these forms of solutions are also modulationally stable.

References

[1] Dalfovo F *et al.*, *Rev. Mod. Phys.* **71** (1999) 463; L. P. Pitaevskii, and S. Stringari, *Bose-Einstein Condensation,* (Oxford University Press, Oxford, 2003)

[2] Gross E, *Phys. Rev.* **106**(1957) 161

[3] Ginzburg V L and Pitaevskii L P, *Sov. Phys. JETP* **7** (1958) 858 ; Pitaevskii L P, *Zh. Eksp. Teor. Fiz.* **40** (1961) 646 .

[4] Akhmediev N and Ankiewicz A *Solitons* (Chapman and Hall, London, 1997)

[5] Skarka V *et al.*, *Phys. Rev. Lett.* **105** (2010) 213901

[6] Mihalache D *et al.*, *Phys. Rev. A* **82** (2010) 023813

[7] Skarka V *et al.*, *Phys. Rev. A* **90** (2014) 023845

[8] Atre R *et al.*, *Phys. Rev. E* **73** (2006) 056611

[9] Zhong W *et al.*, *Phys. Rev. A* **78** (2008) 023821

[10] Belić M *et al.*, *Phys. Rev. Lett.* **101** (2008) 0123904

[11] Petrović N *et al.*, *Opt. Lett.* **34** (2009) 1609

[12] Petrović N *et al.*, *Phys. Rev. E* **81** (2010) 016610

[13] Petrović N *et al.*, *Phys. Rev. E* **83** (2011) 036609

[14] Eiermann B *et al.*, *Phys. Rev. Lett.* **91** (2003) 060402

[15] Eisenberg S *et al.*, *Phys. Rev. Lett.* **85** (2000) 1863

[16] Petrović N *et al.*, *Opt. Exp.* **23** (2015) 10616

[17] Leblond H *et al.*, *Phys. Rev. E* **76** (2007) 026604

[18] Salasnich L *et al.*, *Phys. Rev. A* **65** (2002) 043614

[19] Zaremba E *Phys. Rev. A* **57** (1998) 518

[20] Burger S *et al.*, *Phys. Rev. Lett.* **83** (1999) 5198

[21] Kivshar Y and Agrawal G, *Optical solitons, from fibers to photonic crystals*, (Academic, New York, 2003)

[22] Hasegawa A and Matsumoto M, *Optical Solitons in Fibers* (Springer, 2003)

[23] Malomed B, *Soliton Management in Periodic Systems* (Springer US, 2006)

[24] Petrović N *et al.*, *Phys. Rev. E* **83** (2011) 026604 .

[25] Hong-Yu W *et al.*, *Commun. Theor. Phys. (Beijing, China)* **54** (2010) 55

[26] Malomed B *et al.*, *J. Opt. B* **7** (2005) R53

[27] Petrović M *et al.*, *Laser & Photonics Reviews* **5** 2 (2009) 214

[28] Chen H *et al.*, *Phys. Rev. Lett.* **37** (1976) 0110693

[29] Li Z *et al.*, *Annals of Physics*, **322** (2007) 2545 (2007)

[30] Li Q *et al.*, *Opt. Comm.* **282** (2009) 1676

[31] Yang Q and Zhang J, *Opt. Comm.* **258** (2006) 35

Book of abstracts

PHOTONICA2021

VIII International School and Conference on Photonics

& HEMMAGINERO workshop

23 - 27 August 2021,

Belgrade, Serbia

Editors

Mihailo Rabasović, Marina Lekić and Aleksandar Krmpot

Institute of Physics Belgrade, Serbia

Belgrade, 2021

ABSTRACTS OF TUTORIAL, KEYNOTE, INVITED LECTURES, PROGRESS REPORTS AND CONTRIBUTED PAPERS

of

VIII International School and Conference on Photonics PHOTONICA2021

23 - 27 August 2021

Belgrade Serbia

Editors
Mihailo Rabasović, Marina Lekić and Aleksandar Krmpot

Publisher
Institute of Physics Belgrade
Pregrevica 118
11080 Belgrade, Serbia

Printed by
Serbian Academy of Sciences and Arts

Number of copies
200

ISBN 978-86-82441-53-3

CIP - Каталогизација у публикацији - Народна библиотека Србије, Београд

535(048)

621.37/.39:535(048)

621.37/.39:535]:61(048)

66.017/.018(048)

INTERNATIONAL School and Conference on Photonic (8; 2021; Beograd)

Book of abstracts / VIII International School and Conference on Photonics PHOTONICA2021 & HEMMAGINERO workshop, 23 - 27 August 2021, Belgrade, Serbia; editors Mihailo Rabasović, Marina Lekić and Aleksandar Krmpot. - Belgrade: Institute of Physics, 2021 (Belgrade: SASA). - V, 192 str.: ilustr.; 30 cm

Tiraž 200. - Bibliografija uz većinu apstrakata. - Registar.

ISBN 978-86-82441-53-3

1. Hemmaginero Workshop (2021; Beograd)

а) Оптика -- Апстракти б) Оптички материјали -- Апстракти в) Оптоелектроника -- Апстракти г) Оптоелектроника -- Биомедицина -- Апстракти д) Телекомуникације -- Апстракти

COBISS.SR-ID 44290057

PHOTONICA 2021 (VIII International School and Conference on Photonics - www.photonica.ac.rs) is organized by Institute of Physics Belgrade, University of Belgrade (www.ipb.ac.rs), Serbian Academy of Sciences and Arts (www.sanu.ac.rs), and Optical Society of Serbia (www.ods.org.rs).

PHOTONICA 2021 is organized under auspices and with support of the Ministry of Education, Science and Technological Development, Serbia (www.mpn.gov.rs).

The support of the sponsors of PHOTONICA 2021 is gratefully acknowledged:

Spatio-temporal solitary and traveling wave solutions to the Kundu–Mukherjee–Naskar equation

Nikola Zoran Petrović[1]
[1]*Institute of Physics, Belgrade, Serbia*
e-mail:nzpetr@ipb.ac.rs

Kundu-Mukherjee-Naskar (KMN) equation [1] is an important variant of the (2+1)-dimensional Nonlinear Schrödinger equation for which the transverse Laplacian is replaced with a mixed partial derivative and the derivative with respect to only one of the transverse directions in present in the nonlinear term, thus breaking the symmetry between the two transverse directions. The primary motivation for the development of the KMN equation was to study soliton pulses in (2+1)–dimensions [2]. The KMN equation admits soliton and breather solutions and, due to an infinite number of conserved quantities which can be established through Lax formalism, it can be established that the equation is integrable [1]. Various methods can be used to find exact solutions to the equation, including the extended trial function method [2], the semi-inverse method [3] and the new extended algebraic expansion method [4].

In this work, we generalize the Jacobi Elliptic function expansion method, developed in [5] and [6], to find exact solutions to the KMN equation. An ansatz which takes into account all asymmetries is considered. One obtains both solitary and travelling wave solutions to the KMN equation, both with and without chirp, which to the best of our knowledge was not considered in any of the previous papers. Chirp is, however, only present in the perpendicular direction to the direction of the derivative in the nonlinear term. These solutions could potentially have many practical applications in the continued study of rogue waves [1].

REFERENCES

[1] A. Kundu, A. Mukherjee and T. Naskar, Proc. R. Soc. A. 47020130576 (2014)
[2] M. Ekici et. al., Chinese J. of Phys. 57, 72 (2019)
[3] J. He, Res. In Phys. 17, 103031 (2020)
[4] A. Jhangeer, Res. in Phys. 16, 102816 (2020)
[5] M. Belić, N. Z. Petrović, et. al., Phys. Rev. Lett. 101, 123904 (2008).
[6] N. Z. Petrović et. al., Opt. Lett. 34, 1609 (2009)

3rd CONFERENCE ON NONLINEARITY
4—8.09.2023, Belgrade, Serbia

Main page

General information

Programme

Committees

Conference venue

Speakers/Talks

Proceedings

Participants

Prizes for the best lectures

Application form

Poster

Previous meetings

Sponsors

Nikola Petrovic

Solitary and traveling two-component wave solutions in nematic liquid crystals using the Jacobi elliptic function expansion method

Abstract

We present the two-component solutions to a system of differential equations governing the propagation of two beams in the regime of strong nonlocality and small angle perturbation of the crystals using the Jacobi elliptic function (JEF) expansion method. The principle of harmonic balance can be applied to obtain the largest degree of the JEF depending on the form of the nonlinearity. Both solitary and traveling wave solutions are obtained, as well as solutions which contain chirp.

Book of abstracts

IX International School and Conference on Photonics

PHOTONICA2023

with joint events:

Understanding interaction light - biological surfaces: possibility for new electronic materials and devices

&

Biological and bioinspired structures for multispectral surveillance

&

Quantum sensing integration within microfluidic Lab-on-a Chips for biomedical applications

August 28 - September 01, 2023, Belgrade, Serbia

Editors

Jelena Potočnik, Maja Popović, Dušan Božanić

Vinča Institute of Nuclear Sciences – National Institute of the Republic of Serbia, University of Belgrade

Belgrade, 2023

ABSTRACTS OF TUTORIAL, KEYNOTE, INVITED LECTURES, PROGRESS REPORTS AND CONTRIBUTED PAPERS

of

IX International School and Conference on Photonics

PHOTONICA2023

August 28 - September 01, 2023, Belgrade, Serbia

Editors
Jelena Potočnik, Maja Popović, Dušan Božanić

Publisher
Vinča Institute of Nuclear Sciences – National Institute of the Republic of Serbia, University of Belgrade
Mike Petrovića Alasa 12-14
11351 Vinča, Belgrade, Serbia

Printed by
Serbian Academy of Sciences and Arts

Number of copies
200

ISBN 978-86-7306-165-8

PHOTONICA2023 – IX International School and Conference on Photonics (www.photonica.ac.rs) is organized by Vinča Institute of Nuclear Sciences - National Institute of the Republic of Serbia, University of Belgrade (www.vin.bg.ac.rs), Serbian Academy of Sciences and Arts (www.sanu.ac.rs), and Optical Society of Serbia (www.ods.org.rs).

Other institutions that helped the organization of this event are: Institute of Physics Belgrade, University of Belgrade (www.ipb.ac.rs), Faculty of Physics, University of Belgrade (www.ff.bg.ac.rs), School of Electrical Engineering, University of Belgrade (www.etf.bg.ac.rs), Institute of Chemistry, Technology and Metallurgy, University of Belgrade (www.ihtm.bg.ac.rs), Faculty of Technical Sciences, University of Novi Sad (www.ftn.uns.ac.rs), Faculty of Sciences and Mathematics, University of Niš (www.pmf.ni.ac.rs), Faculty of Biology, University of Belgrade (www.bio.bg.ac.rs) and Faculty of Sciences and Mathematics, University of Kragujevac (www.pmf.kg.ac.rs).

PHOTONICA2023 is organized under auspices and with support of the Ministry of Science, Technological Development and Innovation, Republic of Serbia (www.nitra.gov.rs). PHOTONICA2023 is supported and recognized by Optica (Formerly OSA) - The Optical Society (www.optica.org).

Solutions to nematic liquid crystals systems with cubic-quintic and septic nonlinearities using the Jacobi elliptic function expansion method

N. Petrović[1,2]
[1]*Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia*
[2]*Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar*
e-mail: nzpetr@ipb.ac.rs

Nematic liquid crystals (NLCs) are an important system for studying in nonlinear optics as these crystals allow many nonlinear phenomena to be explored due to a very large nonlinear response via the electro-optic effect [1]. In particular, spatial solitons, also known as nematicons [2], can easily be observed in NLCs and these nematicons have shown to be remarkably stable in the two transverse dimensions [3]. NLCs are generally described by a pair of interconnected nonlinear differential equations governing the behavior of the wave function of light and the angular tilt of the molecules of the crystal [4]. Of further interest is to study systems with cubic-quintic [5] and septic [6] nonlinearities in order to precent the well-known spatiotemporal collapse of the system when there are two or more transverse dimensions [5]. Various methods have been proposed to solve the NLC system of differential equations, including the tan(φ/2)-expansion method [7], the modified simple equation method [8] and others [4,9].

In this work, we generalize the Jacobi elliptic function (JEF) expansion method, developed in [10] and [11], to find exact solutions to the NLC system of equations with cubic-quintic and septic nonlinearities. We apply the principle of harmonic balance to both the wave function and the angular tilt and apply matching conditions to obtain the degrees of these two functions in terms of the JEF. For the cubic-quintic nonlinearity the Jacobi function has a degree of 1, while for the septic nonlinearity it has a degree of 2/3. Solitary and travelling wave solutions to the NLC system of equations with cubic-quintic and septic nonlinearities are obtained, both with and without chirp, which is the quadratic dependence of the phase of the solutions with respect to the transverse variables.

REFERENCES
[1] G. Assanto *et al.*, Opt. Express 15, 5248 (2007).
[2] G. Assanto *et al.*, Opt. Photonics News 14, 44 (2003).
[3] M. Peccianti, G. Assanto, Phys. Rep. 516, 147 (2012).
[4] M. Savescu *et al.*, J. Comput. Theor. Nanosci. 12, 4667 (2015).
[5] A. Desyatnikov *et al.*, Phys. Rev. E 61, 3107 (2000).
[6] Q. Zhou *et al.*, Chin. Phys. Lett. 39, 044202 (2022).
[7] O.A. Ilhan *et al.*, Eur. Phys. J. Plus 135, 313 (2020).
[8] A.H. Arnous *et al.*, Nonlinear Dyn. 88, 2863 (2017).
[9] S. Duran, B. Karabulut, Alex. Eng. J. 61, 1695 (2022).
[10] M. Belić *et al.*, Phys. Rev. Lett. 101, 123904 (2008).
[11] N.Z. Petrović *et al.*, Opt. Lett. 34, 1609 (2009).

Serbian Academy of Sciences and Arts

Vinca Institute of Nuclear Sciences and Institute of Physics

University of Belgrade

XI-th International Conference
SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives (SCT–2024)

Book of abstracts

Belgrade, July 1−5, 2024

SCIENTIFIC PROGRAM COMMITTEE

- Francesco CALOGERO, University of Rome La Sapienza, Rome, Italy (**Chairman**);
- Alexander V. MIKHAILOV, University of Leeds, UK (**Co-Chairman**);
- Jens J. RASMUSSEN, Technical University of Denmark, Lyngby, Denmark;
- Sergei K. TURITSYN, Aston University, Birmingham, UK.

ORGANIZING COMMITTEE

- Evgeny A. KUZNETSOV, Russian Academy of Sciences, Moscow, Russia, email: `evgenii.a.kuznetsov@gmail.com` (**Chairman**);
- Ljupco HADZIEVSKI, Vinca Institute of Nuclear Sciences, University of Belgrade, Belgrade, Serbia, email: `ljupcoh@vin.bg.ac.rs` (**Co-Chairman**);
- Sotiris KONSTANTINOU-RIZOS, P.G. Demidov Yaroslavl State University, Russia, email: `skonstantin84@gmail.com` (**Secretary of the conference**);
- Branislav JELENKOVIC, Serbian Academy of Sciences and Arts, Belgrade, Serbia, email: `branislav.jelenkovic@ipb.ac.rs`;
- Gregory FALKOVICH, Weizmann Institute of Science, Rehovot, Israel, email: `gregory.falkovich@weizmann.ac.il`.

CONTENTS

Agafontsev D. S., Gelash A. A.
Multi-soliton interactions as a mechanism for the emergence of rogue waves 11

Alekseev O. V.
Universality of stochastic Laplacian growth 12

Annenkov S. Y., Shrira V. I.
Wave kinetics with the account for finite non-Gaussianity 13

Antonov E. N., Orlov A. Yu.
Sato approach and $\bar{\partial}$-problem method. Massless and massive fermions 14

Al-Marzoug S. M., Baizakov B. B., Al Khawaja U. and Bahlouli H. Manifestation of superfluidity in atom-number-imbalanced two-component Bose-Einstein condensates 17

Badulin S. I., Kostianoy A. G., Lebedev S. A.
Self-similarity of wind-driven seas in action 19

Bergé L.
From optical wave collapse to plasma-driven terahertz pulse generation: Theory and applications 20

Bogdanov S. A.
Nonlinear Fourier transform for finite-genus solutions of a generic type: application in fibre-optic communications 22

Chernyavsky A., Prinari B., Frantzeskakis D. J., Horikis T. P., Koutsokostas G. N.
Dark-bright soliton perturbation theory for the Manakov system 24

Conte R., Musette M., Tuen Wai Ng, Wu C.
The 11 and only 11 meromorphic solutions of CGL3 and CGL5 25

CONTENTS

Efimov V. B.
Problems of quantum turbulence on accumulation of ultra-cold neutrons in modern installation 26
Gainer A. V., Kochubei S. A., Mazur M. V.
Is the outcome of a single quantum measurement truly unpredictable? The role of nonlinearity in the measurement process 27
Gainer A. V.
Discreteness and local field effects in classical molecular optics. Nonlinear and anisotropic media 28
Galtier S.
Direct evidence of collisionless kinetic Alfven wave (KAW) turbulence in the solar wind 28
Geogjaev V. V.
Coupling Coefficient for 4-wave interactions: properties and asymptotics 30
Gligorić G., Nedić M., Petrovic J., Malucko A.
Topologically protected modes in nonlinear distorted bipartite hexagonal photonic lattice 31
Gelash A. A., Dremov S. V., Mullyadzhanov R. I., Kachulin D.I.
The growth rate of density inhomogeneities in weak turbulence from information theory perspective 32
Falkovich G.
Renormalization of weak turbulence into strong 33
Flach S.
Thermalization slowing down for weakly nonintegrable many-body dynamics 34
Gelash A. A.
Stochastization of condensate solitons 35
Gerdjikov V. S.
Riemann–Hilbert Problems, Lax Pairs and Integrable Equation 35
Grahovski G.
On the N-wave hierarchy with constant boundary conditions 38
Grinevich P., Santini P. M.
Almost degenerate Rieman surfaces in the theory of rogue waves 38

Hetzel S., Aceves A. and Parker R.
Optical solitons in the presence of quartic dispersion . . 39
Kati Y., Ranft J., Lindner B.
Disordered Kuramoto network in the asynchronous state 40
Kazakov A. A.
The black box problem for electrical networks and the geometry of non-negative Grassmannian. 41
Kochetov A. V.
The numerical simulations of reflection index dynamics of incident radio wave coursed by an electromagnetically driven Langmuir turbulence in a smoothly inhomogeneous plasma layer 42
Kochurin E. A., Kuznetsov E. A.
3D Acoustic Turbulence: Random Shocks or Weak Turbulence? 44
Konopelchenko B.
Geometry and integrability. TED equations and Kahler manifolds 45
Korotkevich A. O.
Kolmogorov-Zakharov spectrum of turbulence of capillary waves for finite systems 45
Konstantinou-Rizos S.
Soliton solutions of an integrable discretisation of the NLS equation 47
Kuznetsov E. A., Mikhailov E. A.
Formation of magnetic filaments in convective zone of the Sun 48
Lushnikov P., Zakharov V. E.
Optimal Canonical Variables in Free Surface Dynamics 49
L'vov V.
Nonlinear theory of kinetic instability and Bose-Einstein condensate of magnons 49
Lvov Yu. V.
From wave turbulence to ocean mixing: observations and theory of the forward energy cascade 50
Maluckov A., Leykam D., Smirnova D. A.
Nonlinear signatures of band topology: the role of modulation instability 51

Medvedev S. B., Chekhovskoy I. S., Kachulin D. I., Vaseva I. A., Patrin G. A.
Fast Exponential Splitting Schemes for the direct Nonlinear Fourier Transform 52
Medvedev S. B., Vaseva I.A., Chekhovskoy I. S., Kachulin D. I., Shtyrina O. V., Fedoruk M. V.
Numerical methods for the direct and inverse Zakharov–Shabat problem 54
Meleshko S. V., Pukhnachev V. V.
Generalized helical flows 55
Mikhailov A. V.
Quantisation ideals, deformations of noncommutative algebras and non-Abelian Hamiltonian systems 57
Newell A.
Order parameter equations for patterns 58
Nikolic S. N., Aleksic N. B., Belic M. R.
Optimal Canonical Variables in Free Surface Dynamics 58
Nikolic V., Przulj Z., Chevizovich D. and Ivic Z.
Solitons in Flux-qubit based superconducting quantum metamaterial 59
Petrovic N.
Solitary and traveling wave solutions to the Nonlinear Schrödinger equation describing quantum droplets . . . 60
Pezzutto P.
Reduced Hamiltonian equations for gravity waves in a big box 61
Pogrebkov A. K.
Integrable hierarchies with negative times 63
Pushkarev A. N., Badulin S. I., Geogjaev V. V.
Self-similarity and nonlinear resilience of wind-driven seas 64
Radonjić M., Pelster A., Balaž A.
Effects of quantum depletion and gradient corrections on the emergence of droplets in dipolar condensates . . 65
Roudenko S.
Soliton Stability and Stable Collapse in the NLS and KdV-type equations 66
Santini P. M.
Periodic anomalous (rogue) waves in the $2+1$ dimensional Davey-Stewartson 2 equation 68

Sedov E. V., Chekhovskoy I. S., Fedoruk M. P.
Nonlinear Fourier Transform for Continuous Signal Processing: Novel Windowing Approach 69
Semisalov B. V., Medvedev S. B., Fedoruk M. P.
Simulation of non-linear wave interactions in a random fibre laser 70
Shrira V. I., Oloo J. O.
Collapses in 3-D free-surface boundary layers 71
Todorov M. D.
Dynamics of Bright Soliton Solutions of System of Coupled Nonlinear Schrödinger Equations 73
Vergeles S.
Absorption of inertial waves by shear flow 74
Vladimirova N.
Evolution of precondensate and vortices in two-dimensional Gross-s-Pitaevskii turbulence 75
Vrecica T. S., Toledo Y. I.
On the nonlinear water wave properties in coastal and deep waters 75
Vucelja M.
The growth rate of density inhomogeneities in weak turbulence from information theory perspective 76
Zolnikova N. N., Shkevov R., Mikhailovskaya L. A.
Reflectionless resonance tunneling of electromagnetic wave in nonuniform plasma 77
Zubarev N. M.
Effect of viscosity on the self-similar growth of conic cusps on the surface of a conducting liquid in an electric field 78

The last twenty years have witnessed the emergence of a novell branch in applied science: Quantum technologies. It relies on direct applications of quantum mechanical laws for the design of quantum communication and quantum information devices. Promising platforms for realizing quantum technological devices are quantum metamaterials: engineered media composed of many periodically arranged artificial atoms-qubits. The systems currently extensively used for designing operable quantum metamaterials include superconducting circuits based on Josephson junctions. In this paper, we consider the emergence of solitons in superconducting quantum metamaterial comprised of aligned flux-qubits embedded within the two-stripe superconducting microwave resonator. The propagation of electromagnetic radiation within such a device is described by a set of coupled equations for "atomic" and field variables. Due to the specific long-range interaction of qubits with electromagnetic modes, we eliminate field modes, and system dynamics is described by a set of nonlinear Bloch equations possessing so-called soliton bullet or bubble solutions. The possible practical realization of such solutions would be useful for the achievement of control over light propagation. In particular, it would be of interest for the construction of a delay line for buffering applications.

Solitary and traveling wave solutions to the Nonlinear Schrödinger equation describing quantum droplets

N. Petrovic[1,2]

[1] *University of Belgrade, Belgrade, Serbia;* nzpetr@ipb.ac.rs
[2] *University at Qatar, Doha, Qatar;*

The stabilization of 2-dimensional (2D) and 3D self-trapped localized modes for the nonlinear Schrödinger equation (NLSE) with a cubic nonlinearity, which, unlike in the case of 1D, are unstable due to critical and super-critical collapse [1], is an important topic in nonlinear optics research [2]. A recent study of the Gross-Pitaevskii (GP) equation with the Lee-Huang-Yang correction [3] suggests adding to the NLSE a fourth-order competing term in the 3D and the reduced 2D case [4] and a second-order competing term in the 1D case [5], producing systems whose solutions are called quantum droplets due to their long and flat plateau [2].

We generalize the Jacobi elliptic function (JEF) expansion method, applied to the NLSE in [6], to find novel solutions to the NLSE in the cubic-quartic and quadratic-cubic cases. In the first case, the maximum degree of the JEF is two-thirds, while in the second case it is one, as in the cubic case [6]. Solitary and traveling wave solutions are obtained, which can optionally also contain the second-degree dependence of the phase on the transverse variables known as the chirp [6].

REFERENCES

1. *V. E. Zakharov, S. V. Manakov, S. P. Novikov, and L. P. Pitaevskii,* "Solitons: The Inverse Scattering Method," Nauka Publishers, Moscow (1980); Consultants Bureau, New York (1984) (English translation).
2. *Z.-H. Luo, W. Pang, B. Liu, Y.-Y. Li, B. A. Malomed,* "A new form of liquid matter: Quantum droplets," Front. Phys. 16, No.3, 32201(1–21) (2021).
3. *T. D. Lee, K. Huang, and C. N. Yang,* "Eigenvalues and eigenfunctions of a Bose system of hard spheres and its low-temperature properties," Phys. Rev. 106, 1135–1145 (1957).
4. *E. Shamriz, Z. Chen, B A. Malomed,* "Suppression of the quasi-two-dimensional quantum collapse in the attraction field by the Lee-Huang-Yang effect," Phys. Rev. A, 101, 063628 (1–11) (2020).
5. *D. S. Petrov and G. E. Astrakharchik,* "Ultradilute low-dimensional liquids," Phys. Rev. Lett. 117, 100401 (1–5) (2016).

Reduced Hamiltonian equations for gravity waves in a big box

P. Pezzutto[1]

[1] *Consiglio Nazionale delle Ricerche, IRBIM, Ancona, Italy;*
paolo.pezzutto@cnr.it

The conservative water wave problem has Hamiltonian structure if at the very far away "vertical" boundaries the flux of the velocity potential ϕ cancels out [1]. This integral condition, in practice, imposes a constraint on the boundary conditions. The simplest choice is to impose the integrand to be zero, that is let the volume of fluid to be

Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА
Комисија за стицање научних звања

Број: 660-01-00001/1301
10.06.2020. године
Б е о г р а д

На основу члана 22. став 2. члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка, 18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) и захтева који је поднео

Институт за физику у Београду

Комисија за стицање научних звања на седници одржаној 10.06.2020. године, донела је

ОДЛУКУ
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Никола Петровић

стиче научно звање

Виши научни сарадник

у области природно-математичких наука - физика

О Б Р А З Л О Ж Е Њ Е

Институт за физику у Београду

утврдио је предлог број 563/1 од 17.04.2019. године на седници Научног већа Института и поднео захтев Комисији за стицање научних звања за доношење одлуке о испуњености услова за стицање научног звања ***Виши научни сарадник***.

Комисија за стицање научних звања је по претходно прибављеном позитивном мишљењу Матичног научног одбора за физику на седници одржаној 10.06.2020. године разматрала захтев и утврдила да именовани испуњава услове из члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка,18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) за стицање научног звања ***Виши научни сарадник***, па је одлучила као у изреци ове одлуке.

Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ	**МИНИСТАР**
Др Ђурђица Јововић,	**Младен Шарчевић**
научни саветник	